








Energy Markets Quantitative Analysis

Lecture 3: Volatility of Energy Prices



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Learning objectives

- **Measure volatility correctly**
  - Percent or EUR/MWh changes of the day-ahead price; the weekly calendar removed first
  - What to do when prices approach zero or turn negative
- **Model it**
  - Models with generalised autoregressive conditional heteroskedasticity (GARCH), with Student-t shocks
  - Persistence, half-life of a shock, asymmetry between price rises and falls
- **Recognise regimes and intraday risk**
  - Calm and turbulent regimes, the 2021–2022 energy crisis, the year 2026
  - Intraday variation of hourly prices and the role of solar generation
- **Forecast and evaluate**
  - One day and several weeks ahead; comparison of forecasts with a robust loss function and a significance test
  - The answer promised in Lecture 1: how long a risk desk should keep wider limits after a shock

Abbreviations (1/2)

AIC	Akaike information criterion
ARCH	autoregressive conditional heteroskedasticity
ARCH-LM	Lagrange multiplier test for ARCH effects
BIC	Bayesian (Schwarz) information criterion
BVB	Bucharest Stock Exchange
CEE	Central and Eastern Europe
CET	Central European Time
DCC	dynamic conditional correlation
DE-LU	Germany–Luxembourg bidding zone
DM	Diebold–Mariano test
EGARCH	exponential GARCH (Nelson)
ENTSO-E	European Network of Transmission System Operators for Electricity
EODHD	market data vendor eodhd.com
ES	expected shortfall
ETC	exchange-traded commodity
EU	European Union
EUA	EU emission allowance (1 t CO_2)
EUR	euro
EWMA	exponentially weighted moving average
GARCH	generalised autoregressive conditional heteroskedasticity
GJR	Glosten–Jagannathan–Runkle asymmetric GARCH
H2O	Hidroelectrica (BVB ticker)

Abbreviations (2/2)

HAR	heterogeneous autoregressive model (Corsi)
HU	Hungary (bidding zone)
HUPX	Hungarian Power Exchange
ISE	Fraunhofer Institute for Solar Energy Systems
MAE	mean absolute error
MSE	mean squared error
MW	megawatt
MWh	megawatt-hour
OLS	ordinary least squares
OPCOM	Romanian electricity and gas market operator
PZU	Piața pentru Ziua Următoare (day-ahead market)
QLIKE	quasi-likelihood loss function
RO	Romania (bidding zone)
RON	Romanian leu
RV	realised intraday variation of the price
SNG	Romgaz (BVB ticker)
SNN	Nuclearelectrica (BVB ticker)
SNP	OMV Petrom (BVB ticker)
TTF	Title Transfer Facility (Dutch gas hub)
USD	United States dollar
VaR	value at risk

Data sources: Romanian power, fuels, carbon and energy shares

Series	Frequency	Source
Day-ahead price of Romania (RO), Piața pentru Ziua Următoare (PZU); Hungary (HU) and Germany–Luxembourg (DE-LU) for comparison	daily, hourly	Romanian market operator (OPCOM), Hungarian Power Exchange (HUPX), EPEX SPOT; via energy-charts.info (Fraunhofer ISE) and the European Network of Transmission System Operators for Electricity (ENTSO-E)
Solar generation in Romania	daily	ENTSO-E (Transelectrica data), via energy-charts.info
Brent crude oil, nearest futures contract, in dollars (USD) per barrel	daily	Yahoo Finance (BZ=F)
European Union (EU) emission allowance (EUA), via an exchange-traded commodity (ETC) backed by allowances	daily	Yahoo Finance (SparkChange, CO2.L)
Gas: Title Transfer Facility (TTF), monthly average	monthly	World Bank, Pink Sheet (no free daily series)
Shares on the Bucharest Stock Exchange (BVB): OMV Petrom (SNP), Hidroelectrica (H2O)	daily	data vendor eodhd.com (EODHD), dividend-adjusted

- **Period**
  - Power: 1 Jan 2019 – 31 Oct 2026 (last complete month), 2 861 delivery days in Central European Time (CET), none missing; downloaded 4 Nov 2026
  - Brent: the 95 first trading days of a month are dropped (contract change, not a price move); EUA from 19 Oct 2021

Experiment yourself: notebook §1

Outline

- Measuring volatility
  - Volatility in percent and in EUR/MWh
  - Stylised facts
- The GARCH family
- Regimes and the energy crisis
- Intraday volatility from hourly prices
- Forecasting and evaluating volatility
- Case study: Romanian power in 2026

Definition of volatility

- **In one sentence:** Volatility tells how far from today's price tomorrow's price usually lands.

$$\underbrace{r_t = 100 \ln \frac{P_t}{P_{t-1}}}_{\text{change in \%}} \qquad \underbrace{\Delta P_t = P_t - P_{t-1}}_{\text{change in EUR/MWh}}$$

$$\hat{\sigma} = \sqrt{\tfrac{1}{T-1} \textstyle\sum_{t=1}^{T} (r_t - \bar{r})^2}, \qquad \sigma_{\text{year}} = \hat{\sigma}\sqrt{A}$$

- **where**
  - P_t = daily base price, mean of the day's hourly prices, EUR/MWh
  - T = number of days
  - $A = 365$ for power (an auction every day), 252 for traded assets.

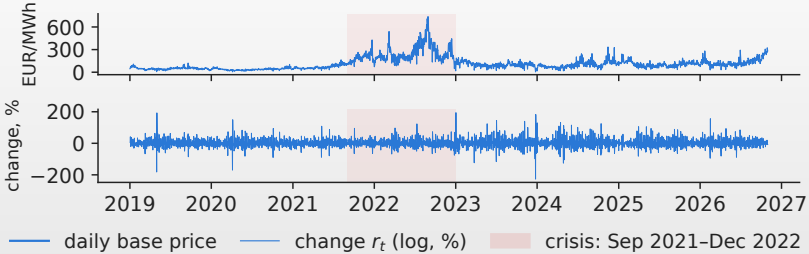
- **On our data**, 2019–2026
  - $\hat{\sigma}$ of $r_t = 28.3\%$ per day; of $\Delta P_t = 30.9$ EUR/MWh
  - Annualised, calendar removed: 382%; Brent 42%, EUA 38%
- **Not an asset return**
  - Electricity cannot be stored: days t and $t-1$ are different products

- **In practice:** a supplier that buys on the PZU every day is exposed exactly to this daily surprise.

(Weron, 2006; Weron, 2014, IJF)

Experiment yourself: notebook §1

Daily price changes alternate calm and turbulent stretches



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E. Data: daily, 2019-01-01 to 2026-10-31, CET delivery days.

- Extreme days
  - Largest rise +194% on 2 Jan 2023, largest fall −227% on 25 Dec 2023 (Christmas)
  - 189 days with $|r_t| > 50\%$ out of 2 860

In practice: a calm month says little about next month: risk comes in clusters.

(OPCOM day-ahead prices)

EMQA_L3_returns_ro

Percentage changes at prices near zero or negative

⊡ **In one sentence:** The logarithm needs positive prices; near zero, ordinary moves become huge percentages.

$$\operatorname{asinh}(x) = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\Delta \operatorname{asinh} P_t \approx \Delta \ln P_t \quad \text{if } P_t \gg 1$$

⊡ **where**

- asinh = inverse hyperbolic sine, defined for any sign
- From 100 to 120 EUR/MWh both formulas give 18.2%
- Near zero both explode.

⊡ **On our data**, 2019–2026

- Romania: base price always positive (minimum 3.97 EUR/MWh)
- But 485 hours with negative prices, on 117 days
- DE-LU: 20 days with base price ≤ 0; from 0.95 to 92.91 EUR/MWh: $r_t = 458\%$

⊡ **Our choice**

- Base price: r_t (%) and ΔP_t (EUR/MWh); hourly prices: EUR/MWh

⊡ **In practice:** a position is in MWh, so its risk is in EUR/MWh

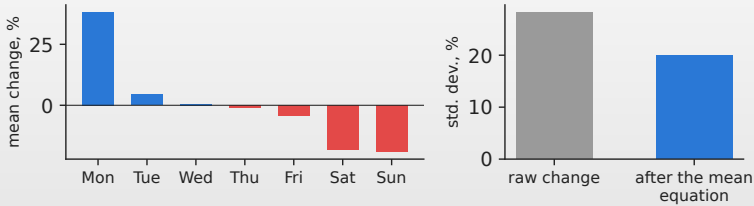
- Use percent only far from zero.

(Uniejewski, Weron & Ziel, 2018, IEEE TPWRS)

Experiment yourself: notebook §1

Half of the daily change is the weekly calendar, known in advance

$$r_t = c + \sum_{k=1}^{7} \phi_k r_{t-k} + \sum_{j=2}^{7} \delta_j D_{j,t} + \varepsilon_t$$



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E. Data: daily, 2019-01-01 to 2026-10-31.

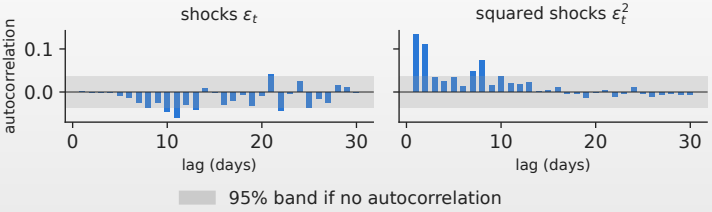
- Mean equation: ϕ_k = weight of the change k days ago, $D_{j,t} = 1$ if day t is weekday j; $R^2 = 0.50$
  - Standard deviation 28.3% before, 20.0% after; models below use ε_t and, in EUR/MWh, u_t

- **In practice:** a Monday price jump is not a risk signal: forecast it, do not hedge it.

(own estimation; Weron, 2014, IJF)

EMQA_L3_dow_effect

Volatility clustering: autocorrelation of shocks and of squared shocks



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E. Data: residuals of the mean equation, 2019-01-01 to 2026-10-31.

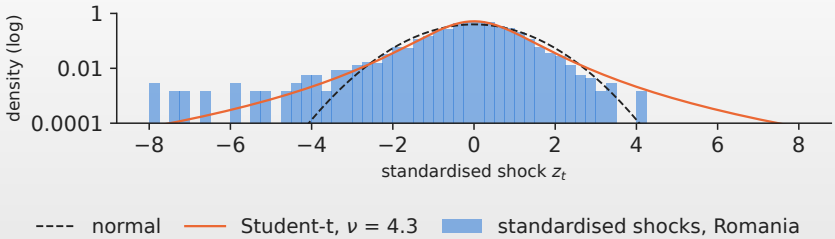
- Ljung–Box $Q(10) = T(T+2)\sum_{k\leq 10}\hat{\rho}_k^2/(T-k)$, $\hat{\rho}_k$ = autocorrelation at lag k: 13.8 for ε_t ($p = 0.18$), 120.0 for ε_t^2 (critical value 18.31)
  - Lagrange multiplier test for autoregressive conditional heteroskedasticity (ARCH-LM), 10 lags: 93.7, $p < 0.001$

- **In practice:** a large shock today announces large shocks in the next days: the subject of this lecture.

(Ljung & Box, 1978; Engle, 1982, Econometrica)

EMQA_L3_acf_clust

Extreme days are far more frequent than the normal law allows



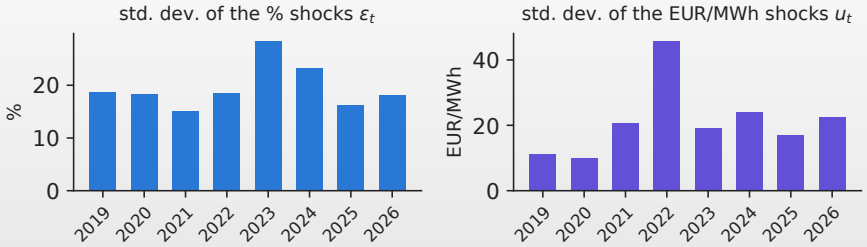
Source: energy-charts.info (OPCOM, via ENTSO-E); own estimates. Data: GARCH(1,1)-t on percent changes, 2019-01-01 to 2026-10-31.

- Shocks ε_t: excess kurtosis 20.0, skewness -1.69 (large falls: holidays, windy days)
  - 15 shocks beyond 4 standard deviations, against 0.18 expected under normality; Student-t with $\nu = 4.3$ degrees of freedom

- **In practice:** a limit built on the normal curve is breached far more often than planned (Lecture 4).

(own estimation; Bollerslev, 1987, RESTAT)

Volatility in 2022: similar in percent, 4 times higher in EUR/MWh



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E. Data: residuals of the mean equation, 2019-01-01 to 2026-10-31.

- Standard deviation of the shocks, 2019 vs 2022
  - In percent: 18.7% vs 18.5% (ratio 0.99)
  - In EUR/MWh: 11.2 vs 45.8 (ratio 4.1); mean price 50 vs 265 EUR/MWh

- **In practice:** margin calls are paid in EUR: collateral rose with the price level although percent volatility did not.

(own estimation)

EMQA_L3_vol_units

Power prices are about 9 times more volatile than oil

Annualised volatility, %	2019	2020	2021	2022	2023	2024	2025	2026
Power, Romania	358	349	287	354	543	444	310	346
Power, Hungary	247	329	289	335	566	547	358	339
EUA	–	–	–	53	32	37	24	28
Brent	30	71	32	46	31	25	29	56
TTF gas (monthly)	42	82	58	100	78	38	28	63
OMV Petrom (SNP)	24	35	22	22	18	17	20	21
Hidroelectrica (H2O)	–	–	–	–	13	14	14	30

- **How to read it**
  - Daily shocks: standard deviation $\times\sqrt{365}$ for power, $\times\sqrt{252}$ for traded assets; TTF: monthly averages $\times\sqrt{12}$ (understates risk)
  - Brent peaks in 2020 and 2026, carbon in 2022; – = series not yet available

In practice: never compare the limits of a power desk with those of an oil desk in the same percent.

(own estimation; World Bank Pink Sheet)

Experiment yourself: notebook §2

Outline

- Measuring volatility
- The GARCH family
  - GARCH(1,1) and its estimation
  - Persistence and half-life
  - Asymmetry
- Regimes and the energy crisis
- Intraday volatility from hourly prices
- Forecasting and evaluating volatility
- Case study: Romanian power in 2026

GARCH updates tomorrow's variance from today's shock

⊡ **In one sentence:** A big shock yesterday raises today's variance; the effect then fades day by day.

$$\sigma_t^2 = \underbrace{\omega}_{\text{constant}} + \underbrace{\alpha\,\varepsilon_{t-1}^2}_{\text{reaction to the shock}} + \underbrace{\beta\,\sigma_{t-1}^2}_{\text{memory}}$$

$$\varepsilon_t = \sigma_t z_t, \qquad \bar{\sigma}^2 = \frac{\omega}{1-\alpha-\beta}$$

⊡ **where**

- σ_t^2 = conditional variance, known on the evening of $t-1$
- z_t = standardised shock (mean 0, variance 1)
- $\omega > 0$, $\alpha, \beta \geq 0$, persistence $\alpha + \beta < 1$, long-run variance $\bar{\sigma}^2$
- $\beta = 0$: ARCH model (Engle, 1982).

⊡ **Romania, shocks in %**

- $\hat{\alpha} = 0.308$, $\hat{\beta} = 0.438$, $\hat{\alpha} + \hat{\beta} = 0.746$
- Long-run $\bar{\sigma} = 20.4\%$ per day

⊡ **Romania, shocks in EUR/MWh**

- $\hat{\alpha} = 0.102$, $\hat{\beta} = 0.892$, $\hat{\alpha} + \hat{\beta} = 0.994$
- Long-run $\bar{\sigma} = 27.6$ EUR/MWh per day

⊡ **In practice:** every evening GARCH gives the risk desk tomorrow's σ, from data it already has.

(Engle, 1982, Econometrica; Bollerslev, 1986, JOE)

Experiment yourself: notebook §3

Parameters are estimated by maximum likelihood with Student-t shocks

- **In one sentence:** Choose the parameters that make the observed shocks most probable; Student-t leaves room for extreme days.

$$\ell(\omega, \alpha, \beta, \nu) = \sum_{t=1}^{T} \left[\ln f_\nu \left(\frac{\varepsilon_t}{\sigma_t} \right) - \ln \sigma_t \right]$$

- **where**
  - f_ν = Student-t density with variance 1 and $\nu > 2$ degrees of freedom
  - Small ν = fat tails, $\nu \to \infty$ = normal
  - Bayesian information criterion (BIC) $= k \ln T - 2\hat{\ell}$, k = number of parameters (lower = better)
  - Appendix A.

GARCH(1,1), shocks in %	normal	Student
log-likelihood	−12 360.3	−12 094.5
BIC	24 744.5	24 220.8
$\hat{\alpha} + \hat{\beta}$	0.927	0.746
half-life, days	9.2	2.4
$\hat{\nu}$	–	4.27

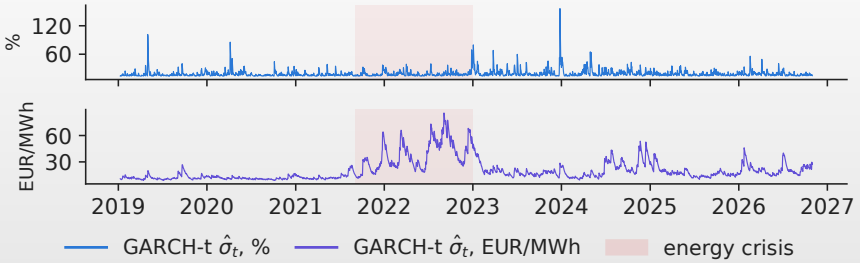
- **Reading**
  - Student wins (lower BIC); the normal model reads every holiday as lasting volatility

- **In practice:** use Student-t shocks
  - With normal shocks the limits stay wide for too long.

(Bollerslev, 1987, RESTAT)

Experiment yourself: notebook §3

GARCH volatility: short spikes in percent, a prolonged rise in EUR/MWh



Source: energy-charts.info (OPCOM, via ENTSO-E); own estimates. Data: GARCH(1,1)-t, 2019-01-01 to 2026-10-31.

- Peaks
  - In percent: 156% per day on 26 Dec 2023, gone within days
  - In EUR/MWh: 85.7 on 4 Sep 2022, against a minimum of 8.8 on 6 Jul 2019; 27.4 at the cut-off

In practice: a supplier's collateral follows the EUR/MWh line, not the percent line.

(own estimation, GARCH(1,1)-t)

EMQA_L3_cond_vol

The half-life counts the days until half of a volatility shock is gone

- **In one sentence:** The extra variance created by a shock shrinks every day by the factor $\alpha + \beta$.

$$\mathbb{E}_t\left[\sigma^2_{t+h}\right] - \bar{\sigma}^2 = (\alpha+\beta)^{h-1}\left(\sigma^2_{t+1} - \bar{\sigma}^2\right)$$

$$h_{1/2} = \frac{\ln 0.5}{\ln(\alpha+\beta)}$$

- **where**
  - $\mathbb{E}_t$ = expectation given the data up to day t
  - h = days ahead
  - $h_{1/2}$ = half-life
  - Derivation in Appendix B.

GARCH(1,1)-t	$\hat{\alpha}+\hat{\beta}$	$h_{1/2}$	unit
Romania, %	0.746	2.4	days
Romania, EUR/MWh	0.994	113.5	days
Hungary, %	0.881	5.5	days
Brent	0.973	25.6	trading days
EUA	0.992	82.8	trading days
OMV Petrom	0.934	10.2	trading days

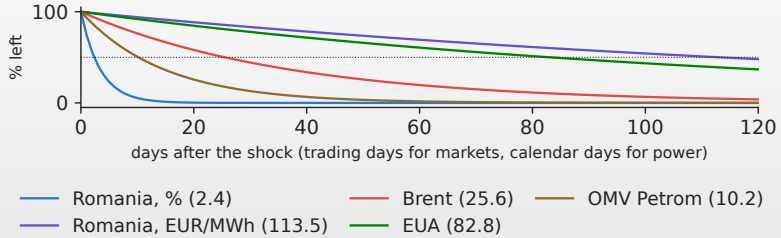
- Samples: power and Brent from 2019, EUA from 19 Oct 2021, OMV Petrom from 2019.

- **In practice:** after an oil shock keep wider limits for about 5 weeks
  - After a percent shock in power, for days.

(Engle, 2001, JEP)

Experiment yourself: notebook §3

Half-life of volatility shocks by market



Source: energy-charts.info (OPCOM, via ENTSO-E); Yahoo Finance (Brent BZ=F, SparkChange EUA ETC); EODHD (BVB shares).
Data: GARCH(1,1)-t, each series over its own sample (table on the slide).

- Curves: $100(\hat{\alpha} + \hat{\beta})^h$ = percent of the extra variance left after h days; half-lives in brackets
  - Brent 26 trading days, about 5 weeks; EUA 83, about 17 weeks
  - Romania: 2.4 days in percent, but 114 days in EUR/MWh, because volatility in money follows the price level

- **In practice:** after a shock, a risk desk keeps wider limits for weeks, not days, as announced in Lecture 1.

(own estimation)

EMQA_L3_halflife

Asymmetric GARCH: rises and falls do not move volatility equally

- **In one sentence:** Two extensions let a rise and a fall of the same size have different effects on tomorrow's variance.

$$\text{GJR:}\quad \sigma_t^2 = \omega + (\alpha + \gamma I_{t-1})\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

$$\text{EGARCH:}\quad \ln\sigma_t^2 = \omega + \alpha\left(|z_{t-1}| - \sqrt{2/\pi}\right) + \gamma z_{t-1} + \beta \ln\sigma_{t-1}^2$$

- **where**
  - $I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$, else 0
  - Glosten–Jagannathan–Runkle (GJR): $\gamma > 0$ = falls raise volatility more (leverage effect of shares), $\gamma < 0$ = rises do
  - Exponential GARCH (EGARCH): opposite sign convention for γ.

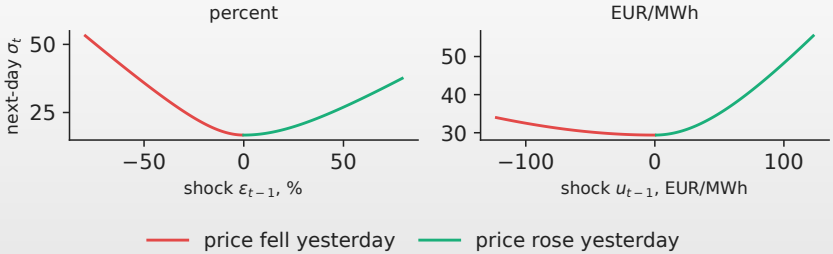
- **Romania, shocks in %**
  - GJR: $\hat{\gamma} = 0.225$ ($t = 2.9$); EGARCH: $\hat{\gamma} = -0.083$ ($t = -3.9$)
  - Falls raise volatility more (rebound after a drop)
- **Romania, shocks in EUR/MWh**
  - GJR: $\hat{\gamma} = -0.127$ ($t = -6.4$); EGARCH: $\hat{\gamma} = 0.104$ ($t = 7.2$)
  - Rises raise volatility more: inverse leverage effect

- **In practice:** for a buyer the dangerous direction is up: after a price jump, widen the limits more than after a fall.

(Glosten et al., 1993, JFIN; Nelson, 1991; Knittel & Roberts, 2005, ENEECO)

Experiment yourself: notebook §4

In EUR/MWh, rises raise volatility more: the inverse leverage effect



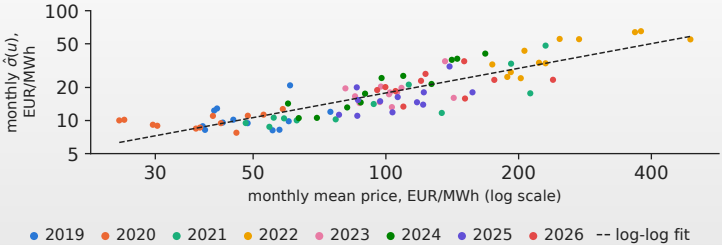
Source: energy-charts.info (OPCOM, via ENTSO-E); own estimates. Data: GJR(1,1)-t, 2019-01-01 to 2026-10-31.

- News impact curve: next-day σ as a function of today's shock (GJR); for a shock of 61 EUR/MWh
  - Next-day $\sigma = 37.6$ EUR/MWh after a rise, 30.6 after a fall
  - In percent (shock of 40%) the order is reversed: 30.3% after a fall, 23.7% after a rise

- **In practice:** an industrial consumer on spot prices should react faster to rises than to falls.

(Engle & Ng, 1993, JFIN; own estimation, GJR(1,1)-t)

Volatility in EUR/MWh grows with the price level



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E. Data: monthly, 2019-01-01 to 2026-10-31.

- Why: at high prices the supply curve (merit order) is steeper, so the same demand shock moves the price more
  - Regression of $\ln \hat{\sigma}_m(u)$ on $\ln \bar{P}_m$, 94 months: slope 0.75, $R^2 = 0.76$; in percent: slope 0.01, $R^2 = 0.00$

- **In practice:** set EUR/MWh limits as a share of the forward price, not as a fixed amount.

(Knittel & Roberts, 2005, ENEECO; own estimation)

EMQA_L3_vol_level

Relative importance of fat tails and asymmetry

Model	$\alpha+\beta$ (%)	BIC (%)	p, z^2 (%)	$\alpha+\beta$ (EUR)	BIC (EUR)	p, z^2 (EUR)
GARCH, normal	0.927	24 744.5	0.328	0.998	24 573.2	0.000
GARCH, Student	0.746	24 220.8	0.367	0.994	24 450.7	0.000
GJR, normal	0.651	24 690.7	0.151	0.998	24 525.1	0.000
GJR, Student	0.727	24 214.1	0.243	0.996	24 411.7	0.000
EGARCH, normal	0.903	24 691.4	0.529	0.995	24 518.0	0.000
EGARCH, Student	0.831	24 219.2	0.532	0.995	24 403.9	0.000

- **Ranking by BIC** (green = best)
  - Every Student-t model beats every normal one; best: GJR-t in %, EGARCH-t in EUR/MWh; Akaike information criterion (AIC) in Appendix C
- **Diagnostic**: Ljung–Box p for z_t^2, the squared standardised shocks
  - In % no clustering is left; in EUR/MWh some is ($p < 0.001$): one equation cannot follow both the price level and the daily shocks

- **In practice:** first get the tails right, then worry about asymmetry.

(Bollerslev, 1987; Glosten, Jagannathan & Runkle, 1993; Nelson, 1991)

Experiment yourself: notebook §4

Outline

- Measuring volatility
- The GARCH family
- Regimes and the energy crisis
- Intraday volatility from hourly prices
- Forecasting and evaluating volatility
- Case study: Romanian power in 2026

A Markov-switching model separates calm and turbulent regimes

In one sentence: Two hidden states, each with its own volatility; for every day the model gives the probability of the turbulent one.

$$u_t = \mu_{s_t} + \sigma_{s_t}\eta_t, \qquad \Pr(s_t = j \mid s_{t-1} = i) = p_{ij}$$

$$\text{expected duration of regime } i = \frac{1}{1 - p_{ii}}$$

where

- u_t = shock in EUR/MWh
- s_t = regime (calm or turbulent)
- η_t = standard normal
- p_{ij} = transition probabilities, by maximum likelihood (Appendix D).

Romania, shocks in EUR/MWh

- Calm: $\sigma = 12.9$, $p_{11} = 0.986$, about 69 days
- Turbulent: $\sigma = 41.9$, $p_{22} = 0.957$, about 23 days; 24% of all days

In percent

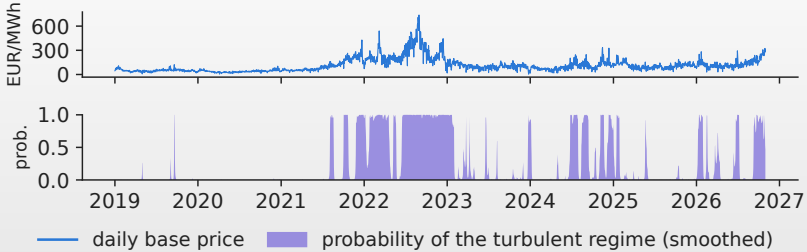
- Turbulent state: only 3.5 days, isolated spikes

In practice: a turbulent-regime probability above 0.5 triggers a review of hedges and collateral.

(Hamilton, 1989, Econometrica)

Experiment yourself: notebook §5

Volatility regimes during the crisis and in 2026



Source: energy-charts.info (OPCOM, via ENTSO-E); own estimates. Data: two-regime Markov-switching model on EUR/MWh shocks, 2019-01-01 to 2026-10-31.

- Turbulent days
  - Longest spell: 15 Jun 2022 – 31 Jan 2023, 231 days
  - Share by year: 2022 86%, 2025 6%, 2026 33% (48% since July)

In practice: Transelectrica and suppliers can use the regime probability as an early warning for balancing costs.

(own estimation; Hamilton, 1989)

EMQA_L3_ms_regime

Persistence depends on the period and on the unit

GARCH(1,1)-t on	$\hat{\alpha}+\hat{\beta}$ (%)	$h_{1/2}$ (%)	$\hat{\alpha}+\hat{\beta}$ (EUR)	$h_{1/2}$ (EUR)	$\bar{\sigma}$, EUR/MWh
2019–2020 (calm)	0.568	1.2	0.208	0.4	10.5
2021–2022 (crisis)	0.882	5.5	1.000	∞	–
2023 – Oct 2026	0.727	2.2	0.927	9.1	20.9
whole sample	0.746	2.4	0.994	113.5	27.6

- **In EUR/MWh**
  - In 2019–2020 a shock halves in under a day; in the crisis $\hat{\alpha}+\hat{\beta}$ reaches 1, because volatility rose with the price month after month
  - The whole-sample value mixes regimes, and mixing regimes overstates persistence
- **In percent**
  - Persistence stays well below 1 in every period: the crisis moved the level, not percent volatility

- **In practice:** re-estimate after a regime change
  - A model fitted on the crisis keeps limits wide for too long.

(Lamoureux & Lastrapes, 1990, JBES; own estimation)

Experiment yourself: notebook §5

Outline

- Measuring volatility
- The GARCH family
- Regimes and the energy crisis
- Intraday volatility from hourly prices
- Forecasting and evaluating volatility
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Intraday variation measures how much the price moves within a day

In one sentence: Sum the squared hour-to-hour changes of one delivery day; this is the variation a battery or a flexible consumer earns from.

$$\mathrm{RV}_d = \sqrt{\sum_{h=1}^{H_d-1} \left(P_{d,h} - P_{d,h-1}\right)^2}, \qquad \text{relative} = \frac{\mathrm{RV}_d}{\bar{P}_d}$$

where

- $P_{d,h}$ = price of hour h of day d (from 1 October 2025, mean of the four quarter-hours)
- H_d = 24 hours (23 or 25 on the 8 + 8 clock-change days)
- $\bar{P}_d$ = daily base price
- In EUR/MWh, because hourly prices can be negative.

Realised intraday variation (RV), yearly mean

- 2019: 36; 2022: 186; 2026: 135 EUR/MWh
- Relative to the price (median): 0.64 in 2019, 1.24 in 2024

A different risk

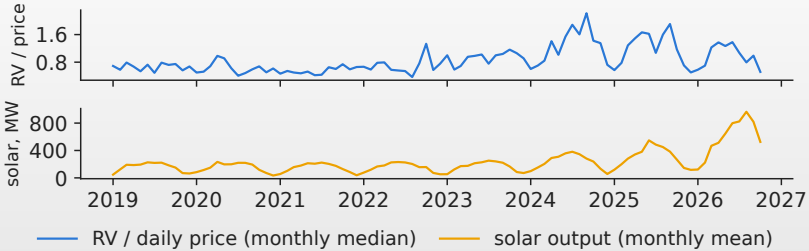
- Correlation between RV_d and $|u_t|$: only 0.46
- Not the realised volatility of finance: the 24 hours are 24 different products

In practice: a battery developer's revenue scales with intraday variation, not with the daily average price.

(Andersen & Bollerslev, 1998, IER; Weron, 2014, IJF)

Experiment yourself: notebook §6

Solar generation has made the price move more within the day



Source: energy-charts.info (Fraunhofer ISE); OPCOM prices; ENTSO-E solar generation (Transelectrica data for Romania). Data: hourly prices, monthly values, 2019-01-01 to 2026-10-31.

- Relative intraday variation (median)
  - 0.64 in 2019, peak 1.24 in 2024, 1.01 in 2026
  - Mean solar output: 157 MW in 2019, 593 MW in Jan – Oct 2026; monthly correlation 0.46

- **In practice:** more solar means cheaper middays and dearer evenings: the business case of storage.

(own estimation; ENTSO-E solar generation)

EMQA_L3_rv_solar

Intraday variation has a long memory: day, week and month

In one sentence: Three averages over different horizons give a simple long-memory regression.

$$\ln \mathrm{RV}_d = b_0 + b_D \ln \mathrm{RV}_{d-1} + \tfrac{b_W}{7} \sum_{j=1}^{7} \ln \mathrm{RV}_{d-j} + \tfrac{b_M}{30} \sum_{j=1}^{30} \ln \mathrm{RV}_{d-j} + e_d$$

where

- Heterogeneous autoregressive (HAR) model, by ordinary least squares (OLS)
- b_D, b_W, b_M = weights of yesterday, the last 7 and 30 days
- Forecast $= \exp(\widehat{\ln \mathrm{RV}}_d + s^2/2)$, s^2 = residual variance.

Estimates, before the test year

- $\hat{b}_D = 0.40$, $\hat{b}_W = 0.36$, $\hat{b}_M = 0.22$ (all $t > 6$), $R^2 = 0.82$

Test year

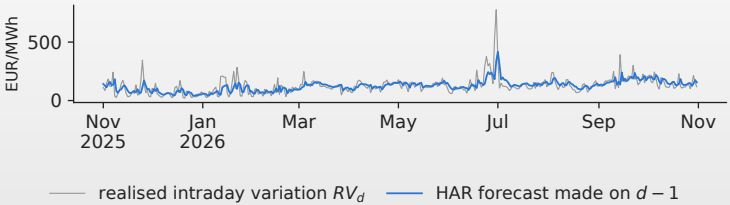
- Mean absolute error (MAE): HAR 35.6, yesterday's value 39.1 EUR/MWh (9.1% less)

In practice: a battery operator schedules maintenance on days with low forecast variation.

(Corsi, 2009, JFEC)

Experiment yourself: notebook §6

HAR forecasts track the level of intraday variation but smooth out isolated spikes



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E. Data: test year 2025-11-01 to 2026-10-31; HAR re-estimated daily.

- Test year, forecast made on $d-1$
  - HAR re-estimated every day on the data available on $d-1$; MAE 35.6 EUR/MWh

- **In practice:** use the forecast for planning
  - Keep reserves for the spikes it cannot see.

(own estimation)

EMQA_L3_har_fc

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A volatility forecast is judged against a noisy proxy

- **In one sentence:** We compare the forecast with a noisy proxy, the squared shock, using a loss that still ranks models correctly.

$$\text{QLIKE}_t = \frac{\varepsilon_t^2}{\hat{\sigma}_t^2} - \ln \frac{\varepsilon_t^2}{\hat{\sigma}_t^2} - 1, \qquad \text{MSE}_t = \left(\varepsilon_t^2 - \hat{\sigma}_t^2\right)^2$$

- **where**
  - Quasi-likelihood loss (QLIKE), mean squared error (MSE)
  - ε_t = out-of-sample shock of day t
  - $\hat{\sigma}_t^2$ = forecast made on $t-1$
  - MSE is dominated by a few extreme days
  - Diebold–Mariano (DM) test on the daily loss differences.

- **Example**: true $\sigma = 20$
  - Forecast 10: QLIKE = 1.61; forecast 40: QLIKE = 0.64
  - Underestimating costs 2.5 times more; MSE says the opposite
- **Design**, test year 1 Nov 2025 – 31 Oct 2026
  - Five models re-estimated daily on data up to $t-1$

- **In practice:** ask your risk vendor for QLIKE on your own days, out of sample.

(Patton, 2011, JOE; Hansen & Lunde, 2005, JAE)

Experiment yourself: notebook §7

GARCH-type forecasts compared with 30-day historical volatility

One-day-ahead variance	QLIKE, %	p vs 30-day	QLIKE, EUR	p vs 30-day
30-day historical variance	1.523	–	1.428	–
exponentially weighted average (EWMA)	1.499	0.082	1.398	0.026
GARCH(1,1)-t	1.459	0.170	1.378	0.040
GJR(1,1)-t	1.445	0.145	1.357	0.032
EGARCH(1,1)-t	1.452	0.120	1.349	0.027

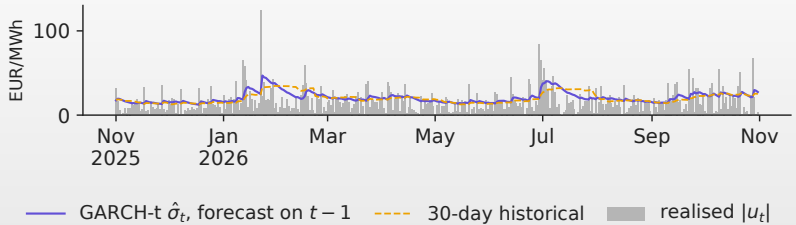
- **Reading**
  - Lower QLIKE = better (green); p = one-sided DM test (Newey–West, 7 lags), 365 days
  - Best: GJR-t in % (5.1% below the 30-day loss), EGARCH-t in EUR/MWh (5.5%)
- **Significance at 5%**
  - EUR/MWh: all GARCH-type models beat the history; percent: none does; asymmetry adds nothing significant ($p = 0.08$)

- **In practice:** use GARCH-t as the default
  - Asymmetric models add little out of sample.

(Diebold & Mariano, 1995, JBES; Patton, 2011, JOE; RiskMetrics, 1996)

Experiment yourself: notebook §7

GARCH and 30-day historical volatility around the largest shock



Source: energy-charts.info (OPCOM, via ENTSO-E); own estimates. Data: test year 2025-11-01 to 2026-10-31; models re-estimated daily.

- Largest shock of the test year: +124 EUR/MWh on 22 Jan 2026
  - Next day: GARCH $\hat{\sigma}$ from 25 to 47, history only 33 EUR/MWh
  - 30 days later the shock leaves the window: the history drops from 32 to 23 overnight

- **In practice:** a limit that jumps 30 days after a shock, with no news, is a warning sign of a historical-window model.

(own estimation, test year 1 Nov 2025 – 31 Oct 2026)

EMQA_L3_fc_test

Over several weeks, volatility returns to its long-run level

- **In one sentence:** High volatility is expected to fall and low volatility to rise, so $\sigma\sqrt{h}$ misstates the risk over h days.

$$\hat{\sigma}^2_{T+h|T} = \bar{\sigma}^2 + (\alpha+\beta)^{h-1}\left(\hat{\sigma}^2_{T+1} - \bar{\sigma}^2\right)$$

$$\mathrm{Var}_T\left(\sum_{k=1}^{h} r_{T+k}\right) = \sum_{k=1}^{h} \hat{\sigma}^2_{T+k|T} \;\neq\; h\,\hat{\sigma}^2_{T+1}$$

- **where**
  - T = forecast day
  - Second formula: returns without autocorrelation (oil, carbon), not power
  - 90% band = $\pm 1.645\sqrt{\cdot}$.

- **Brent on 9 Apr 2026**, after the 2026 oil shock
  - $\hat{\sigma}_{T+1} = 93\%$ a year, long-run 39%
  - 20-day band: GARCH $\pm 38.5\%$, $\sigma\sqrt{20}$ $\pm 43.1\%$ (12% too wide)
- **Brent on 30 Oct 2026**, calm again
  - $\hat{\sigma}_{T+1} = 32\% < 40\%$: now $\sigma\sqrt{20}$ is too narrow

- **In practice:** an industrial buyer budgeting next month's fuel should add the GARCH forecasts, not scale today's σ.

(Engle, 2001, JEP; own estimation)

Experiment yourself: notebook §7

Outline

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Case study: price level and volatility in 2026 compared with 2022

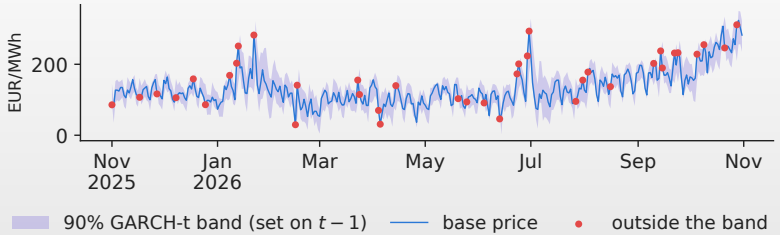
- **What happened to prices**
  - Romanian base price: 150 EUR/MWh in January, 239 in October 2026 (monthly means)
  - Brent: 69 USD/bbl in February, 100 in March, 108 in October; TTF gas: 32 → 87 EUR/MWh (February → October)
- **What happened to volatility**
  - In percent, nothing: mean monthly $\hat{\sigma}$ 17.4% in 2026, 17.4% in 2022, 16.6% in 2019
  - In EUR/MWh it doubled from 2019 (10.9 → 21.8), but its peak month, 34.7 (Jan 2026), stays far below 2022 (65.1, Sep 2022)
  - Turbulent regime: 33% of the days of 2026 (2022: 86%); probability 0.47 on 31 Oct 2026
  - Brent: GARCH volatility up to 100% annualised (9 Apr 2026), against 30% on average in 2025
- **Verdict**
  - A shock to the price level, with a turbulent regime about half of the time since July; not (yet) a 2022

In practice: a supplier renewing contracts in November 2026 should price risk on the higher level, not on a new volatility regime.

(own estimation; World Bank Pink Sheet; Yahoo Finance)

Experiment yourself: notebook §8

Case study: the one-day GARCH-t band held over the test year



Source: energy-charts.info (OPCOM, via ENTSO-E); own estimates. Data: test year 2025-11-01 to 2026-10-31.

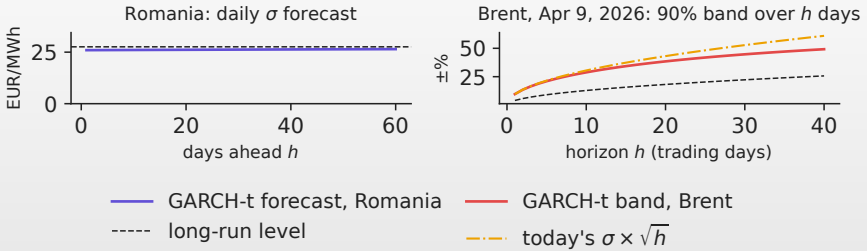
- Band = forecast price $\pm$ Student quantile $\times \hat{\sigma}_t$, set on $t-1$
  - 38 of 365 days outside (10.4%, 10% expected): 9.4% from November to April, 11.4% from May to October
  - Mean half-width 33 EUR/MWh in the first half, 36 in September–October

- **In practice:** a supplier can set its daily purchase limit from this band
  - The formal tests of such bands come in Lecture 4.

(own estimation, test year 1 Nov 2025 – 31 Oct 2026)

EMQA_L3_case_band

Case study: November 2026 limits and the April oil shock



Source: energy-charts.info (OPCOM, via ENTSO-E); Yahoo Finance (Brent BZ=F, SparkChange EUA ETC); EODHD (BVB shares). Data: GARCH(1,1)-t forecasts made with data up to 2026-10-31.

- Romania, 31 Oct 2026: $\hat{\sigma} = 26.0$ EUR/MWh, close to the long-run 27.6
  - One-day 90% limit about ± 43 EUR/MWh for each day of November; Brent: the band grows slower than $\sigma\sqrt{h}$

In practice: set limits from the forecast term structure, not from today's volatility.

(own estimation)

EMQA_L3_term_structure

Case study C3: weak link between Romanian energy shares and oil

- **What the paper did** (Grecu, Cramer, Pele & Lessmann, 2025)
  - Data: stock indices of the 27 European Union countries, Brent and TTF futures, 2010–2023
  - Method: GARCH volatility for each series, then a dynamic conditional correlation (DCC) for each day; a Markov model for regimes of that correlation
  - Result: the correlation with oil is mostly between 0 and 0.4; Central and Eastern Europe (CEE) is the least correlated region; regimes persist above 98%
- **Our replication** (OMV Petrom, Romgaz (SNG), Nuclearelectrica (SNN) vs Brent, 2 860 days)
  - DCC parameters $a = 0.021$, $b = 0.950$: a slow, very persistent correlation
  - Mean correlation 0.13; maximum 0.54 in 2020, minimum -0.32 in 2022: the calendar of the paper
  - Hedging the basket with Brent, out of sample: variance change -1.7% (dynamic), -6.9% (static); negative = more risk
- C3 numbers are computed by the C3 case notebook on our data; the paper's own results are in the case brief.

- **In practice:** the risk of Romanian energy shares is regulatory and local
  - A Brent hedge does not reduce it.

(Grecu, Cramer, Pele & Lessmann, 2025, NAJEF; case study C3)

Quiz 1: half-life of an oil shock

- For Brent, GARCH(1,1)-t gives $\hat{\alpha} + \hat{\beta} = 0.973$
  - After a shock, how long until half of the extra variance is gone?

Answer

- $h_{1/2} = \ln 0.5 / \ln 0.973 = 25.6$ trading days, about 5 weeks: limits stay wider for weeks, not days.

Quiz 2: percent or euro

- In 2022 the percent volatility of the Romanian price was the same as in 2019
  - Did the risk of a supplier's open position of 10 MW change?

Answer

- Yes: the position loses or gains in EUR, and the EUR/MWh volatility of the shocks rose 4.1 times, with the price level.

Quiz 3: a Monday jump

- The PZU price rises by 35% from Sunday to Monday
  - Should the risk desk widen the limits?

Answer

- No: on average Monday is $+38\%$ above Sunday
  - The calendar is forecast by the mean equation, and only the shock ε_t (here almost zero) enters GARCH.

Quiz 4: choosing a volatility model

- A vendor's volatility model has the lowest mean squared error on your test year, thanks to three extreme days
  - What do you ask for?

Answer

- QLIKE on the same days and a DM test against GARCH-t and the 30-day history
  - On our test year the best QLIKE in EUR/MWh is EGARCH-t, not significantly better than GARCH-t.

Quiz 5: after a rise or after a fall

- Yesterday the price moved by 61 EUR/MWh
  - For a buyer, is tomorrow riskier if it was a rise or a fall?

Answer

- After a rise: the GJR model in EUR/MWh gives $\sigma = 37.6$ after a rise, 30.6 after a fall (inverse leverage effect).

Wrap-up

- Measuring volatility
- The GARCH family
- Regimes and the energy crisis
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Measure

- Remove the weekly calendar first; in percent the crisis was not more volatile, in EUR/MWh it was 4.1 times more

Model and forecast

- GARCH-t is the default; oil shocks halve in about 5 weeks; next: risk measures for energy companies (Lecture 4)

Research from our group on volatility and risk

- **Energy prices and EU stock markets** (Grecu, Cramer, Pele & Lessmann, 2025, NAJEF)
  - What it did: GARCH volatility, daily DCC correlations and Markov regimes for 27 national indices against oil and gas, 2010–2023
  - Result: European markets follow oil more than gas, with persistent regimes (case study C3)
- **Entropy and market risk** (Pele, Lazar & Dufour, 2017, Entropy)
  - What it did: measured the entropy of intraday euro–yen returns and related it to intraday value at risk (VaR) and expected shortfall (ES)
  - Result: a negative relation between entropy and intraday VaR and ES; intraday entropy is then used to forecast the daily VaR
- **High-frequency entropy for daily risk** (Pele & Mazurencu-Marinescu-Pele, 2019, Entropy)
  - What it did: used the entropy of intraday Bitcoin returns to forecast the daily VaR, against classical GARCH models
  - Result: the entropy-based VaR forecast performs best in the backtests: intraday data carry information on daily risk
- **Energy Financial Risk Meter** (Toma, Dima, Grecu, Pele & Lessmann, 2026, SSRN)
  - What it did: a systemic-risk indicator for the EU energy sector; preview of Lecture 4
- Findings as stated in the papers; numbers on the other slides come from our own data.

(Bucharest University of Economic Studies; Romanian Academy, Institute for Economic Forecasting)

AIDA 2026: geopolitical crises turn the energy link of EU stocks negative

- **What the work did** (A.-A. Cramer, AIDA (AI and Digital Assets in Energy and Finance) conference, 3 June 2026)
  - Data: main stock index of each EU country, by region and by industrialisation; Brent and TTF futures, 2010–2026
  - Models: GARCH, then a daily DCC correlation (as in case study C3); Markov regimes; responses to shocks
- **What it found**
  - Correlation with oil mostly positive, in most cases between 0 and 0.4; with gas generally lower than with oil
  - Oil matters most in Southern and Western Europe, gas in the North and West; industrialised countries follow oil more
  - Crises turn the correlation negative: Crimea 2014 (gas), Ukraine 2022 (gas: strongest in CEE), the Middle East 2026
  - After a shock, the most industrialised countries are hit more in the first months, then the effect fades
- Findings as stated by the authors. Talk: slides; paper: doi:10.1016/j.najef.2025.102420.

- **In practice:** whoever hedges EU shares with oil or gas futures must expect the correlation to flip sign in a crisis.

(Cramer, AIDA conference, Bucharest, 3 June 2026; Grecu, Cramer, Pele & Lessmann, 2025, NAJEF)

AIDA 2026: spillovers of energy tokens are mostly short-run

- **What the work did** (S. Ben Amor & M. Horváth, AIDA conference, 4 June 2026)
  - Data: energy tokens (Power Ledger, SunContract, Energy Web), cryptocurrencies, clean-energy stocks, oil, gas, carbon allowances, green bonds; Dec 2020 – Jan 2026
  - Model: a time-varying vector autoregression; spillovers split into short-run (transitory) and long-run (persistent) parts
- **What it found**
  - Connectedness: about 60% of the total in the short run, about 5% in the long run
  - Crises (crypto turmoil 2021, energy crisis 2022) amplify short-run shocks; the long-run structure stays flat
  - Power Ledger and SunContract are the main short-run transmitters; Energy Web matters mainly in stress episodes
  - Diversification is token-specific: minimum-variance portfolio, Sharpe ratio 0.127 without tokens, 0.143 with SunContract
- Findings as stated by the authors. Talk: slides.

- **In practice:** shocks pass within days
  - Judge the diversification value token by token.

(Ben Amor & Horváth, AIDA conference, Bucharest, 4 June 2026)

Further reading and resources

- **R. F. Engle, GARCH 101** (2001): the clearest short introduction, by the Nobel laureate who invented ARCH — doi:10.1257/jep.15.4.157.
- **R. Weron, Modeling and Forecasting Electricity Loads and Prices** (Wiley, 2006): stylised facts and volatility of power prices — doi:10.1002/9781118673362.
- **The `arch` Python package** (K. Sheppard), used for every model of this lecture: arch.readthedocs.io.
- **This course**: code, data, notebooks — github.com/danpele/EMQA.
- **Case study C3**: the DCC model and the Brent hedge for Romanian energy shares, step by step.
- **Experiment yourself**: the notebook reproduces every chart and number of this lecture — notebooks/.

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Appendix A: the Student-t likelihood of GARCH

$$\ell(\theta) = \sum_{t=1}^{T} \left[\ln \Gamma\left(\tfrac{\nu+1}{2}\right) - \ln \Gamma\left(\tfrac{\nu}{2}\right) - \tfrac{1}{2} \ln\big(\pi(\nu-2)\big) - \tfrac{1}{2} \ln \sigma_t^2 - \tfrac{\nu+1}{2} \ln\left(1 + \frac{\varepsilon_t^2}{(\nu-2)\sigma_t^2}\right)\right]$$

- $\theta = (\omega, \alpha, \beta, \nu)$
  - Γ = gamma function
  - The factor $\nu - 2$ gives the Student variable variance 1.
- Numerical maximisation under $\omega > 0$, $\alpha, \beta \geq 0$, $\alpha + \beta < 1$, $\nu > 2$
  - Robust standard errors
  - Package `arch`.
- Normal case: $\ell = -\frac{1}{2} \sum_t \left[\ln(2\pi) + \ln \sigma_t^2 + \varepsilon_t^2/\sigma_t^2\right]$, the limit $\nu \to \infty$.

(Bollerslev, 1987, RESTAT)

GARCH(1,1)-t	%	EUR/MWh
ω	105.677 (35.482)	4.635 (1.773)
α	0.308 (0.081)	0.102 (0.020)
β	0.438 (0.150)	0.892 (0.021)
ν	4.266 (0.370)	6.149 (0.655)

Estimate (standard error).

Appendix B: multi-step forecast and half-life

- **Step 1**
  - Write $\varepsilon_t^2 = \sigma_t^2 + v_t$ with $\mathbb{E}_{t-1}[v_t] = 0$; then $\sigma_{t+1}^2 = \omega + (\alpha + \beta)\sigma_t^2 + \alpha v_t$
- **Step 2**
  - Take expectations at T: $\mathbb{E}_T[\sigma_{T+h}^2] = \omega + (\alpha + \beta)\mathbb{E}_T[\sigma_{T+h-1}^2]$ for $h \geq 2$
  - Subtract $\bar{\sigma}^2 = \omega + (\alpha + \beta)\bar{\sigma}^2$: the deviation from $\bar{\sigma}^2$ is multiplied by $\alpha + \beta$ every day
- **Step 3**
  - Hence $\mathbb{E}_T[\sigma_{T+h}^2] - \bar{\sigma}^2 = (\alpha + \beta)^{h-1}(\sigma_{T+1}^2 - \bar{\sigma}^2)$
  - Half of the deviation is left when $(\alpha + \beta)^h = 1/2$, so $h_{1/2} = \ln(1/2)/\ln(\alpha + \beta)$
- **GJR and EGARCH**
  - GJR with a symmetric z_t: persistence $\alpha + \gamma/2 + \beta$, because $\Pr(\varepsilon < 0) = 1/2$
  - EGARCH: $\ln \sigma_t^2$ follows an autoregression with coefficient β; the persistence is β

(Engle, 2001, JEP; Bollerslev, 1986, JOE)

Appendix C: all six models, both units

Model	log-L, %	AIC, %	BIC, %	log-L, EUR	AIC, EUR	BIC, EUR
GARCH, normal	−12 360.3	24 726.6	24 744.5	−12 274.7	24 555.4	24 573.2
GARCH, Student	−12 094.5	24 197.0	24 220.8	−12 209.4	24 426.8	24 450.7
GJR, normal	−12 329.4	24 666.8	24 690.7	−12 246.6	24 501.3	24 525.1
GJR, Student	−12 087.2	24 184.3	24 214.1	−12 185.9	24 381.9	24 411.7
EGARCH, normal	−12 329.8	24 667.6	24 691.4	−12 243.1	24 494.2	24 518.0
EGARCH, Student	−12 089.7	24 189.4	24 219.2	−12 182.1	24 374.1	24 403.9

- log-L = log-likelihood $\hat{\ell}$; AIC $= 2k - 2\hat{\ell}$, BIC $= k \ln T - 2\hat{\ell}$, k = number of parameters; 2 853 days, 1 Jan 2019 – 31 Oct 2026.
- Asymmetry $\hat{\gamma}$ and $\hat{\nu}$: GJR-t 0.225, 4.41 (%); −0.127, 6.50 (EUR/MWh).

(Glosten, Jagannathan & Runkle, 1993; Nelson, 1991)

Appendix D: the Hamilton filter and the robustness of QLIKE

- **Hamilton filter** (Markov switching)
  - Predict: $\Pr(s_t = j \mid \mathcal{F}_{t-1}) = \sum_i p_{ij} \Pr(s_{t-1} = i \mid \mathcal{F}_{t-1})$
  - Update: multiply by the normal density of u_t in regime j and normalise; the log of the normaliser summed over t is the log-likelihood
  - Smoothed probabilities (the chart) use all the data, filtered ones only the past; a real-time warning must use the filtered ones
- **Why QLIKE**
  - With $\mathbb{E}_{t-1}[\varepsilon_t^2] = \sigma_t^2$, the expected QLIKE is minimised only at $\hat{\sigma}_t^2 = \sigma_t^2$, whatever the noise of ε_t^2
  - The same holds for MSE; losses such as $|\varepsilon_t| - \hat{\sigma}_t$ or ln-errors can rank models wrongly (Patton, 2011)
  - DM statistic: mean daily loss difference over its Newey–West standard error (7 lags), asymptotically normal

(Hamilton, 1989; Patton, 2011, JOE; Diebold & Mariano, 1995)