








Energy Markets Quantitative Analysis

Lecture 4: Risk Management for Energy Prices



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Learning objectives

- Measure the price risk of an energy position
  - Value-at-risk (VaR) and expected shortfall (ES) as quantiles of the loss distribution
  - Why the normal curve understates crisis days (promised in Lecture 1)
- Estimate VaR and ES in four ways on Romanian prices
  - Historical simulation (HS), normal and Student-t, filtered historical simulation (FHS), extreme value theory (EVT)
- Check a risk model after the fact (backtesting)
  - Kupiec and Christoffersen tests, the Basel traffic light, a test for ES
- Turn quantiles into decisions
  - Position limits, exchange margins and the day-ahead guarantee, for a supplier, a producer and a trader

Abbreviations (1/2)

APC	anti-procyclicality tools of a CCP
AUC	area under the curve (0.5 = no skill, 1 = perfect)
BCBS	Basel Committee on Banking Supervision
CARDI	carbon risk dynamic indicator
CCP	central counterparty (clearing house)
CEE	Central and Eastern Europe
CET	Central European Time
CFaR	cash-flow-at-risk
CISS	Composite Indicator of Systemic Stress (ECB)
DE-LU	Germany–Luxembourg bidding zone
ECB	European Central Bank
EFRM	Energy Financial Risk Meter
EMIR	European Market Infrastructure Regulation (EU) No 648/2012
ENTSO-E	European Network of Transmission System Operators for Electricity
EPEX	EPEX SPOT power exchange
ES	expected shortfall
EU	European Union
EUR	euro
EVT	extreme value theory
FHS	filtered historical simulation
FRM	Financial Risk Meter
FRTB	Fundamental Review of the Trading Book (Basel market-risk rules)

Abbreviations (2/2)

GARCH	generalised autoregressive conditional heteroskedasticity
GPD	generalised Pareto distribution
HS	historical simulation
ICE	Intercontinental Exchange (ICE Futures Europe)
IM	initial margin
ISE	Fraunhofer Institute for Solar Energy Systems
LASSO	least absolute shrinkage and selection operator
LR	likelihood ratio
MLE	maximum likelihood estimation
MW	megawatt
MWh	megawatt-hour
OPCOM	Romanian electricity and gas market operator
POT	peaks over threshold
PZU	Piața pentru Ziua Următoare (day-ahead market)
QR	quantile regression
QRA	quantile regression averaging (Lecture 2)
RO	Romania (bidding zone)
RTS	regulatory technical standards
TTF	Title Transfer Facility (Dutch gas hub)
USD	United States dollar
VaR	value-at-risk
VM	variation margin

Data sources: prices and positions analysed

Series	Frequency	Source
Day-ahead base price of Romania (RO), Piața pentru Ziua Următoare (PZU), and of the Germany–Luxembourg zone (DE-LU)	daily, hourly	the Romanian market operator (OPCOM) and the European Power Exchange (EPEX SPOT), via energy-charts.info (Fraunhofer ISE) and the European Network of Transmission System Operators for Electricity (ENTSO-E)
Romanian load (shape of a supplier's portfolio)	hourly	ENTSO-E (Transelectrica data), via energy-charts.info
Brent crude oil futures, front month (a margined contract)	daily	Yahoo Finance (BZ=F)
Dutch gas hub price, Title Transfer Facility (TTF)	monthly	World Bank, Pink Sheet

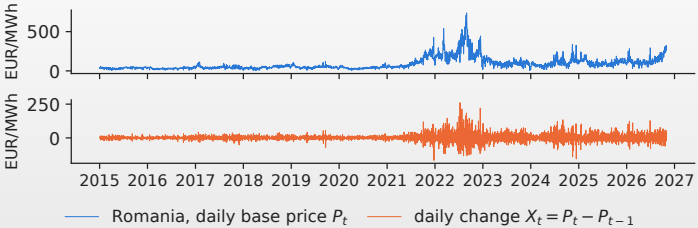
- **Period**
  - 1 Jan 2015 – 31 Oct 2026 (last complete month); downloaded 4 Nov 2026; daily price = mean of the delivery day in Central European Time (CET)
  - 5 missing days of the Romanian price stay missing; changes next to them are not computed
- **Backtest**
  - Every day from 1 Jan 2017 to 31 Oct 2026 (3 587 valid days), each forecast made with data up to the previous day

Experiment yourself: notebook §1

Outline

- Price risk: losses, quantiles, VaR and ES
- Estimating VaR and ES: four methods
- Backtesting VaR and ES
- Margins, guarantees and position limits
- Price risk for suppliers and producers
- Case study: the price rise of autumn 2026

Daily price changes before and after 2021



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E; own estimates. Data: daily, 2015-01-01 to 2026-10-31 (CET delivery days).

- $X_t = P_t - P_{t-1}$ = change of the daily base price, EUR/MWh
  - Prices can be negative, so we use changes, not log returns
  - Largest rise 260.77 (11 Jul 2022), largest fall −166.59 EUR/MWh (24 Dec 2021)

- **In practice:** a supplier that buys 100 MW flat on the PZU and budgets at yesterday's price loses 2,400 EUR for every EUR/MWh of rise.

(OPCOM; EPEX SPOT; own computation)

EMQA_L4_price_changes

Value-at-risk is a quantile of tomorrow's loss

- **In one sentence:** VaR is the loss that is exceeded only on a small, chosen share of days, e.g. one day in a hundred.

$$\underbrace{L_t = Q \cdot 24 \cdot X_t}_{\text{loss, EUR}}, \qquad \underbrace{\mathrm{VaR}_{\alpha,t} = \min\{\ell :\ \Pr(L_t \le \ell \mid \mathcal{F}_{t-1}) \ge \alpha\}}_{\text{the } \alpha\text{-quantile of the loss}}$$

- **where**
  - Q = position in MW (100 MW flat: $Q \cdot 24 = 2{,}400$ MWh per day)
  - X_t = price change, EUR/MWh
  - α = confidence level, 0.95 or 0.99
  - $\mathcal{F}_{t-1}$ = information up to day $t-1$ (prices known at 12:00 CET, gate closure).

- **On our data**, last 365 days
  - 99% quantile of X_t: 86.63 EUR/MWh, i.e. $\mathrm{VaR}_{0.99} = 207\,922$ EUR for 100 MW
  - 95% quantile: 50.39 EUR/MWh
- **What VaR does not say**
  - How large the loss is on the worst 1% of days
  - For that we need the ES, next slide

- **In practice:** the desk reports the 99% VaR every morning, before bidding at 12:00 CET.

(Jorion, 2007; McNeil, Frey & Embrechts, 2015)

Experiment yourself: notebook §2

Expected shortfall: the average loss beyond VaR

- **In one sentence:** When a bad day comes, how large is the loss on average?

$$\mathrm{ES}_{\alpha,t} = \mathbb{E}\left[L_t \mid L_t \geq \mathrm{VaR}_{\alpha,t}\right] = \frac{1}{1-\alpha}\int_{\alpha}^{1} \mathrm{VaR}_{u,t}\, du$$

$$\underbrace{\mathrm{ES}(L_1 + L_2) \leq \mathrm{ES}(L_1) + \mathrm{ES}(L_2)}_{\text{subadditivity: diversification never adds risk}}$$

- **where**
  - ES is coherent, VaR may break subadditivity
  - The Fundamental Review of the Trading Book (FRTB) uses ES at 97.5% instead of VaR at 99%.

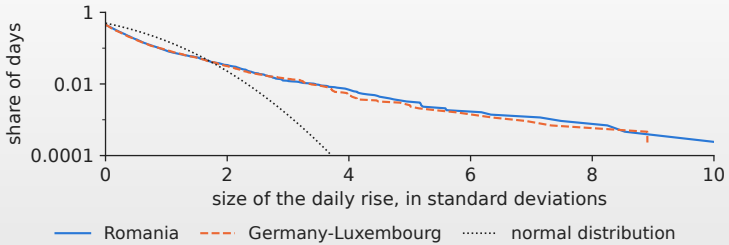
- **On our data**, last 365 days, EUR/MWh
  - $\mathrm{VaR}_{0.975}$ 61.96, $\mathrm{ES}_{0.975}$ 86.91 (mean of the 10 worst days)
- **Romania + Germany–Luxembourg**, 100 MW in each
  - 99% VaR: 86.63 + 91.76 separately, 150.77 together
  - Correlation 0.55: the joint risk is smaller

- **In practice:** size the cash buffer with ES, not with VaR.

(Artzner et al., 1999, Math. Finance; Acerbi & Tasche, 2002, JBF; BCBS, 2019)

Experiment yourself: notebook §2

Large price rises are far more frequent than the normal curve allows



Source: energy-charts.info (Fraunhofer ISE); OPCOM (RO) and EPEX SPOT (DE-LU) day-ahead prices, via ENTSO-E. Data: daily changes, 2015-01-01 to 2026-10-31.

- Excess kurtosis 12.0 in Romania, 12.1 in Germany–Luxembourg (0 for a normal curve)
  - Rises above 3 standard deviations: 52 days observed vs 5.8 expected under normality
  - The largest rise is 10.1 standard deviations: under normality, once in more than 10^{20} years

- **In practice:** a limit built on the normal curve understates how often a crisis day comes (Lecture 1).

(own computation, 1 Jan 2015 – 31 Oct 2026)

EMQA_L4_tails

A normal-curve limit set in 2020 was breached 19 times more often than planned in 2021–2022

In one sentence: A static limit assumes that tomorrow's risk looks like the past; in a crisis it does not.

$$\underbrace{\ell_{0.99} = \hat{\mu} + z_{0.99}\,\hat{\sigma},}_{\text{normal 99\% limit}} \qquad z_{0.99} = 2.326$$

where

- $\hat{\mu}$, $\hat{\sigma}$ = mean and standard deviation of X_t in 2015–2020
- $z_{0.99}$ = 99% quantile of the standard normal distribution
- A correct limit is exceeded on 1% of days.

The limit, from 2 182 days of 2015–2020

- $\hat{\sigma} = 10.21$, limit = 23.77 EUR/MWh
- Already in 2015–2020: 45 breaches vs 21.8 expected

Breaches afterwards

- 2021–2022: 139 vs 7.3 expected
- 2023–2026: 226 vs 14.0 expected (16 times more)

In practice: a limit must move with volatility

- The next section shows four ways to make it move.

(own computation; Lecture 1)

Experiment yourself: notebook §2

Outline

- Price risk: losses, quantiles, VaR and ES
- Estimating VaR and ES: four methods
  - Historical simulation and parametric models
  - Filtered historical simulation and extreme values
  - Comparing the methods
- Backtesting VaR and ES
- Margins, guarantees and position limits
- Price risk for suppliers and producers
- Case study: the price rise of autumn 2026

Historical simulation with a one-year window

⊡ **In one sentence:** Sort the last 365 daily changes; the 4th largest is the 99% VaR.

$$\widehat{\mathrm{VaR}}^{\mathrm{HS}}_{\alpha,t} = \hat{q}_\alpha\left(X_{t-W},\ldots,X_{t-1}\right)$$
$$\widehat{\mathrm{ES}}^{\mathrm{HS}}_{\alpha,t} = \text{mean of } \{X_s \geq \widehat{\mathrm{VaR}}^{\mathrm{HS}}_{\alpha,t}\}$$

⊡ **where**

- $W = 365$ days (the window)
- $\hat{q}_\alpha$ = empirical quantile
- No distribution is assumed.

⊡ **On our data**, October 2026

- $\mathrm{VaR}_{0.99}$ 86.63, $\mathrm{ES}_{0.975}$ 87.59 EUR/MWh
- 2017–2026: exceeded on 2.04% of days, not 1%

⊡ **Weakness**

- A new regime needs months to fill the window
- A crisis day counts for 365 days, then drops out at once

⊡ **In practice:** HS is the default in many energy companies, and it lags the market.

(Jorion, 2007; Christoffersen, 2012)

Experiment yourself: notebook §3

Parametric VaR: normal and Student-t distributions

In one sentence: Fit a curve to the last year of changes and read its quantile; the Student-t curve has fatter tails.

$$\mathrm{VaR}_\alpha^N = \mu + \sigma z_\alpha, \qquad \mathrm{ES}_\alpha^N = \mu + \sigma \frac{\varphi(z_\alpha)}{1-\alpha}$$

$$\mathrm{VaR}_\alpha^t = m + s\, t_{\nu,\alpha}, \qquad \mathrm{ES}_\alpha^t = m + s \frac{g_\nu(t_{\nu,\alpha})}{1-\alpha} \cdot \frac{\nu + t_{\nu,\alpha}^2}{\nu - 1}$$

where

- μ, σ = mean, standard deviation
- z_α, φ = normal quantile, density
- m, s, ν = location, scale, degrees of freedom, by maximum likelihood estimation (MLE)
- $t_{\nu,\alpha}$, g_ν = Student-t quantile, density (Appendix A).

On our data, October 2026, EUR/MWh

- Normal: $\mathrm{VaR}_{0.99}$ 69.64, $\mathrm{ES}_{0.975}$ 69.98
- Student-t: $\mathrm{VaR}_{0.99}$ 76.66, $\mathrm{ES}_{0.975}$ 80.05

Backtest, days above the 99% VaR

- Normal 3.21%, Student-t 1.98% (target 1%)
- At 95% the t curve is too narrow: 8.75%

In practice: the normal VaR is the easiest to explain and the most dangerous to trust.

(McNeil, Frey & Embrechts, 2015)

Experiment yourself: notebook §3

Filtered historical simulation: past shocks rescaled to current volatility

- **In one sentence:** Divide past changes by their volatility, then rescale the shocks by today's volatility.

$$e_t = X_t - \mu_{d(t)}, \quad \sigma_t^2 = \omega + a\, e_{t-1}^2 + b\, \sigma_{t-1}^2, \quad z_t = e_t/\sigma_t$$

$$\mathrm{VaR}_{\alpha,t}^{\mathrm{FHS}} = \mu_{d(t)} + \sigma_t\, \hat{q}_\alpha(z)$$

- **where**
  - $\mu_{d(t)}$ = mean change of that weekday (prices fall at weekends)
  - σ_t = generalised autoregressive conditional heteroskedasticity (GARCH) volatility, Lecture 3, refitted every 21 days on 730 days
  - ES: σ_t times the mean of z above $\hat{q}_\alpha(z)$.

- **On our data**, GARCH(1,1)
  - σ_t from 4.06 (29 Mar 2021) to 100.86 EUR/MWh (12 Jul 2022)
  - October 2026: $\mathrm{VaR}_{0.99}$ 67.50 EUR/MWh
- **Backtest**
  - 99%: 1.51% of days
  - 95%: 5.58%, Kupiec $p = 0.120$

- **In practice:** FHS raises the limit within days of a volatility jump
  - HS needs months.

(Barone-Adesi, Giannopoulos & Vosper, 1999; Bollerslev, 1986)

Experiment yourself: notebook §3

Extreme value theory: model only the tail, with a generalised Pareto curve

- **In one sentence:** Above a high threshold, the excesses of almost any distribution follow the same two-parameter curve.

$$\Pr(Z - u \le y \mid Z > u) \approx 1 - \left(1 + \frac{\xi y}{\beta}\right)^{-1/\xi}$$

$$\mathrm{VaR}_\alpha = u + \frac{\beta}{\xi}\left[\left(\frac{n}{N_u}(1-\alpha)\right)^{-\xi} - 1\right], \quad \mathrm{ES}_\alpha = \frac{\mathrm{VaR}_\alpha + \beta - \xi u}{1 - \xi}$$

- **where**
  - Peaks over threshold (POT): $u = 90\%$ quantile of z, exceeded by N_u of n residuals
  - Generalised Pareto distribution (GPD): shape ξ (tail heaviness), scale β, by MLE
  - Then multiplied by σ_t as in FHS (Appendix B).

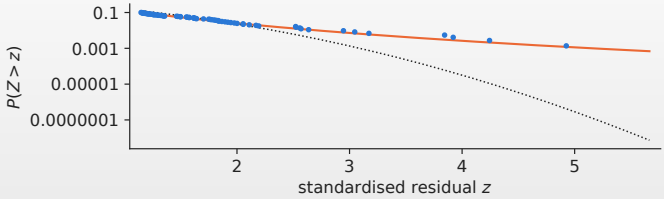
- **Raw changes**, threshold = 95% quantile
  - $\hat{\xi} = 0.164$: a heavy tail
  - Moments above order 6.1 do not exist
- **After the GARCH filter**
  - Median $\hat{\xi} = 0.011$: the filter absorbs most of the tail
  - 99% backtest: 1.37% of days, the best of five

- **In practice:** use EVT for the one-in-ten-years day that no window contains.

(Pickands, 1975; McNeil & Frey, 2000; Chan & Gray, 2006, IJF)

Experiment yourself: notebook §3

Above the threshold the Pareto curve fits the data; the normal curve does not



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E; own estimates. Data: GARCH-filtered changes, the 730 days before 2026-10-31.

- Last refit (11 Oct 2026): $\hat{\xi} = 0.182$, $\hat{\beta} = 0.569$
  - $\hat{\xi} > 0$ in 53% of the refits: the standardised tail is sometimes heavy, sometimes not

- **In practice:** a 4-sigma day is rare but not impossible
  - The normal curve calls it impossible.

(Balkema & de Haan, 1974; McNeil & Frey, 2000)

EMQA_L4_gpd_tail

VaR and ES on the same day by five methods

Method	$\text{VaR}_{0.95}$	$\text{VaR}_{0.99}$	$\text{ES}_{0.975}$	$\text{VaR}_{0.99}$, kEUR	$\text{ES}_{0.975}$, kEUR
HS	50.38	86.63	87.59	208	210
Normal	49.33	69.64	69.98	167	168
Student-t	45.77	76.66	80.05	184	192
FHS	36.22	67.50	67.44	162	162
EVT	35.66	64.33	68.54	154	164

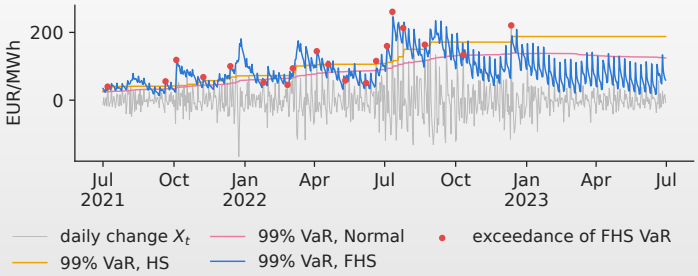
- Mean of the 31 forecasts made for October 2026; first three columns in EUR/MWh; last two for 100 MW flat (2,400 MWh per day).
- **Reading the table**
  - HS and Student-t still carry the volatile days of the past year
  - FHS and EVT use today's volatility, below the yearly average in October 2026

- **In practice:** report two methods side by side and explain the gap.

(own estimates)

Experiment yourself: notebook §3

In the crisis FHS adapted within days; HS and the normal VaR lagged



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E; own estimates. Data: forecasts for 2021-07-01 to 2023-06-30.

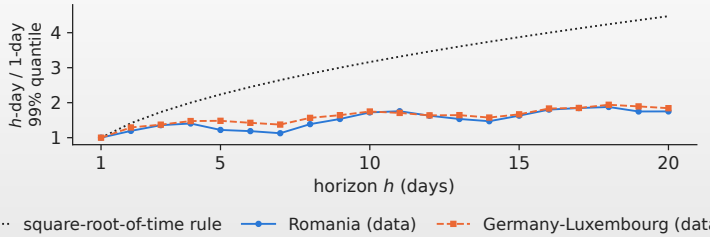
- Exceedances of the 99% VaR, 1 Jul 2021 – 30 Jun 2023 (730 days, 7.3 expected)
  - HS 28, normal 45, Student-t 29, FHS 19, EVT 17

- **In practice:** in a crisis, a model that reacts within days protects the cash.

(own estimates)

EMQA_L4_var_paths

The square-root-of-time rule overstates the 10-day risk



Source: energy-charts.info (Fraunhofer ISE); OPCOM (RO) and EPEX SPOT (DE-LU) day-ahead prices, via ENTSO-E. Data: daily base prices, 2015-01-01 to 2026-10-31.

- 10-day 99% quantile $= 1.72 \times$ the 1-day one (RO; 1.75 in DE-LU), not $\sqrt{10} = 3.16$
  - Variance ratio at 10 days 0.33 (1 = random walk); autocorrelation of X_t: -0.076 at lag 1, 0.474 at lag 7

- **In practice:** for a 10-day horizon, measure 10-day changes directly
  - Multiplying by $\sqrt{10}$ wastes capital.

(own computation; Weron, 2014, IJF)

EMQA_L4_horizon

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Kupiec test of the exceedance frequency

- **In one sentence:** A 99% VaR promises to be exceeded on 1% of days; count the exceedances and test whether the share differs from 1%.

$$I_t = \mathbf{1}\{L_t > \mathrm{VaR}_{\alpha,t}\}, \qquad x = \sum_{t=1}^{n} I_t, \qquad \hat{p} = x/n$$

$$\mathrm{LR}_{uc} = -2 \ln \frac{(1-p)^{n-x}\, p^{x}}{(1-\hat{p})^{n-x}\, \hat{p}^{\,x}} \;\sim\; \chi^2_1$$

- **where**
  - I_t = exceedance indicator
  - $p = 1 - \alpha$ = promised share
  - Likelihood ratio (LR) test of unconditional coverage
  - Reject at 5% if $\mathrm{LR}_{uc} > 3.84$.

- **On our data**, 99% VaR, 3 587 days
  - Expected 35.9 exceedances
  - Normal 115, LR_{uc} 111.5; FHS 54, LR_{uc} 8.0
  - EVT 49, $p = 0.037$
- **Power of the test**
  - With one year at 99% we expect only 2–3 exceedances; the test can hardly reject
  - Ten years of daily data make it sharp

- **In practice:** ask for the exceedance count before you trust anyone's VaR.

(Kupiec, 1995, J. Derivatives)

Experiment yourself: notebook §4

Christoffersen test of independent exceedances

- **In one sentence:** A good model is not surprised twice in a row: an exceedance today should not make one tomorrow more likely.

$$\pi_{01} = \Pr(I_t = 1 \mid I_{t-1} = 0), \qquad \pi_{11} = \Pr(I_t = 1 \mid I_{t-1} = 1)$$

$$\mathrm{LR}_{ind} = -2 \ln \frac{(1-\hat{\pi})^{n_{00}+n_{10}} \hat{\pi}^{\,n_{01}+n_{11}}}{(1-\hat{\pi}_{01})^{n_{00}} \hat{\pi}_{01}^{\,n_{01}} (1-\hat{\pi}_{11})^{n_{10}} \hat{\pi}_{11}^{\,n_{11}}} \sim \chi^2_1$$

- **where**
  - n_{ij} = days with $I_{t-1} = i$, $I_t = j$
  - $\hat{\pi}$ = overall share
  - H_0: $\pi_{01} = \pi_{11}$
  - Joint test $\mathrm{LR}_{cc} = \mathrm{LR}_{uc} + \mathrm{LR}_{ind} \sim \chi^2_2$.

- **On our data**, 99% VaR, independence p-values
  - HS 0.682, normal 0.868, FHS 0.199
  - No method fails it: exceedances rarely fall on consecutive days
- **But they cluster in months**
  - Normal: 7.26% of days in 2021–2022, 2.33% in 2017–2020
  - A first-order test misses it; the traffic light sees it

- **In practice:** look at the timing of exceedances, not only at their number.

(Christoffersen, 1998, IER)

Experiment yourself: notebook §4

Over ten years only the filtered methods pass at 95%; at 99% every method has too many exceedances

Romania	95%: share	Kupiec p	99%: count	share, %	Kupiec p	Christoffersen p	red days, %
HS	6.16	0.002	73	2.04	<0.001	0.682	16.7
Normal	7.36	<0.001	115	3.21	<0.001	0.868	29.5
Student-t	8.75	<0.001	71	1.98	<0.001	0.630	18.3
FHS	5.58	0.120	54	1.51	0.005	0.199	6.2
EVT	5.55	0.139	49	1.37	0.037	0.244	2.8

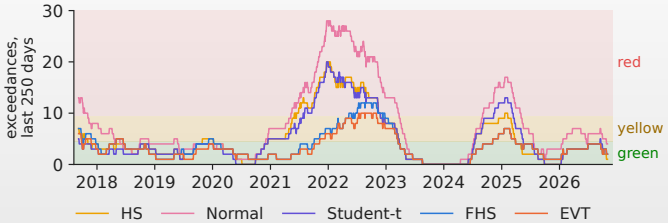
- Backtest 1 Jan 2017 – 31 Oct 2026; expected: 5% and 1% of 3 587 days (35.9 at 99%); green = close to the target, red = far.
- Red days = share of days whose last 250 days held 10 or more exceedances (next slide); Germany–Luxembourg: Appendix D.

- **In practice:** accept a model at 95%, but hold extra capital for the 99% tail that none of them captures fully.

(Kupiec, 1995; Christoffersen, 1998; own estimates)

Experiment yourself: notebook §4

The Basel traffic light: the normal VaR spent 30% of days in the red zone



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E; own estimates. Data: backtest 2017-01-01 to 2026-10-31, 99% VaR.

- Count of 99% exceedances in the last 250 days: green 0–4, yellow 5–9, red 10 or more by the Basel Committee on Banking Supervision (BCBS)
  - A correct model is green with probability 89.2%, red with 0.025%
  - Red days: HS 16.7%, FHS 6.2%, EVT 2.8%

- **In practice:** yellow means recalibrate, red means stop using the model for limits.

(BCBS, 1996)

EMQA_L4_traffic

Exceedance rates by method in 2021–2022 and since 2023

99% VaR, % of days	RO 17–20	RO 21–22	RO 23–26	DE-LU 17–20	DE-LU 21–22	DE-LU 23–26
HS	1.58	4.93	1.00	1.16	3.84	1.07
Normal	2.33	7.26	2.00	2.40	5.89	2.07
Student-t	1.30	4.66	1.29	1.37	3.97	1.29
FHS	1.37	2.88	0.93	1.44	2.47	1.36
EVT	1.24	2.47	0.93	1.51	2.19	1.07

- **The crisis**
  - Even FHS was exceeded on 2.88% of days in Romania: the gas shock moved the whole distribution
- **After the crisis**
  - FHS 0.93%, EVT 0.93%, HS 1.00%, normal 2.00%

- **In practice:** test every model on 2021–2022 before you trust it in a crisis.

(own estimates)

Experiment yourself: notebook §4

Backtesting ES: the size of losses beyond VaR

⊡ **In one sentence:** Divide each exceedance loss by its ES forecast; if ES is right, the ratios average 1.

$$Z_2 = \frac{1}{T(1-\alpha)} \sum_{t=1}^{T} \frac{L_t\, I_t}{\mathrm{ES}_{\alpha,t}} - 1, \qquad \mathbb{E}_{H_0}[Z_2] = 0$$

⊡ **where**

- $\alpha = 0.975$
- I_t = exceedance of $\mathrm{VaR}_{0.975}$
- $Z_2 > 0$ = ES too small
- p-value from 2,000 paths simulated from each model (Appendix C).

Z_2	all	p	2021–22	p	2023–26	p
HS	0.628	<0.001	2.694	<0.001	−0.026	0.761
Normal	1.425	<0.001	4.170	<0.001	0.697	<0.001
Student-t	0.908	<0.001	2.910	<0.001	0.329	0.052
FHS	0.335	0.004	1.164	<0.001	−0.083	0.759
EVT	0.279	0.009	1.183	<0.001	−0.114	0.722

⊡ **Reading**: every ES was too small in the crisis

- Since 2023, FHS, EVT and HS pass.

⊡ **In practice:** judge ES on several years, never on one quarter (about 9 tail days a year).

(Acerbi & Székely, 2014; Pele & Mazurencu-Marinescu-Pele, 2026)

Experiment yourself: notebook §4

AIDA 2026: a conformal shift measures and corrects the miscalibration of VaR

- **What the work did** (D. T. Pele, AIDA (AI and Digital Assets in Energy and Finance) conference, 4 June 2026)
  - Score $s_t = \hat{q}_t - r_t$ (> 0 on an exceedance day); $\hat{q}_V = 99\%$ quantile of past scores; corrected VaR = VaR + $\hat{q}_V$, fixed (audit) or rolling over 250 days
  - 1% VaR of 10 models (foundation models, GARCH-type, HS) on 24 assets, incl. oil and gas: 240 pairs; Kupiec, Christoffersen, traffic light
- **What it found**
  - $\hat{q}_V$ from -0.007 (too prudent) to $+0.102$ (no tail signal); it explains 53.4% (partial R^2) of the gain in quantile score
  - Green-zone pairs: 63/240 (26.2%) raw, 203/240 (84.6%) fixed, 234/240 (97.5%) rolling
  - The exceedance rate is restored, their clustering is not
- **Link to this lecture**
  - Kupiec says whether VaR fails, $\hat{q}_V$ by how much; our guarantee widening $\hat{c}_t$ (Section 4) is the same shift
- Findings as stated by the authors; talk: slides; paper submitted.

In practice: after a failed backtest, widen VaR by the conformal shift and keep testing for clusters.

(Pele, Bolovăneanu, Ginavar, Lessmann & Härdle, AIDA, ASE, 4 June 2026)

Outline

- Price risk: losses, quantiles, VaR and ES
- Estimating VaR and ES: four methods
- Backtesting VaR and ES
- Margins, guarantees and position limits
  - Exchange margins
  - Guarantees and limits
- Price risk for suppliers and producers
- Case study: the price rise of autumn 2026

Exchange margins: initial margin as a VaR, variation margin paid daily

- **In one sentence:** The clearing house holds a deposit that covers a 99% two-day loss and settles gains and losses in cash every evening.

$$\underbrace{\mathrm{IM}_t = KF_t\,\hat{q}_{0.99}\left(|r^{(2)}|\right)}_{\text{initial margin}}, \qquad \underbrace{\mathrm{VM}_t = nK(F_t - F_{t-1})}_{\text{variation margin}}$$

- **where**
  - Initial margin (IM), variation margin (VM), set by the central counterparty (CCP)
  - F_t = futures price
  - K = 1,000 barrels per Brent lot
  - $r^{(2)}$ = two-day return
  - n = lots sold (+) or bought (−).

- **European Market Infrastructure Regulation (EMIR)**, regulatory technical standards (RTS) 153/2013
  - Futures: 99%, 2-day liquidation, at least 12 months of data (Art. 24–26)
  - Anti-procyclicality (APC), Art. 28: a 25% buffer, or 25% weight on stress, or a 10-year floor
- **Intercontinental Exchange (ICE) Brent**, 100 lots sold: largest one-day VM 0.88 million USD (16 Sep 2019)

- **In practice:** a hedged producer gains on the physical sale later, but pays the margin today.

(EU Delegated Regulation 153/2013; ICE Futures Europe)

Experiment yourself: notebook §5

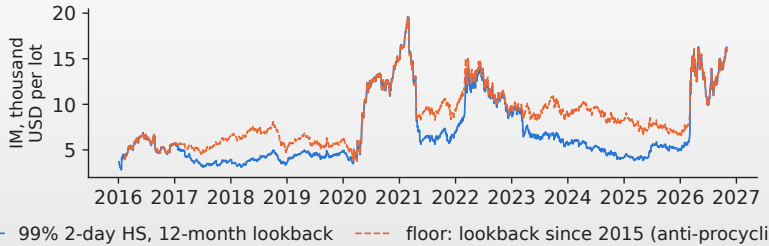
Margin calls and liquidity shortfalls of hedged energy companies in 2022

- **What happened**, according to the European Central Bank (ECB), November 2022
  - TTF gas futures reached 227 EUR/MWh on 7 March 2022, about 12 times the level of early 2021
  - By mid-2022 about twice as much initial margin was posted as in late 2021
  - The firms hurt were mostly utilities and producers that had sold forward: hedgers, not speculators
- **How the European Union (EU) responded**
  - Delegated Regulation (EU) 2022/2311: public guarantees accepted as collateral until 29 November 2023
- **In Romania**, on the day-ahead market
  - OPCOM accepts a buy order at a positive price only up to the participant's guarantee (a bank letter)

- **In practice:** the risk that sinks a hedged company is liquidity, not the price itself.

(ECB, 2022; Delegated Regulation (EU) 2022/2311; OPCOM, 2025)

Margins are procyclical: in 2026 the Brent margin rose 2.7-fold in two months



Source: Yahoo Finance (ICE Brent front-month BZ=F); own computation. Data: trading days 2016-01-01 to 2026-10-30; contract-roll days dropped; 1 lot = 1,000 bbl.

- Brent rose from 72.48 USD (27 Feb 2026) to 118.35 USD (31 Mar 2026)
  - IM with 12 months of history: 5 970 → 16 313 USD per lot; with the floor since 2015: ×2.07 instead of ×2.73
  - Losses beyond IM: 1.80% of days with 12 months, 0.66% with the floor (target 1%)

In practice: plan your cash for the margin a crisis would demand, not for today's margin.

(own computation; EMIR RTS, Art. 28; ICE Brent) EMQA_L4_brent_im

The day-ahead guarantee is set from a quantile forecast of tomorrow's price

In one sentence: To be sure its order clears, a supplier bids up to a high price, and the guarantee must cover price times volume.

$$G_t \;\geq\; Q \cdot 24 \cdot \hat{q}_{\tau,t}, \qquad \hat{q}_{\tau,t} = \beta_0 + \beta_1 P_{t-1} + \beta_2 P_{t-7} + \sum\nolimits_j \delta_j D_{j,t}$$

$$\tilde{q}_{\tau,t} = \hat{q}_{\tau,t} + \hat{c}_t, \qquad \hat{c}_t = \hat{q}_\tau\left(P_s - \hat{q}_{\tau,s},\ s = t-365, \ldots, t-1\right)$$

where

- G_t = guarantee, EUR
- $\hat{q}_{\tau,t}$ = quantile regression (QR) on the last 365 days, as in quantile regression averaging (QRA), Lecture 2
- $D_{j,t}$ = weekday dummies
- $\hat{c}_t$ = conformal correction from past errors.

On our data, $\tau = 0.99$, 2017–2026

- Days with the price below the forecast: QR 96.90%, widened 98.47%, FHS 98.49%

Cost, 100 MW flat

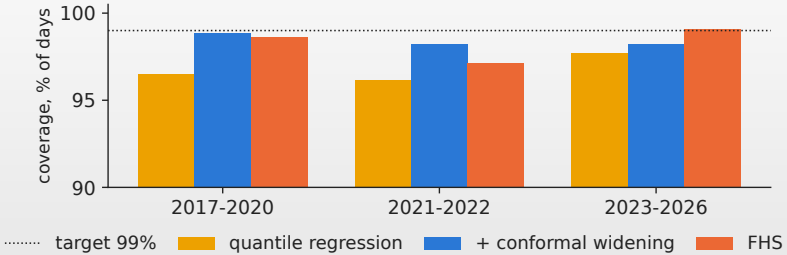
- 0.371 million EUR per day on average, +7.5% for the widening
- 2017–2020: 0.172; 2021–2022: 0.649 million EUR per day

In practice: size the bank line behind the guarantee on the crisis quantile, before the crisis.

(OPCOM, 2025; Koenker & Bassett, 1978; Romano et al., 2019)

Experiment yourself: notebook §6

Heavier tails bring the 99% quantile close to its nominal coverage



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E; own estimates. Data: forecasts 2017-01-01 to 2026-10-31, 99% quantile.

- Kupiec p at 99%: QR <0.001, widened 0.003, FHS 0.005
  - At 95% the widened QR passes ($p = 0.053$), as Lecture 2 suggested for the tails

In practice: a forecast used for collateral must be calibrated in the tail, not only accurate on average.

(Nowotarski & Weron, 2018, RSER; Romano et al., 2019)

EMQA_L4_coverage

A position limit in MW should shrink when VaR grows

- **In one sentence:** The board sets a daily risk budget in EUR; the desk divides it by today's VaR per MW.

$$Q_t^{\max} = \frac{B}{24 \cdot \mathrm{VaR}_{0.99,t}^{\mathrm{FHS}}}$$

- **where**
  - B = daily VaR budget, here 250,000 EUR (an assumption for the example)
  - VaR in EUR/MWh
  - $Q_t^{\max}$ in MW of open flat position.

- **On our data**, FHS 99% VaR
  - 2019 (median): VaR 20.27 EUR/MWh, limit 514 MW
  - 12 Jul 2022: VaR 245.80, limit 42 MW
  - 31 Oct 2026: VaR 53.15, limit 196 MW
- **Why it matters**
  - A fixed MW limit lets the money at risk grow with volatility
  - A fixed EUR budget keeps it constant

- **In practice:** set limits in EUR of VaR and convert them to MW every day.

(own estimates)

Experiment yourself: notebook §6

Outline

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A supplier's monthly cost-at-risk under the normal distribution

In one sentence: A supplier prices next month at this month's cost; the risk is a jump of the load-weighted price.

$$\bar{P}^L_m = \frac{\sum_h P_h L_h}{\sum_h L_h}, \qquad o_m = \frac{\bar{P}^L_m}{\bar{P}^L_{m-1}} - 1$$
$$\mathrm{CFaR}_{0.95} = \hat{q}_{0.95}(o_m)$$

where

- P_h = hourly price
- L_h = Romanian load (portfolio shape)
- o_m = cost overrun
- Cash-flow-at-risk (CFaR) = its quantile
- In EUR for 100 MW on average.

On our data, 140 months, Mar 2015 – Oct 2026

- Normal 99% limit (2015–2020): +50.5%, exceeded 3 times since 2021 (0.7 expected)
- Worst month Nov 2024: +86.6%, 5.96 million EUR

Empirical, all months

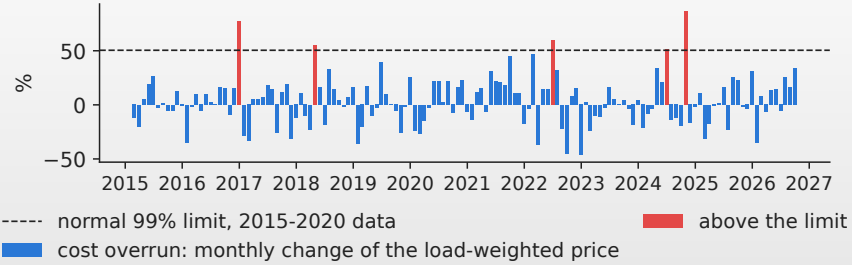
- $\mathrm{CFaR}_{0.95}$ = +39.9%, i.e. 2.84 million EUR
- $\mathrm{ES}_{0.95}$ = +60.2%, i.e. 6.49 million EUR

In practice: price a fixed retail offer with an ES-based premium, or hedge it forward (Lecture 5).

(own computation; Assignment 2)

Experiment yourself: notebook §7

Since 2021 the normal 99% monthly limit was exceeded 3 times (0.7 expected)



Source: energy-charts.info (Fraunhofer ISE); OPCOM prices; ENTSO-E load (Transelectrica data for Romania). Data: monthly, 2015-03 to 2026-10.

- Red bars exceed the limit computed in 2020
  - The largest overrun came in Nov 2024, after the summer of 2024.

- **In practice:** re-estimate the monthly limit every quarter and stress it with the 2022 months.

(own computation)

EMQA_L4_supplier_month

Loss tails: the supplier's tail is heavier than the producer's

- **In one sentence:** A supplier loses when the price rises, a producer when it falls; prices jump up more than down.

$$L_t^{\text{buyer}} = +Q\,24\,X_t, \qquad L_t^{\text{seller}} = -Q\,24\,X_t$$

- **where**
  - One distribution of X_t, two tails: upper (supplier), lower (producer selling spot)
  - Skewness of X_t: 1.02.

- **Daily**, EUR/MWh, 1 Jan 2015 – 31 Oct 2026
  - 99% quantile (supplier): +88.21; 1% quantile (producer): −70.74
  - $\text{ES}_{0.975}$: 91.29 (supplier) vs 74.92 (producer)
- **Monthly base price**
  - 95% quantile of the change +42.4%, 5% quantile −32.3%

- **In practice:** the producer's price risk is smaller, but a dry year adds volume risk (next slide).

(own computation)

Experiment yourself: notebook §7

Case study C4: a systemic risk meter and two risks specific to energy companies

- **The paper** (Toma, Dima, Grecu, Pele & Lessmann, 2026)
  - Data and method: 100 European energy companies, 2016–2026; 5% quantile regressions with the least absolute shrinkage and selection operator (LASSO); the mean penalty is the Energy Financial Risk Meter (EFRM)
  - Result: the EFRM captures 2020 and 2022; with the ECB's Composite Indicator of Systemic Stress (CISS) it predicts volatile months with area under the curve (AUC) 0.731
- **Our replication** on three Romanian companies (OMV Petrom, Romgaz, Nuclearelectrica)
  - AUC 0.38, against 0.61 for past volatility: a systemic meter needs many firms, as in the paper
- **Energy-specific risks**
  - Price–volume premium of the household profile: 5.2% of the base price in 2025
  - Dry-year buy-back of a hydro fleet that sold its median output forward: 394 million EUR (2022)

In practice: put the systemic meter, the price–volume premium and the dry-year cost on one dashboard.

(Toma, Dima, Grecu, Pele & Lessmann, 2026, SSRN; case study C4)

AIDA 2026: systemic tail risk in EU energy broke upward in July 2021

- **What the work did** (F.-M. Toma, AIDA conference, 3 June 2026; paper of case study C4)
  - 100 European energy firms, April 2016 – March 2026: each firm's 5% return quantile regressed by LASSO on the other 99 firms, oil, gas and stock indices
  - EFRM = mean LASSO penalty: more links needed to explain the tail = more systemic exposure
- **What it found**
  - EFRM breaks: February 2020 (pandemic) and July 2021 (gas run-up), seven months before the invasion
  - Geopolitical risk moved the EFRM only after the invasion: +1.52 percentage points (time-varying model)
  - Effect on Stoxx 600 stocks about 60% weaker since, on 5-year credit default swaps still positive
  - Volatile months: AUC 0.731 for EFRM + CISS, 0.708 EFRM alone, 0.673 CISS alone
- Findings as stated by the authors; talk: slides; paper: doi:10.2139/ssrn.6985301.

- **In practice:** a firm's own VaR cannot see contagion
  - Read a sector meter next to it.

(Toma, AIDA, ASE, 3 June 2026; Toma, Dima, Grecu, Pele & Lessmann, 2026, SSRN)

AIDA 2026: the tail risk of carbon-heavy firms predicts the low-carbon premium

- **What the work did** (Prof. Wolfgang Karl Härdle, AIDA, 5 June 2026; with R. Wang and L. Zhang)
  - Carbon premium puzzle: do high-carbon firms earn more (risk compensation) or low-carbon firms (green preference)?
  - China, July 2014 – February 2025: 50 high-carbon and 50 low-carbon firms; LASSO regressions as for the EFRM, 63 days, 5% quantile
  - Carbon risk dynamic indicator (CARDI) = Financial Risk Meter (FRM, mean penalty λ), high-carbon / low-carbon firms
- **What it found**
  - High-risk regimes of CARDI coincide with unexpected or early-stage climate-policy events
  - A rising CARDI predicts a higher factor-adjusted low-carbon premium next month (0.022, 118 months); the conditional VaR ratio does not
  - Long low-carbon, short high-carbon only when CARDI rises (2015–2024): 115.84%, Sharpe 0.72 (always on: 89.03%, 0.49)
- **Link to this lecture**: the EFRM of the previous frame, split by carbon intensity
- Findings as stated by the authors; talk: slides; paper: doi:10.2139/ssrn.5757742.

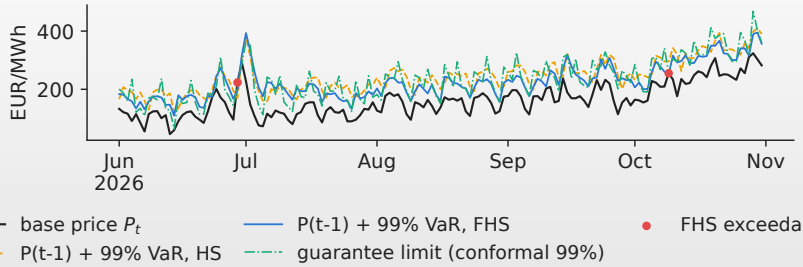
- **In practice:** in an oil and gas firm, watch CARDI: when it rises, investors favour low-carbon peers.

(Wang, Härdle & Zhang, AIDA, ASE, 5 June 2026; 2025, SSRN)

Outline

- Price risk: losses, quantiles, VaR and ES
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Case study: in 2026 the price doubled gradually, without jumps



Source: energy-charts.info (Fraunhofer ISE); OPCOM day-ahead prices for Romania, via ENTSO-E; own estimates. Data: daily, 2026-06-01 to 2026-10-31.

- Monthly mean 122.99 EUR/MWh in June, 239.32 in October; peak 323.76 (29 Oct 2026)
  - 99% VaR exceedances in 153 days (1.5 expected): FHS 2, HS 1, normal 3

In practice: a slow rise passes the VaR tests and still drains cash: watch the price level too.

(own estimates; OPCOM)

EMQA_L4_case2026

Case study: market risk and liquidity needs in 2026

- **Market risk barely moved**
  - FHS 99% VaR: 61.64 EUR/MWh in June, 67.50 in October
  - ES test since 2023, FHS: $Z_2 = -0.083$, not rejected
- **Cash needs doubled**
  - Day-ahead guarantee for 100 MW (widened 99% quantile): 0.416 → 0.776 million EUR per day
  - Orders above the guarantee limit: 3 days (6 without widening)
- **The supplier's monthly bill**, 100 MW
  - August +25.1% (2.38 million EUR), September +16.2% (1.86), October +34.0% (4.68)
  - Each month below the 99% limit of +50.5%, but three in a row
- **Drivers**
  - TTF gas 44.94 → 86.81 EUR/MWh (June → October)
  - Brent 108.34 USD in October; Germany–Luxembourg 169.48 EUR/MWh

In practice: a risk report needs a liquidity line next to VaR: guarantees, margins and the cumulative monthly overrun.

(own estimates; OPCOM; World Bank; Yahoo Finance)

Experiment yourself: notebook §8

A risk desk checklist for energy prices

- **Measure**
  - Define the position and the sign of the loss (buyer or seller)
  - Use only data known at gate closure
  - Report VaR and ES from two methods, one of them filtered (FHS or EVT)
- **Check**
  - Kupiec, Christoffersen and the traffic light on several years, including 2021–2022
  - Z_2 for ES; never rank models on a few tail days
- **Decide**
  - Limits in EUR of VaR, converted daily to MW
  - Guarantees and credit lines from calibrated high quantiles
  - A cash buffer for the margin a crisis would demand
- **Report**
  - In EUR and in MW, by month, with the liquidity next to the market risk

In practice: Assignment 2 (Task 5) applies this checklist to the cash flow of your own role.

(BCBS, 2019; EMIR RTS 153/2013; Assignment 2)

Quiz 1: reading a VaR

- Your 99% one-day VaR for 100 MW is 162 thousand EUR
  - What does it promise, and what does it not say?

Answer

- On about 1 day in 100 the loss should exceed it
  - It says nothing about how large the loss is on those days, which is what ES (162 thousand EUR at 97.5%) measures.

Quiz 2: seven exceedances

- Over the last 250 days your 99% VaR was exceeded 7 times
  - What zone is it in, and what does Kupiec say?

Answer

- Yellow (5–9)
  - $\mathrm{LR}_{uc} = 5.50$, $p = 0.019$: rejected at 5%
  - Recalibrate (e.g. move from HS to FHS) and keep watching.

Quiz 3: ten-day VaR

- Your chief financial officer computes the 10-day VaR as $\sqrt{10}$ times the 1-day VaR
  - Is that prudent or wasteful for power?

Answer

- Wasteful: power prices revert, and the empirical 10-day 99% quantile is only 1.72 times the 1-day one in Romania, not 3.16.

Quiz 4: a margin call

- You sold 100 lots of Brent to hedge your production
  - Today the price rises by 5 USD per barrel
  - What happens tonight?

Answer

- You pay a variation margin of $100 \times 1{,}000 \times 5 = 500{,}000$ USD in cash
  - The physical oil you sell later is worth more, but the cash leaves today.

Quiz 5: a guarantee that is too small

- Your 99% quantile forecast of tomorrow's price was exceeded on 111 days out of 3 585
  - What do you do?

Answer

- Widen the tail with a conformal correction from recent errors: coverage rises from 96.90% to 98.47% for 7.5% more guarantee.

Wrap-up

- Price risk: losses, quantiles, VaR and ES
- Estimating VaR and ES: four methods
  - Historical simulation and parametric models
  - Filtered historical simulation and extreme values
  - Comparing the methods
- Backtesting VaR and ES
- Margins, guarantees and position limits
  - Exchange margins
  - Guarantees and limits
- Price risk for suppliers and producers
- Case study: the price rise of autumn 2026

- VaR is a quantile, ES the mean beyond it
  - On power prices the normal curve fails, filtered methods (FHS, EVT) come closest, and none was right in 2021–2022.
- The same quantiles set limits, margins and guarantees
  - The 2026 case shows that liquidity, not VaR, is the first casualty.
  - Next: hedging with derivatives (Lecture 5).

Research from our group on energy risk

- **Energy Financial Risk Meter** (Toma, Dima, Grecu, Pele & Lessmann, 2026, SSRN)
  - A systemic-risk indicator for the EU energy sector from LASSO quantile regressions on 100 companies; it captures the 2020 and 2022 shocks (case study C4)
- **Foundation models for VaR and ES** (Jheng, Tak, Găman, Mazurencu-Marinescu-Pele & Pele, 2026)
  - Data: ten daily return series from five Central and Eastern European (CEE) markets, including Romania, out of sample 2018–2025
  - Method: three pre-trained time-series models, used without fine-tuning, against classical benchmarks; Kupiec, Christoffersen, Acerbi–Székely tests and the Basel traffic light
  - Result: green-zone rate 90% for the best raw model, 96.7% after a rolling conformal calibration, the highest of all
- **How precise is an ES comparison?** (Pele & Mazurencu-Marinescu-Pele, 2026, Mathematics)
  - Turns the $(n\alpha)^{-1/2}$ precision limit of ES into four practical diagnostics, including a sample-size rule
  - Result: on global financial assets about one in five pairwise ES comparisons cannot be trusted at regulatory tail levels
- Findings as stated in the papers; numbers on the other slides come from our own data.

(Bucharest University of Economic Studies; Romanian Academy, Institute for Economic Forecasting)

Further reading and resources

- **McNeil, Frey & Embrechts**, *Quantitative Risk Management*, 2nd ed.: VaR, ES, EVT, backtesting.
- **Christoffersen**, *Elements of Financial Risk Management*, 2nd ed.: FHS and backtests, with exercises.
- **ECB**, Financial stability risks from energy derivatives markets (2022): ecb.europa.eu.
- **This course**: code, data, notebooks — github.com/danpele/EMQA.
- **Assignment 2**, Task 5: VaR and ES of your role's cash flow, with backtests.
- **OPCOM**, day-ahead guarantee procedure: opcom.ro.
- **Experiment yourself**: the notebook reproduces every chart and number of this lecture — notebooks/.

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Appendix A: ES of the normal and Student-t distributions

$$\mathrm{ES}_\alpha^N = \frac{1}{1-\alpha}\int_{z_\alpha}^{\infty}(\mu+\sigma z)\varphi(z)\,dz = \mu + \sigma\frac{\varphi(z_\alpha)}{1-\alpha}$$

- Because $\int_c^\infty z\varphi(z)\,dz = \varphi(c)$ (the derivative of φ is $-z\varphi$).
- At $\alpha = 0.975$, $\mathrm{ES}^N = \mu + 2.338\sigma$, almost $\mathrm{VaR}_{0.99}^N = \mu + 2.326\sigma$: this is why FRTB swapped one for the other.

(McNeil, Frey & Embrechts, 2015, ch. 2)

$$\mathrm{ES}_\alpha^t = m + s\,\frac{g_\nu(t_{\nu,\alpha})}{1-\alpha}\cdot\frac{\nu + t_{\nu,\alpha}^2}{\nu-1}, \quad \nu > 1$$

- From $\int_c^\infty x\,g_\nu(x)\,dx = g_\nu(c)\,(\nu + c^2)/(\nu-1)$
  - For $\nu \leq 1$ the mean, hence ES, does not exist.
- We fit (m, s, ν) by MLE every day
  - ν is kept at least 2.2 so that the variance exists.

Appendix B: VaR and ES from the generalised Pareto tail

$$\Pr(Z > z) \approx \frac{N_u}{n}\left(1 + \xi\frac{z-u}{\beta}\right)^{-1/\xi}, \quad z > u$$

$$\Pr(Z > \mathrm{VaR}_\alpha) = 1 - \alpha \;\Rightarrow$$

$$\mathrm{VaR}_\alpha = u + \frac{\beta}{\xi}\left[\left(\frac{n(1-\alpha)}{N_u}\right)^{-\xi} - 1\right]$$

- Pickands–Balkema–de Haan: for a high u, excesses over u are approximately GPD for almost all distributions.

(Pickands, 1975; Balkema & de Haan, 1974; McNeil & Frey, 2000)

$$\mathbb{E}[Z - v \mid Z > v] = \frac{\beta + \xi(v-u)}{1-\xi}$$

$$\Rightarrow \; \mathrm{ES}_\alpha = \frac{\mathrm{VaR}_\alpha + \beta - \xi u}{1-\xi}$$

- The mean excess is linear in v for the GPD (needs $\xi < 1$)
  - $\xi > 0$ = heavy tail, $\xi = 0$ = exponential, $\xi < 0$ = bounded tail.
- Conditional EVT: apply this to $z_t = e_t/\sigma_t$, then $\mathrm{VaR}_t = \mu_{d(t)} + \sigma_t \mathrm{VaR}^z_\alpha$.

Appendix C: the three backtests in full

- **Kupiec and Christoffersen**
  - Under H_0 each I_t is Bernoulli(p), independent; LR_{uc} compares p with $\hat{p}$
  - LR_{ind} compares a first-order Markov chain with independence; $\mathrm{LR}_{cc} = \mathrm{LR}_{uc} + \mathrm{LR}_{ind} \sim \chi^2_2$, 5% critical value 5.99
- **Basel traffic light**
  - $x \sim$ Binomial(250, 0.01) for a correct model
  - $\Pr(x \leq 4) = 89.22\%$, $\Pr(5 \leq x \leq 9) = 10.76\%$, $\Pr(x \geq 10) = 0.025\%$
- **Acerbi–Székely Z_2**
  - Since $\mathbb{E}[L_t I_t] = (1-\alpha)\mathrm{ES}_t$ when VaR and ES are correct, $\mathbb{E}[Z_2] = 0$
  - Z_2 mixes a VaR error (wrong count of I_t) and an ES error (wrong size); ES alone is not elicitable, so it is tested together with VaR
  - Our p-value: 2,000 simulated paths, each day drawn from that model's predictive distribution (HS: the window; FHS: $\mu + \sigma z^*$; EVT: GPD above u)

(Kupiec, 1995; Christoffersen, 1998; BCBS, 1996; Acerbi & Székely, 2014)

Appendix D: the same backtest for Germany–Luxembourg

DE-LU	95%: share	Kupiec p	99%: count	share	Kupiec p	Chr. p	red, %	Z_2	p
HS	6.38	<0.001	60	1.67	<0.001	0.998	9.2	0.529	<0.001
Normal	6.66	<0.001	107	2.98	<0.001	0.332	16.3	1.250	<0.001
Student-t	8.33	<0.001	67	1.87	<0.001	0.174	9.0	0.617	0.001
FHS	5.82	0.028	58	1.62	<0.001	0.948	2.7	0.305	0.009
EVT	5.93	0.013	53	1.48	0.007	0.811	0.7	0.283	0.007

- Backtest 1 Jan 2017 – 31 Oct 2026, 3 591 days; same methods and windows as for Romania.
- The ranking is the same as in Romania: normal worst, filtered methods best.

(own estimates; EPEX SPOT)