








Modelling Financial Markets

Chapter 0: Financial Markets in 2026



Daniel Traian PELE

Bucharest University of Economic Studies
IDA Institute Digital Assets
Blockchain Research Center
AI4EFin Artificial Intelligence for Energy Finance
Romanian Academy, Institute for Economic Forecasting
MSCA Digital Finance

Which Financial Comparisons Can We Trust?



Andrew W. Lo. Photo: David Shankbone (2012); CC BY 3.0; Wikimedia Commons

"The building blocks of the Sharpe ratio—expected returns and volatilities—are unknown quantities that must be estimated statistically and are, therefore, subject to estimation error. This raises the natural question: How accurately are Sharpe ratios measured?"

— *Lo (2002)*

- Sharpe ratios, Jan 2015 – Sep 2026: Bitcoin 1.05, S&P 500 (Standard & Poor's) with dividends 0.82, gold 0.76
  - standard error (SE) of each: about 0.29
- S&P 500 minus gold Sharpe ratio: $+0.06$
  - 95% interval $[-0.65,\ +0.88]$: the data do not rank the two assets
- SE of the S&P 500 annualised mean return over 11.7 years: 5.1 percentage points (pp)
- None of the 10 pairs of non-crypto assets stays significant after a Bonferroni correction

▸ Central chart: Risk and Return, 2015–2026

Which Financial Comparisons Can We Trust? (1/2)

Markets, Data and Models

- **Guiding questions**
  - which financial comparisons can we trust?
  - how precisely do we know them?
- Part I: market structure and the history of exchanges and models
  - the course data, measurement and its precision
- Part II: the 2000–2026 evidence and the Romanian market
  - digital assets, passive investing and two case studies
- Closing: one full pass of the research loop assisted by artificial intelligence (AI)

- Detailed examples and advanced topics are in Further Topics, after the Conclusions
  - the footer button leads there, and the Back button returns to the slide you came from

Which Financial Comparisons Can We Trust? (2/2)

What you should be able to do (Seminar 0 exercises)

- compute returns, CAGR, annualised volatility and maximum drawdown (A1–A4, B2)
  - CAGR: compound annual growth rate
- interpret a price series before using it (B1)
  - price field, observation time, bad ticks
  - every cleaning decision justified
- compute the change in the correlation of two assets between two periods, on common days (B6)
- compute a mean return and a Sharpe ratio with a standard error and a 95% interval (A7, B2 Extended)
- interpret an empirical claim and spot a misleading one, including an AI-generated one (C1)
  - missing uncertainty, an ignored rival explanation, a wrong formula or reference
- Further Topics and Appendix: reading for familiarity

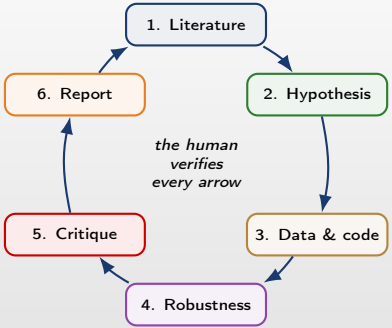
Course Organisation

- Master's programme **Applied Statistics and Data Science**, 2026/2027
  - Faculty of Cybernetics, Statistics and Economic Informatics
  - Bucharest University of Economic Studies (ASE)
- Lecture: Prof. Daniel Traian Pele, danpele@ase.ro
- Seminar: Antoaneta Amza, PhD candidate, antoaneta.amza@csie.ase.ro
- Team project (GitHub, presentation, oral defence): **70%**
- Quizzes and seminar activity: **20%**
  - one quiz per chapter on the course website: 20 questions drawn from a bank, each answer explained
- Attendance: **10%**, at least **4 lectures** and **4 seminars**
  - the code of the day shown in the room (QR, quick-response, code) or the Attendance form on the website

The Team Project and Its Outputs

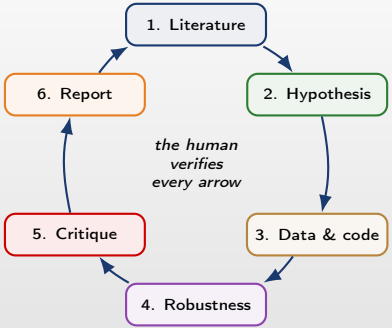
- Teams of 3–4 students, one research question on real market data
  - presented in Chapter 19
- One public GitHub repository per team
  - README (question, data, how to run) and weekly commits
- Every figure as a Quantlet: one folder with code, `Metainfo.txt` and the chart
- A result exists only if someone else can reproduce it from the repository
  - data loaded by code, every reported number produced by a script, a fixed seed
- Tools: Python, Google Colab, the course repository github.com/danpele/MFM
  - Python libraries: pandas, numpy, scipy, statsmodels, scikit-learn, matplotlib
- AI use is allowed and declared
  - every use in `AI_USE.md`
  - every caught AI error in `AI_ERRORS.md`

The AI-Assisted Research Loop (1/2)



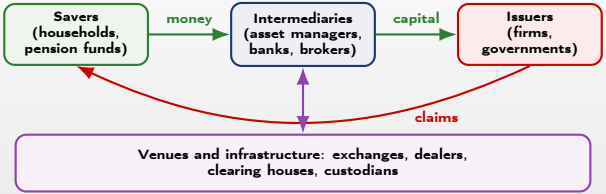
- An AI assistant can draft every step
  - e.g. an LLM (large language model) such as Claude, ChatGPT or Gemini
  - literature map, competing hypotheses, code, robustness plans, a hostile referee report
- The human owns the question, the identifying assumption, the test and every reported number
- Verification checklist
  - every reference: the DOI (Digital Object Identifier) resolves and the title matches
  - every number: reproduced from the data by your own code
  - H_0, test, sample and multiple-testing correction fixed before seeing results

The AI-Assisted Research Loop (2/2)



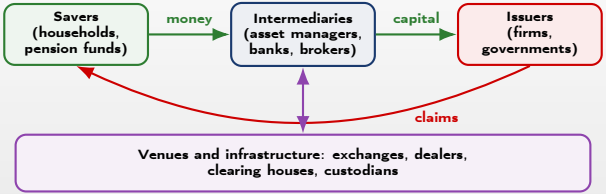
- Typical LLM failures
  - invented references
  - plausible but wrong formulas
  - look-ahead in code (information from the future)
  - searching until something is "significant"
- Each failure has a check in the list
  - the final section runs the loop on a new question

What Markets Do and Who Trades (1/2)



- **This section answers**
  - who trades what, and where?
  - how is the price we download formed?
- **Financial market**: a mechanism to exchange claims on future cash flows
  - shares, bonds, currencies, contracts, tokens

What Markets Do and Who Trades (2/2)



- Functions: **capital allocation**, **risk sharing**, **payments and settlement**
- **Liquidity**: converting assets to cash quickly
- **Price discovery**: prices aggregate dispersed information
- **Buy side**: investors and asset managers
  - **passive** (replicate an index) or **active** (try to beat it)
- **Sell side**: brokers (execute orders) and dealers (trade from inventory)
  - **market maker**: a dealer who continuously quotes a buy and a sell price

Asset Classes

Class	Examples	Cash flow	Main risks
Equities	shares, equity ETFs	dividends, residual claim	market, firm-specific
Fixed income	government and corporate bonds	coupons, principal	interest rate, credit
Money market	T-bills (< 1 year), repo (secured loan)	short-term interest	liquidity
FX	EUR/USD, EUR/RON	interest differential	currency, policy
Commodities	gold, oil futures	none (roll yield)	supply, storage
Derivatives	futures, options, swaps	contingent payoffs	leverage, counterparty
Digital assets	Bitcoin, Ether, stablecoins	none / protocol-specific	volatility, regulation

- Terms in the table
  - **ETF** (exchange-traded fund): a fund whose shares trade like a stock
  - **FX**: the currency market; EUR (euro)/RON = lei per euro
  - **roll yield**: the gain or loss from replacing an expiring futures contract
- One statistical toolbox for all of them: returns, volatility, dependence and tails
  - full list of terms in the Appendix

The Limit Order Book and a Market Order (1/2)



- **Limit order book**: all waiting buy (**bid**) and sell (**ask**) orders, sorted by price
- **Bid–ask spread**: best ask minus best bid, here $100.01 - 99.99 = 0.02$, the price of trading at once
- **Depth**: the quantity near the best prices
- A resting **limit order** provides liquidity, a **market order** consumes it
- A **stop order** becomes a market order at a trigger price
  - clustered stops can accelerate a fall

The Limit Order Book and a Market Order (2/2)



- **Questions for the room**: a market buy of 5 units arrives
  - at what average price is it executed?
  - what is the spread afterwards?

The Limit Order Book and a Market Order (2/2)



- **Questions for the room**: a market buy of 5 units arrives
  - at what average price is it executed?
  - what is the spread afterwards?
- **Answer**: it consumes the book (orange)
  - average price: $(2 \times 100.01 + 3 \times 100.02)/5 = 100.016$
  - the best ask moves to 100.02 and the spread widens to 0.03
  - this is **market impact** (Chapter 10)

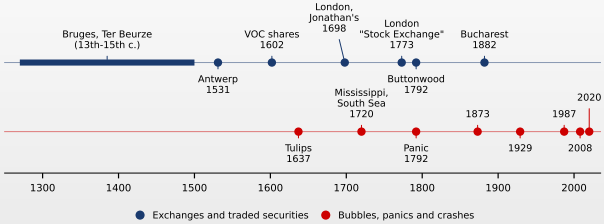
Observation Times and Price Noise (1/2)

- A "daily price" is a price observed at a specific time
  - exchange-traded assets: the closing auction (S&P 500: 20:00 or 21:00 UTC, coordinated universal time); in a **call auction**, orders are collected and matched at one price
  - auctions versus continuous trading, and why speed is rewarded: ▸ Appendix
  - Bitcoin: no close; the price of day t is taken at 24:00 UTC, the end of day t
  - EUR/RON: the reference rate of the BNR (Banca Națională a României, the National Bank of Romania), fixed around 13:00 Bucharest time
- The venue decides what is recorded (venue table in the ▸ Appendix)
  - exchanges, OTC (over-the-counter) dealers, dark pools and 24/7 crypto platforms show different prices and trades

Observation Times and Price Noise (2/2)

- Different observation times split the same news across two days (Appendix)
  - correlations of assets with different closes are attenuated
- **Bid–ask bounce**: trades alternate between bid and ask
  - observed price changes are negatively autocorrelated, even when the efficient price is a random walk
  - the Roll model: consecutive price changes have a negative covariance, $\mathrm{Cov}(\Delta p_t, \Delta p_{t-1}) = -s^2/4$ (Roll (1984); derivation in the Appendix)
  - p_t: the observed trade price; $\Delta p_t = p_t - p_{t-1}$: its change; s: the bid–ask spread; a wider spread gives a more negative covariance
- Spreads, impact and fragmentation add noise to observed prices
  - most visible at high frequency (Chapters 9 and 10)

A Timeline of Exchanges and Crashes



- **This background section answers**
  - how did exchanges, crashes and models grow together?
  - what did each wave of models add?
- The chart: the horizontal axis shows years
  - top (blue): exchanges and the first traded securities
  - bottom (red): bubbles, panics and crashes
- Shares have traded continuously since the 1600s, and bubbles and crashes followed almost at once
- The Bucharest Stock Exchange opened in 1882 (Appendix)

MFM_ch0_history ▶ Interactive

The First Exchanges: from Bruges to New York (1/2)



The Amsterdam exchange of Hendrick de Keyser, 1612

Engraving: Claes Jansz. Visscher (1612); public domain; Stadsarchief Amsterdam, Wikimedia Commons

- **Bruges**, 13th–15th centuries: merchants met on the square before an inn (Gelderblom (2013))
  - the inn of the Van der Beurse family, hence *bourse*, *beurs*, *bursă*
- **Antwerp**, 1531: a building purpose-built for daily trading (Gelderblom (2013))
- **Amsterdam**, 1602: the VOC issues transferable shares
  - VOC: Vereenigde Oostindische Compagnie, the Dutch East India Company
  - within a decade: continuous trading, forwards, options, short selling (Gelderblom & Jonker (2004); Petram (2014))

The First Exchanges: from Bruges to New York (2/2)



The Amsterdam exchange of Hendrick de Keyser, 1612

Engraving: Claes Jansz. Visscher (1612); public domain; Stadsarchief Amsterdam, Wikimedia Commons

- **London**, 1698–1773 (Neal (1990))
  - John Castaing publishes price lists at Jonathan's Coffee House
  - in 1773 the brokers call their new building the Stock Exchange
- **New York**, 17 May 1792 (Sylla, Wright & Cowen (2009))
  - 24 brokers sign the Buttonwood Agreement, the origin of the NYSE (New York Stock Exchange)

The First Bubbles, Panics and Crashes

- **Tulip mania**, Holland 1636–37: prices of bulb contracts rose many-fold, then collapsed in February 1637
  - later research shows a smaller episode than the legend, with few bankruptcies (Garber (1989); Goldgar (2007); Appendix)
- **1720**: the Mississippi Company and the South Sea Company rise and collapse within months
  - the Mississippi Company in Paris (John Law), the South Sea Company in London
  - a bubble around a real innovation: Atlantic trade and insurance (Frehen, Goetzmann & Rouwenhorst (2013))
- **Panic of 1792**, New York: the Treasury acts as a lender of last resort (Sylla, Wright & Cowen (2009))
  - the Treasury under Alexander Hamilton
- **1873**: crashes in Vienna (May) and New York (September) (NBER business-cycle dates)
  - the NYSE closes for ten days
  - the US (United States) contraction dated by the NBER (National Bureau of Economic Research) lasts until 1879
- **1929**: Black Tuesday, the New York stock-market crash (Appendix)
- **19 October 1987**: the DJIA falls 22.6% in one day (Roll (1988))
  - DJIA: Dow Jones Industrial Average
  - large falls in every major market
- Recurring ingredients: a new asset or technology, credit and leverage, then forced selling (Chapter 17)

Financial Models (1/4): Predictability and Priced Risk




Louis Bachelier; William F. Sharpe, 2007

Bachelier: unknown author; public domain; Sharpe: Larry D. Moore (2007); CC BY 4.0; Wikimedia Commons

- **Can past prices predict future prices?** Random walk and efficiency (Appendix)
  - a random walk for the log price: $\ln P_t = \ln P_{t-1} + \mu + \varepsilon_t$, with iid (independent and identically distributed) zero-mean shocks ε_t
  - Bachelier (1900): Brownian motion; Mandelbrot (1963): far heavier tails than under the Normal distribution
  - EMH (efficient market hypothesis, Fama (1970)): prices fully reflect available information
- **Which risks should earn a reward?** Portfolios and equilibrium (Appendix)
  - Markowitz (1952): choose weights by mean and variance; diversification depends on covariances
  - CAPM (capital asset pricing model, Sharpe (1964)): only market risk is priced
- In this course: Chapters 1–4

Financial Models (2/4): Option Pricing and Risk Measurement





Fischer Black, Myron Scholes, Robert C. Merton

Black: Dalmatine; public domain; Scholes: Nobellaureatesphotographer (2008); CC BY-SA 3.0; Merton: Digarnick (2006); public domain; Wikimedia Commons

- **What is an option worth?** (Black & Scholes (1973); Merton (1973); Appendix)
  - a continuously rebalanced hedge replicates the option, so its price does not depend on the stock's expected return
- **Is volatility forecastable?** (Engle (1982); Bollerslev (1986); Appendix)
  - ARCH (AutoRegressive Conditional Heteroskedasticity) and GARCH (generalised ARCH): today's variance depends on past shocks, which explains volatility clustering
- **How much capital should a bank hold against market risk?** (BCBS (1996); BCBS (2019); Appendix)
  - VaR 1% (Value-at-Risk): the loss exceeded with probability 1%
  - later ES 2.5% (expected shortfall): the mean loss in the worst 2.5% of cases
- In this course: Chapters 5–12

Financial Models (3/4): Model Risk and the 2008 Crisis

- **Lesson of 2008**: an example of model risk (Appendix)
  - CDO (collateralised debt obligation): a bond paid from a pool of loans
  - one Gaussian-copula parameter priced every tranche of a CDO
  - the parameter could not reproduce the joint defaults of the crisis
- Interest-rate models, from Vasicek to the yield curve: Appendix

Financial Models (4/4): Learning from Big Data and Text

- **Problem**: hundreds of firm signals and millions of documents do not fit a small parametric model
- **Machine learning** (ML; Appendix): flexible functions fitted to data
  - selected by out-of-sample performance rather than by theory
  - Gu, Kelly & Xiu (2020): trees and neural networks beat linear models on about 30,000 US stocks and 94 firm characteristics
  - the monthly out-of-sample R^2 stays below 1%: small but economically valuable
- **Generative AI** (Appendix): the Transformer (Vaswani et al. (2017)) underlies today's LLMs
  - Lopez-Lira & Tang (2026): ChatGPT scores of news headlines predict next-day stock returns out of sample
  - foundation models for time series forecast new series without retraining (Ansari et al. (2024))
- Main risks (Chapters 13–15)
  - overfitting
  - look-ahead: the model may have read texts written after the forecast date
  - many trials on the same data

Where the Course Data Come From, and Survivorship Bias

Data	Content	Format
Daily market data from EODHD (EOD Historical Data), saved in the course repository	indices, stocks, ETFs, FX, crypto, yields	CSV
FRED (Federal Reserve Economic Data)	US rates, exchange rates, macro series	public CSV
BNR	official RON reference exchange rates	public XML
DefiLlama	stablecoin supply, DeFi (decentralised finance)	public JSON

- **This section answers**
  - which data feed every chapter?
  - which price field and which calendar should you use?
- **Survivorship bias** (Brown, Goetzmann, Ibbotson & Ross (1992))
  - samples rebuilt from today's survivors make past returns look too good

Which Price and Which Calendar

- Price type
  - stocks and ETFs: the adjusted close (dividends, splits)
  - indices, FX and crypto: the close
  - bonds: the closing quoted yield
- Calendar
  - one series: returns on its own calendar
  - several series: join the *prices* on common days first, then take returns
- Annualise with the observed frequency, the number of returns per year in the data
  - about 365 for crypto, 260 for spot gold, 252 for equities

Simple and Log Returns

- **This section answers**
  - how do we measure growth and risk?
  - how precisely do we know the answer?
- The simple return R_t and the log return r_t

$$R_t = \frac{P_t}{P_{t-1}} - 1, \qquad r_t = \ln \frac{P_t}{P_{t-1}} = \ln(1 + R_t)$$

  - P_t: the closing price (or index level) on day t
- Simple returns aggregate across assets: a portfolio with weights w_i has $R_{p,t} = \sum_i w_i R_{i,t}$
- Log returns aggregate over time: the h-day log return is the sum $r_{t+1} + \cdots + r_{t+h}$
- For small moves the two nearly coincide, $r_t = R_t - R_t^2/2 + \ldots$
  - for a 50% fall they differ a lot: $\ln 0.5 = -69\%$

Compounding and the CAGR

- A **price index** (S&P 500, BET) ignores dividends
  - BET: the main index of the Bucharest Stock Exchange
  - the **total-return index** (BET-TR) and the **adjusted close** reinvest them
- **Compounding**: daily simple returns multiply, log returns add

$$P_T = P_0 \prod_{t=1}^{T} (1 + R_t) = P_0 \, e^{r_1 + \cdots + r_T}$$

- **CAGR**: the constant yearly rate that turns the first price into the last one

$$\mathrm{CAGR} = \left(\frac{P_{\text{end}}}{P_{\text{start}}} \right)^{1/Y} - 1, \qquad \ln(1 + \mathrm{CAGR}) = \frac{1}{Y} \sum_{t=1}^{T} r_t$$

  - CAGR: compound annual growth rate
  - Y: the sample length in years, calendar days/365.25; the ratio $P_{\text{end}}/P_{\text{start}}$ is the **multiple**
  - the second form: the CAGR is the sum of log returns per year, expressed as a simple rate
- Example: S&P 500 (price index), 2 Jan 2015 – 18 Sep 2026
  - $Y = 4\,277/365.25 = 11.71$
  - the multiple $\times 3.72$ gives CAGR $= 3.72^{1/11.71} - 1 = 11.9\%$

Annualised Volatility

- The subscript d marks a daily quantity, the subscript A an annualised one
  - A is also the number of returns per year actually observed
- **Annualised volatility**: $\hat{\sigma}_d$ scaled by $\sqrt{A}$

$$\hat{\sigma}_A = \hat{\sigma}_d \sqrt{A}$$

  - $\hat{\sigma}_d$: the sample standard deviation of daily log returns
  - why $\sqrt{A}$: the one-year log return is the sum of A daily returns; if these are uncorrelated with constant variance, the variance of the sum is $A\sigma_d^2$
  - with autocorrelated daily returns the scaling is no longer exact (variance ratio, Appendix)
- Example: SPY has $T = 2\,944$ daily returns over $Y = 11.71$ years, so $A = T/Y = 251.4$
  - SPY: the SPDR (Standard & Poor's Depositary Receipts) S&P 500 ETF, with dividends

Arithmetic Mean, CAGR and Volatility Drag

- The annualised arithmetic mean $\hat{\mu}_A = A\,\bar{R}$ exceeds compound growth

$$\ln(1 + \mathrm{CAGR}) \approx \hat{\mu}_A - \hat{\sigma}_A^2/2$$

  - $\bar{R}$: the sample mean of daily simple returns
  - the reason: $r_t \approx R_t - R_t^2/2$, and the mean of R_t^2 is about σ_d^2
  - **volatility drag**: at the same arithmetic mean, higher volatility lowers compound growth
- SPY, 2015–2026: $\hat{\mu}_A = 14.39\%$ and $\hat{\sigma}_A = 17.53\%$ (from simple returns)
  - so $14.39\% - 17.53\%^2/2 = 12.85\%$
- SPY's CAGR is 13.7%, and $\ln(1.137) = 12.84\%$
  - the approximation reproduces the observed compound growth
- The price index has a CAGR of 11.9%: a gap of 1.8 percentage points (pp) a year
  - the gap comes mainly from reinvested dividends, net of the ETF's fees and tracking differences

Drawdown and Maximum Drawdown

- Volatility does not say how much an investor lost from the best earlier moment
  - the order of returns matters
- **Drawdown**: the percentage distance below the running peak

$$DD_t = \frac{P_t}{\max_{s \le t} P_s} - 1 \le 0, \qquad \mathrm{MDD} = \min_t DD_t$$

  - MDD (maximum drawdown): the deepest fall from a peak over the whole sample
  - duration of a drawdown: the days until the price exceeds the earlier peak again
- Example (Seminar 0, A4): prices 100, 120, 90, 110, 130, 100
  - the running peak 100, 120, 120, 120, 130, 130 gives DD_t: 0, 0, -25%, -8.3%, 0, -23.1%
  - MDD $= -25\%$, although the last price is above the first

How Precisely Do We Know an Expected Return? (1/2)

- Assumption: the daily simple returns R_t are iid
  - true mean μ_d and standard deviation σ_d
  - $T = AY$ observations
- Estimator: $\hat{\mu}_A = A\,\bar{R}$, with $\bar{R} = \frac{1}{T}\sum_{t=1}^{T} R_t$
- **Standard error** (SE): the typical distance between $\hat{\mu}_A$ and the true annual mean

$$\mathrm{SE}(\hat{\mu}_A) = \frac{A\,\sigma_d}{\sqrt{T}} = \frac{\sigma_A}{\sqrt{Y}}, \qquad \sigma_A = \sigma_d\sqrt{A}$$

  - the SE falls only with the square root of the number of years
  - 95% confidence interval: $\hat{\mu}_A \pm 1.96$ SE

How Precisely Do We Know an Expected Return? (2/2)

- SPY, 2015–2026: $\hat{\mu}_A = 14.4\%$, $\sigma_A = 17.5\%$, $Y = 11.71$
  - σ_A from simple returns; 17.6% from log returns
  - SE $= 17.5\%/\sqrt{11.71} = 5.1$ pp
  - 95% confidence interval [4.3%, 24.4%]
- **Question for the room**: to halve this SE, which would you rather have?
  - four times as many observations (hourly instead of daily data)
  - or four times as many years

How Precisely Do We Know an Expected Return? (2/2)

- SPY, 2015–2026: $\hat{\mu}_A = 14.4\%$, $\sigma_A = 17.5\%$, $Y = 11.71$
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  - SE $= 17.5\%/\sqrt{11.71} = 5.1$ pp
  - 95% confidence interval [4.3%, 24.4%]
- **Question for the room**: to halve this SE, which would you rather have?
  - four times as many observations (hourly instead of daily data)
  - or four times as many years
- **Answer**: four times as many years (about 47 instead of 11.7)
  - the SE depends only on the **span** Y, not on the sampling frequency (Merton (1980))

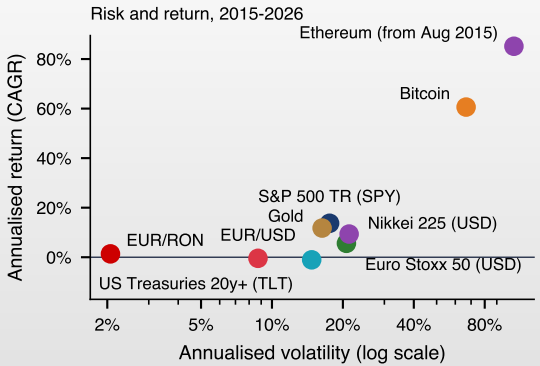
The Sharpe Ratio Is an Estimate

- **Sharpe ratio (SR)**: the mean return per unit of risk, with the risk-free rate set to 0
  - per day $\mathrm{SR}_d = \mu_d/\sigma_d$ (mean and standard deviation, SD, of daily simple returns); annualised $\mathrm{SR}_A = \sqrt{A}\,\mathrm{SR}_d$
  - $\widehat{\mathrm{SR}}_A$: the value estimated from the sample
- Its standard error, for iid returns following the Normal distribution (Lo (2002)):

$$\mathrm{SE}(\widehat{\mathrm{SR}}_A) = \sqrt{\frac{1 + \mathrm{SR}_A^2/(2A)}{Y}} \approx \frac{1}{\sqrt{Y}}$$

  - with daily data the term $\mathrm{SR}_A^2/(2A)$ is tiny ($0.82^2/504 \approx 0.001$), so the SE depends almost only on the number of years Y

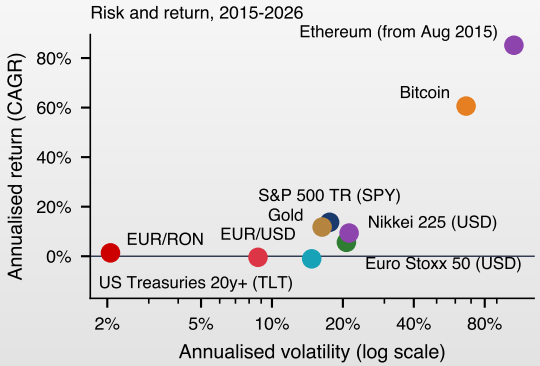
Risk and Return, 2015–2026 (1/2)



- **This section answers**
  - what return and what risk did the main asset classes deliver?
  - is any difference between them statistically significant?
- One window for every asset, 2 Jan 2015 – 18 Sep 2026
  - different windows would compare different shocks
- Vertical axis: CAGR, the realised compound growth
- Horizontal axis: annualised volatility on a log scale
  - 2% to 4% looks as wide as 20% to 40%
- All in USD (US dollar), except EUR/RON
  - TR: total return
  - TLT: the ETF of US Treasuries with more than 20 years to maturity

MFM_ch0_cross_asset ▶ Interactive

Risk and Return, 2015–2026 (2/2)



- S&P 500 with dividends: CAGR 13.7%, volatility 17.6%
- Gold: CAGR 11.8%, volatility 16.3% (weekday quotes)
- Bitcoin: CAGR 60.6%, volatility 66.7%
- SPY and TLT include dividends and coupons, but Euro Stoxx 50 and Nikkei 225 are price indices, without dividends
  - comparisons with them are not tests between total returns of equivalent investments
- Ethereum (shorter sample), EUR/USD and EUR/RON are shown but not tested
  - an exchange rate's "return" omits the interest earned on the currency
- The tests that follow concern the arithmetic mean and the Sharpe ratio, not the CAGR

MFM_ch0_cross_asset

Is Any Ranking in the Risk–Return Chart Significant?

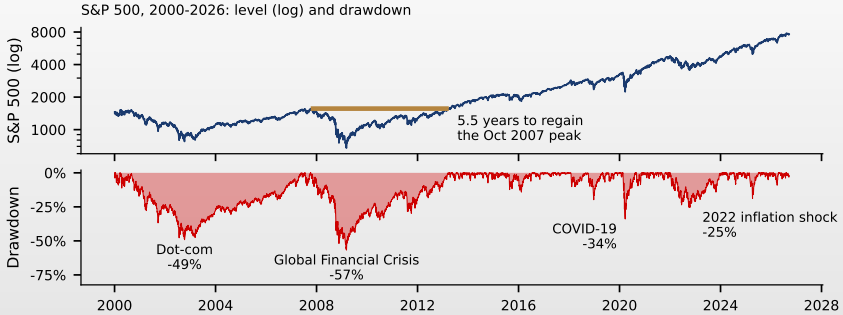
- SE of the annualised mean return, $\sigma_A/\sqrt{Y}$ with $Y = 11.7$
  - S&P 500 with dividends 5.1 pp, gold 4.8 pp, Euro Stoxx 50 6.0 pp, Nikkei 225 6.2 pp, Bitcoin 19.4 pp
- S&P 500 minus gold on common days: $+1.9$ pp a year, SE 6.8 pp, $t = 0.28$
- Sharpe ratios, each with SE ≈ 0.29
  - Bitcoin 1.05, S&P 500 0.82, gold 0.76, Nikkei 225 0.53, Euro Stoxx 50 0.37, TLT 0.00
  - the SE requires a finite fourth moment (tails not too heavy), which Chapter 1 questions
- Overlapping intervals do not test a difference
  - bootstrap (resample the observed days) the paired difference of the two Sharpe ratios on common days (Seminar 0, B2 Extended)
  - S&P 500 minus gold: $+0.06$, 95% interval $[-0.65,\ +0.88]$
  - S&P 500 minus Euro Stoxx 50: $+0.46$, $[0.09,\ 0.85]$, significant alone
  - none of the $\binom{5}{2} = 10$ pairs of the non-crypto assets survives a Bonferroni correction (each test at $5\%/10 = 0.5\%$)
- **Question for the room**: the S&P 500 minus gold interval contains zero
  - does this show that the two Sharpe ratios are equal?

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- **Question for the room**: the S&P 500 minus gold interval contains zero
  - does this show that the two Sharpe ratios are equal?
- **Answer**: no, the comparison is inconclusive
  - the data are compatible with gold ahead by 0.65 and with the S&P 500 ahead by 0.88
  - the chart describes the sample without ranking expected returns
- Type of evidence: description (it shows what happened in the data, without testing a cause)

MFM_ch0_cross_asset

Crises and Drawdowns (1/2)



- Upper panel: the S&P 500 level (price index), log scale
  - the bar: 5.5 years to regain the October 2007 peak
- Lower panel: the drawdown DD_t, in % below the running peak

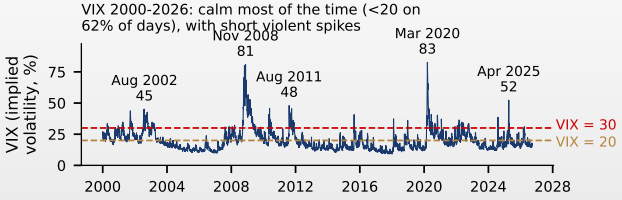
MFM_ch0_drawdowns_vix ▶ Interactive

Crises and Drawdowns (2/2)

- Four S&P 500 troughs since 2000
  - dot-com: −49% (Oct 2002)
  - global financial crisis: −57% (Mar 2009), from 1 565.15 to 676.53, regained only in Mar 2013
  - COVID-19: −34% (Mar 2020), the fastest fall
  - inflation shock: −25% (Oct 2022)
- The MDD deepens with the sample length, so compare MDDs over the same horizon (Appendix)

MFM_ch0_drawdowns_vix

Volatility Regimes: the VIX



- VIX: the S&P 500 volatility over the next 30 days **implied** by option prices
  - VIX: CBOE Volatility Index; CBOE: Chicago Board Options Exchange; in % per year
- Since 2000: below 20 on 62% of days, above 30 on 9%
  - peaks 82.7 (16 Mar 2020), 80.9 (20 Nov 2008), 52.3 (8 Apr 2025)
- Long calm periods, short intense stress
  - volatility comes in regimes (Chapters 5 and 9)
- The VIX exceeded the volatility realised over the next 21 trading days on 83% of days
  - once volatility is realised (ex post): gap = variance risk premium + forecast error (Appendix)
  - premium: what investors pay for protection against high variance; defined on variances, not volatilities

MFM_ch0_drawdowns_vix ▶ Interactive

Diversification Benefits Change over Time



- 252-day rolling correlations of daily log returns, prices joined on common days first
- Stocks vs long Treasuries: mean rolling -0.43 (2010–2020), $+0.08$ (2022–2026)
- Stocks vs Bitcoin: mean rolling 0.01 (2016–2019), 0.38 (2022–2026)
- Fixed-correlation portfolio models miss such shifts (Chapters 4 and 6)
- Two traps for crypto–equity correlations
  - asynchronous closes (Appendix)
  - the volatility bias (Appendix)

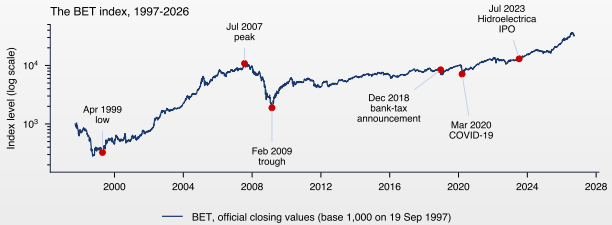
MFM_ch0_correlations ▶ Interactive

Further Topics: testing the correlation change

The Bucharest Stock Exchange Today

- **This background section answers**
  - what do the same measurements show on the Romanian market?
- BVB (Bursa de Valori București, the Bucharest Stock Exchange) (Appendix)
  - opened in 1882, closed in 1948, re-established in 1995
- **BET** (Bucharest Exchange Trading index): the most traded Romanian companies (BVB (indices))
  - launched on 19 Sep 1997 at 1 000 points
  - **BET-TR**: its total-return version, with reinvested dividends
- Largest companies
  - banks (Banca Transilvania, BRD), energy (OMV Petrom, Romgaz, Hidroelectrica), utilities (Transgaz)
- Classified as **Secondary Emerging** by FTSE Russell (FTSE: Financial Times Stock Exchange), the index provider, since September 2020 (FTSE Russell (2026))
- Hidroelectrica: the largest IPO in BVB (Bursa de Valori București, the Bucharest Stock Exchange) history, EUR 1.9 bn, July 2023 (BVB (history))
  - IPO: initial public offering
- **TVBETETF**: an ETF on BVB that replicates BET

The BET Index, 1997–2026: a Long Recovery

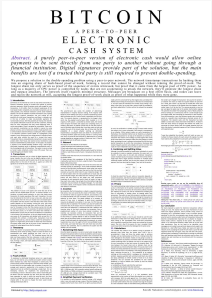


- Official closing values, log scale
  - from the peak of 24 Jul 2007 to the trough of 25 Feb 2009: −82.5%
  - regained only on 16 Mar 2021, after **13.6 years**
- 19 Dec 2018: −11.2% in one day, after the bank-tax announcement
  - 18 Sep 2026: ×32.8 since launch, CAGR 12.8% over 29 years
- The charts since 2015 are in the Appendix
  - BVB equities with dividends (▸ Appendix)
  - the EUR/RON rate (▸ Appendix)
  - Romanian versus German government yields (▸ Appendix)

MFM_ch0_bvb_history ▶ Interactive

▸ Further Topics: the 2018 bank tax

Bitcoin, Ethereum and Tokens (1/2)



BITCOIN

A PEER-TO-PEER ELECTRONIC CASH SYSTEM

Abstract. A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution. Digital signatures provide part of the solution, but the main benefits are lost if a trusted third party is still required to prevent double-spending.

Bitcoin white paper (2008)

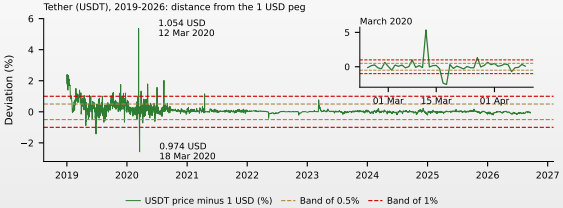
Text: Satoshi Nakamoto (2008); poster: DailyCoinPost (2018); CC0; Wikimedia Commons

- **This background section answers**
  - what are crypto assets and stablecoins?
  - what do their prices measure?
- 31 Oct 2008: *Bitcoin: A Peer-to-Peer Electronic Cash System* Nakamoto (2008)
  - by the pseudonymous Satoshi Nakamoto
- **Blockchain**: a shared ledger of transactions, without a trusted intermediary
  - copied on many computers and extended block by block
- **Bitcoin** (2009): a fixed supply rule, at most 21 million coins
  - **proof of work**: adding a block costs computing power, so rewriting history is expensive

Bitcoin, Ethereum and Tokens (2/2)

- **Ethereum**: a blockchain that runs programs (**smart contracts**)
  - its coin is Ether
- A **token** is a unit on a blockchain representing a coin, a claim or a right
- Härdle, Harvey & Reule (2020): statistical properties, pricing and regulation
- Liu & Tsyvinski (2021): cryptocurrency returns have low exposures to stock, currency and commodity factors
  - they are explained by crypto-specific factors: momentum (the continuation of recent returns) and investor attention
- Pele et al. (2023): three statistical factors (tails, memory, moments of returns) set cryptos apart from other asset classes
- In 2024–2026 Bitcoin was still about 3.2 times as volatile as the S&P 500 (Appendix)
- Since January 2024 spot Bitcoin ETFs trade in the US (Appendix)
  - how ETFs work: next section

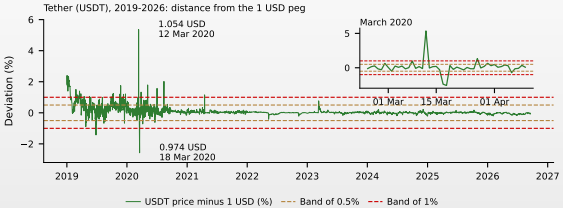
Stablecoins and the Peg (1/2)



- **Stablecoin**: a token designed to keep a fixed value (**peg**), usually 1 USD
  - **reserve-backed** (cash, short-term T-bills) and redeemable at 1 USD, or **algorithmic** (a second token and trading rules)
- **Depeg**: the price leaves the peg
  - the algorithmic TerraUSD lost its peg and collapsed in May 2022

MFM_ch0_digital_assets ▶ Interactive

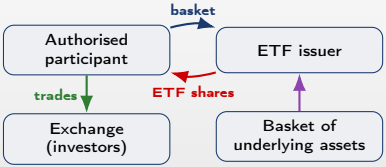
Stablecoins and the Peg (2/2)



- The chart: the USDT (Tether, a US dollar stablecoin) price minus 1 USD, in %
  - dashed lines: the $\pm 0.5\%$ and $\pm 1\%$ bands
  - inset: March 2020
- Tether (USDT), 2019–2026: within $\pm 0.5\%$ of 1 USD on 93% of days
  - within $\pm 1\%$ on 98%
- Supply is not the peg (▸ Appendix)
  - the circulating value can fall while the price stays at 1 USD
  - tokenisation and regulation: ▸ Appendix

MFM_ch0_digital_assets

ETFs: Creation and Redemption



- **This background section answers**
  - how does an ETF keep its price near its assets?
  - what does cap-weighting imply for investors?
- Most ETFs are **passive**: they replicate an index at low cost
- **Net asset value (NAV)**: the fund's assets minus its liabilities, per share, computed once a day
- **Authorised participant (AP)**: a large dealer allowed to deal directly with the issuer
- **Creation** and **redemption** keep the market price close to the NAV
  - creation: the AP delivers the basket and receives new ETF shares
  - redemption: the reverse

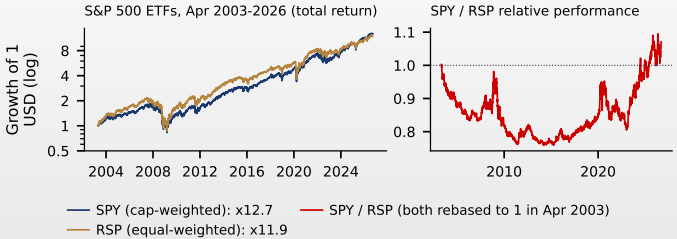
Cap-Weighted versus Equal-Weighted Indices

Definition 1 (Index weights and weight concentration)

- Capitalisation weights: $w_i = MC_i / \sum_{j=1}^{n} MC_j$, with n stocks in the index
  - MC_i: the market capitalisation of stock i, price $\times$ number of shares
- Equal weights: $w_i = 1/n$, rebalanced periodically
- Effective number of holdings: $N_{\text{eff}} = 1/\sum_i w_i^2$ (inverse Herfindahl index), a measure of weight concentration
  - n for equal weights; 1 if one stock holds all the weight
  - far fewer than n when a few firms dominate
  - it uses only the weights: the diversification of risk also depends on the covariances of the returns

- A cap-weighted index is more concentrated than its number of stocks suggests
  - the passive funds that track it carry the concentration of the largest firms (Chapters 3, 4 and 17)

Cap-Weighted versus Equal-Weighted Performance: SPY and RSP (1/2)



- The chart
  - left: the value of 1 USD invested in April 2003, log scale
  - right: the SPY/RSP ratio, equal to 1 in April 2003
- SPY: the S&P 500 with capitalisation weights
- RSP: the same stocks, each 1/500
  - RSP: Invesco S&P 500 Equal Weight ETF

MFM_ch0_concentration ▶ Interactive

Cap-Weighted versus Equal-Weighted Performance: SPY and RSP (2/2)

- Total return (adjusted close) since RSP's launch in April 2003
  - SPY $\times$12.7, RSP $\times$11.9; the ratio ends at 1.07
- SPY/RSP rose by 31% since January 2023
  - a handful of very large companies drove the index
- The ratio measures relative performance
  - it is not a direct measure of concentration or of passive ownership

MFM_ch0_concentration

Case Study: What Happens When Passive Investing Grows? (1/2)

- **This section asks**
  - what does a landmark paper find about the effect of passive investing on prices?
- Haddad, Huebner & Loualiche (2025), *American Economic Review* 115(3), 975–1018
- **The paper's question**: when some investors turn passive, do the others trade more aggressively?
  - and do they undo the effect on prices?
- **Demand elasticity** $\mathcal{E}_i$ of investor i
  - the percentage fall in the shares demanded after a 1% price rise
  - a passive investor has $\mathcal{E}_i = 0$

Case Study: What Happens When Passive Investing Grows? (2/2)

- **Aggregate elasticity** $\mathcal{E}_{agg}$: the holdings-weighted average of the $\mathcal{E}_i$ (eq. 2)

$$\Delta p = \mathcal{E}_{agg}^{-1} \Delta \bar{d} \quad \text{(eq. 3)}$$

  - $\Delta \bar{d}$: the average demand shock, the percentage change in the shares demanded at the old price
  - Δp: the change in the log price
  - a lower $\mathcal{E}_{agg}$ lets the same shock move the price more: prices more volatile and less informative (Section II.A)
- Link: stocks with higher ETF ownership are more volatile (Ben-David, Franzoni & Moussawi (2018))

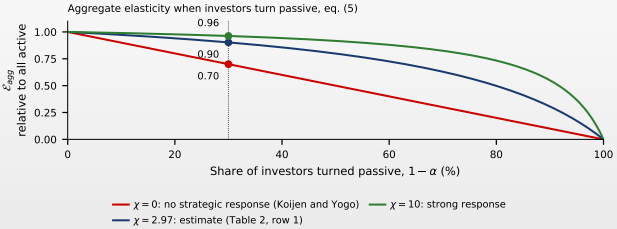
Case Study: an Illustrative Scenario, 30% Turn Passive (1/2)

- **Strategic response** χ: $\mathcal{E}_i = \bar{\mathcal{E}}_i - \chi\,\mathcal{E}_{agg}$ (eq. 4)
  - $\bar{\mathcal{E}}_i$: the investor's own component
  - $\chi = 0$: nobody reacts; $\chi \to \infty$: the others fully offset any change
- Eq. (5): $\mathcal{E}_{agg}$ after a share $1 - \alpha$ turns passive

$$\alpha + (1 - \alpha)\,\frac{\chi\alpha}{1 + \chi\alpha}$$

  - relative to the all-active market ($\alpha = 1$) with the same own elasticities
  - α: the share of holdings still active; first term: the direct fall
  - second term: the part offset by active investors, 0 when $\chi = 0$
- **Question for the room**: 30% turn passive ($\alpha = 0.7$) and nobody reacts ($\chi = 0$)
  - what share of the aggregate elasticity is left?

Case Study: an Illustrative Scenario, 30% Turn Passive (2/2)



- Horizontal axis: the share of investors turned passive, $1 - \alpha$
- Vertical axis: $\mathcal{E}_{agg}$ relative to the all-active market, eq. (5)
- **Answer**: 0.70 if nobody reacts ($\chi = 0$)
- With the paper's $\chi = 2.97$, 0.90 is left
  - two-thirds of the fall is offset
- With $\chi = 10$, the aggregate elasticity left is 0.96
- At 100% passive all curves reach 0
  - nobody absorbs shocks

MFM_ch0_passive_elasticity ▶ Interactive

▸ Further Topics: deriving Eq. (5)

Case Study: Estimate and Identification

- **Data**: quarterly holdings of US institutional managers from Form 13F, 2001–2020
  - Form 13F: filed with the SEC (Securities and Exchange Commission)
  - about 80% of US market capitalisation
- **Estimate**: strategic response $\chi = 2.97$ (SE 0.47; Table 2, row 1)
  - a 95% interval of about [2.0, 3.9]
  - investors react: the estimate is far from both 0 and ∞
- Type of evidence: causal identification, conditional on the identifying assumption below
- **Identifying assumption** (Theorem 1)
  - the same investor meets a different mix of other investors in each stock
  - this variation separates its own elasticity from χ
  - instruments, GMM (generalised method of moments) and the demand system: Appendix

Case Study: What the Estimate Implies for Passive Investing

- **Implication**: the median active share fell from 81.4% (2001) to 59.5% (2020)
  - i.e. $\ln(81.4/59.5) = 31.3\%$
- **Pass-through** (eq. 28): the % change in $\mathcal{E}_{agg}$ per 1% change in α

$$\frac{1}{1+\chi\alpha} = \frac{1}{1+2.97\times 0.68} = 33\%$$

  - α: the active share
  - $\alpha = 0.68$: the time average of the active share in the paper (p. 1006)
  - the pass-through depends on α: 0.29 at $\alpha = 0.814$ (2001) and 0.36 at $\alpha = 0.595$ (2020)
  - so stocks are about $0.33 \times 31.3\% \approx 10\%$ less elastic (the paper's abstract: 11%)
  - inelastic stocks are more volatile and less liquid (Table 5; robustness and reduced-form checks: Appendix)

Case Study: One Limitation and an Open Question

- **One limitation**: one χ for all investors, with no fund flows (Section I.B)
  - all stated limits and what the course data can check: Appendix
- **Open question**: does passive investing change price formation?
  - Sammon (2025): higher passive ownership makes prices anticipate earnings announcements less
  - passive ownership is, however, endogenous: it depends on firm characteristics, so the link need not be causal
- **Question for the room**: with $\chi = 2.97$, what pass-through would an active share of $\alpha = 0.5$ imply?
  - why is it larger than at $\alpha = 0.68$?

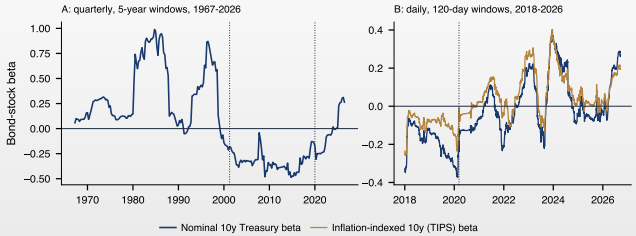
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  - why is it larger than at $\alpha = 0.68$?
- **Answer**: $1/(1 + 2.97 \times 0.5) = 0.40$
  - with fewer active investors left, there are fewer to offset each further switch to passive

Case Study: When Are Treasury Bonds a Hedge?

- **The question**: when do US Treasury bonds protect an equity portfolio?
- Pflueger (2025), *Journal of Financial Economics* 167, 104027
- **The measure**: the bond's stock market beta, $\beta_b = \mathrm{Cov}(r_b, r_s)/\mathrm{Var}(r_s)$
  - r_b: the excess return on the 10-year Treasury bond
  - r_s: the excess return on the US stock market
  - $\beta_b < 0$: the bond gains when stocks lose, so it hedges
  - $\beta_b > 0$: the bond loses together with stocks, so it is a risky asset
- **The paper's claim**: the positive nominal bond beta of the 1980s required two conditions at once
  - volatile supply shocks
  - a monetary policy that reacts quickly and strongly to inflation

Case Study: Treasury Bond–Stock Betas, 1967–2026



- Left: nominal β_b over rolling windows of 20 quarters (5 years); dotted lines at 2001Q2 and 2020Q1
- Right: daily β_b over 120-day windows, for the nominal bond and for TIPS
  - TIPS (Treasury Inflation-Protected Securities): inflation-indexed US Treasury bonds
- Nominal β_b on the course data: 0.27 in 1979Q4–2001Q1 and -0.34 in 2001Q2–2019Q4
  - in the paper (NBER version, Table 2): 0.24 and -0.31
- The 5-year β_b turned positive again in 2023Q4 and is 0.26 in 2026Q2

MFM_ch0_bond_beta ▶ Interactive

Case Study: The Paper's Method and the Course Replication

- **The paper's model**: a three-equation New Keynesian model
  - the consumption Euler equation, the Phillips curve and the monetary policy rule
  - habit formation preferences, hence time-varying risk premia
  - a supply shock raises inflation and lowers output (stagflation): nominal bonds fall together with stocks
- **Calibration**: separately for 1979Q4–2001Q1 and 2001Q2–2019Q4
  - the 2001Q2 break: the correlation between inflation and the output gap (actual minus potential output) turns positive (Campbell, Pflueger & Viceira (2020))
  - bond betas are not targeted in the calibration; the model reproduces them
- **What the course replication does differently**
  - bonds: FRED (Federal Reserve Economic Data) constant-maturity yields (DGS10, DFII10), duration fixed at 10 years
  - $r_b \approx -10\,\Delta y_t + y_t/4$ per quarter, with y_t the 10-year yield; the paper uses zero-coupon curves
  - stocks: the cap-weighted US market (CRSP, Center for Research in Security Prices), from the Kenneth French library; daily, SPY
  - quarterly excess returns: over the 3-month Treasury bill (T-bill), as in the paper
  - TIPS only from 2003 (the paper fills the earlier period with UK inflation-linked bonds)
  - the sample is extended to September 2026

Case Study: What the Data Show after 2020

- **Type of evidence**: replication (the 2001 sign change, reproduced on the course data)
  - nominal β_b 0.27 (SE 0.14, 86 quarters) in 1979Q4–2001Q1
  - nominal β_b −0.34 (SE 0.06, 75 quarters) in 2001Q2–2019Q4
  - the explanation through supply shocks and monetary policy comes from the model; the regression alone does not identify it
- **2020Q1–2026Q2**: nominal β_b 0.00 (SE 0.17), TIPS β_b 0.19 (SE 0.13)
  - since 11 March 2020 the daily nominal beta is below the TIPS beta on 84% of the days
  - in the paper, a nominal beta below the real one signals little stagflation risk
- **Limits**
  - 120-day windows are noisy
  - the bond duration is approximated
  - the conclusion rests on one calibrated model
- **Question for the room**: in 2026 (up to 18 September) the nominal β_b estimated on daily returns is 0.22 and the TIPS β_b 0.17
  - what signal would the paper read in this ordering?

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- **Question for the room**: in 2026 (up to 18 September) the nominal β_b estimated on daily returns is 0.22 and the TIPS β_b 0.17
  - what signal would the paper read in this ordering?
- **Answer**: a rise in stagflation risk
  - the difference, 0.05 (SE 0.02), is positive for the first time in a calendar year since 2018
  - one partial year does not, however, show a return to the 1980s

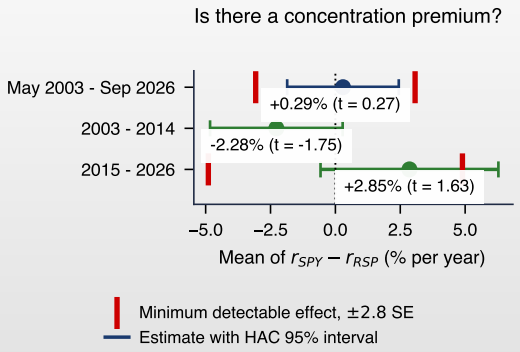
Hypothesis: Is There a Concentration Premium? (1/2)

- **This section asks**
  - how does one pass of the AI-assisted research loop look on a new question?
- The SPY–RSP comparison is descriptive
  - it cannot identify the case study's passive-ownership mechanism
- A typical assistant proposal
  - "passive flows favour the largest firms, so cap weights beat equal weights"
- Discovery loop in one pass
  - **literature**: peer-reviewed papers, every DOI resolved yourself
  - **hypothesis**: three competing explanations (passive flows, sector composition, earnings growth of the largest firms), each with one implication the others do not share
  - **data and code**: SPY and RSP adjusted (total-return) prices, joined on common days

Hypothesis: Is There a Concentration Premium? (2/2)

- Test: $d_t = r_{\mathrm{SPY},t} - r_{\mathrm{RSP},t}$
  - $H_0 : \mathbb{E}[d_t] = 0$ against $H_1 : \mathbb{E}[d_t] \neq 0$
  - $r_{\mathrm{SPY},t}$, $r_{\mathrm{RSP},t}$: the daily log returns of the two ETFs on day t; $\mathbb{E}[d_t]$: the true mean daily spread
- Checks fixed in advance
  - the price type, the calendar join, the windows (full sample and two halves)
  - the t-statistic $\bar{d} / \mathrm{SE}(\bar{d})$ with the HAC Newey–West standard error (▸ Appendix)
  - HAC: heteroskedasticity and autocorrelation consistent; it stays valid when d_t is autocorrelated or its variance changes

Mini-Case: Estimate, Interval and MDE



- May 2003 – Sep 2026 (5 884 days)
  - mean spread +0.29% a year, $t = 0.27$
- Halves fixed in advance
  - May 2003 – Dec 2014, −2.28% ($t = -1.75$)
  - Jan 2015 – Sep 2026, +2.85% ($t = 1.63$)
- The chart: dots, the mean of d_t
  - bars, HAC 95% intervals; red, ±MDE
- **Minimum detectable effect (MDE)** (Appendix)
  - the smallest true effect a two-sided 5% test detects with 80% probability (its power)
  - 2.8 × SE, i.e. 3.1% a year for the full sample
- Every interval contains 0 and the sign flips between the halves
  - the "premium" is a feature of the chosen window
- "Not significant" is not "no effect"
- Type of evidence: description

MFM_ch0_ai_discovery ▶ Interactive

Your Turn: A Project Seed, the Specification

- **Question**: does the SPY − RSP spread survive after controlling for sector returns?
  - regress d_t on the daily log returns of the SPDR sector ETFs
  - test H_0: intercept $= 0$ with HAC standard errors
- **Regressors and window**, fixed before estimation
  - main specification: the nine Select Sector SPDRs launched in December 1998 (XLB, XLE, XLF, XLI, XLK, XLP, XLU, XLV, XLY), which cover the whole mini-case window
  - XLRE (real estate, 2015) and XLC (communication services, 2018) only in a second, shorter specification, with its window reported
- A join that keeps only days on which every regressor exists silently cuts the sample
  - to 2018–2026 if XLC is included
  - the intercept would then estimate the spread of a different period
- RSP is rebalanced to equal weights each quarter
  - so the sector loadings are not exactly constant

What We Learned: Estimation Uncertainty

- A comparison is trustworthy only with the right data, the right calendar and an honest interval
- A "daily price" depends on the venue and on the observation time
- Each wave of models solved a new problem with new data and computing power
  - without replacing the earlier ones
- Bubbles and crashes have recurred for four centuries
- The mean and the Sharpe ratio are known only to the precision the number of years allows
  - an interval that contains zero means "inconclusive", not "equal"
- Cap-weighted passive investing carries market concentration
- A claim, human or AI-generated, needs an estimate, an interval, a rival explanation and a stated limitation

Review Questions and Chapter 1

Retrieval questions

- Why does the SE of a mean return not fall when you use daily instead of monthly data?
- Why must prices be joined on common days before computing Bitcoin – S&P 500 returns?
- A Sharpe-ratio difference has a 95% interval $[-0.65,\ +0.88]$: what may you conclude?
- Why is the SPY/RSP ratio not a measure of passive ownership?

Chapter 1: stylised facts

- How heavy are the tails of daily returns, and is the kurtosis even finite?
- Does volatility have long memory?
- At which horizon do returns approach the Normal distribution?
- Do the facts hold for the S&P 500, Bitcoin, BVB stocks and EUR/RON?

- **Further study**: core reading and the Quantinar courses (▸ Appendix)

The Practical Decision

- **What I choose**
  - every mean and every Sharpe ratio with an SE and a 95% interval, on the same window and on common days
  - the iid SE ($\sigma_A/\sqrt{Y}$; about $1/\sqrt{Y}$ for a Sharpe ratio) as a first calculation, next to a block bootstrap (resampling blocks of consecutive days, which keeps autocorrelation)
  - the difference of two assets tested directly on common days, not through overlapping intervals
- **Under which conditions**
  - the iid SE assumes uncorrelated returns with constant variance
  - the SE of a Sharpe ratio or of a correlation requires a finite fourth moment, which Chapter 1 questions
  - several comparisons need a multiple-testing correction (Bonferroni: none of the 10 pairs stays significant)
- **What result would make me drop it**
  - the iid SE: a block-bootstrap interval much wider than the iid one (stock–bond correlation: 1.59 times)
  - a "premium": a sign that flips between windows fixed in advance (SPY − RSP: −2.28%, then +2.85% a year)

Financial Models (1/2): Quantum Computing, Principles



IBM Quantum System One, Ehningen (Germany), 2021

Photo: IBM (International Business Machines, Corporation) Research (2021); CC BY 2.0; Wikimedia Commons

- **Problem**: Monte Carlo pricing and risk need millions of simulated scenarios
- **Idea** (Orús, Mugel & Lizaso (2019); Herman et al. (2023); Appendix)
  - a quantum computer works with qubits, which can hold a superposition of 0 and 1
  - interference makes the right answer more likely
- **Proven gain** (Montanaro (2015); Appendix)
  - error ε: about $1/\varepsilon^2$ classical samples against $1/\varepsilon$ quantum steps, e.g. 10^8 against 10^4 for $\varepsilon = 10^{-4}$
  - the gain is quadratic, not exponential

Financial Models (2/2): Quantum Computing, Limits

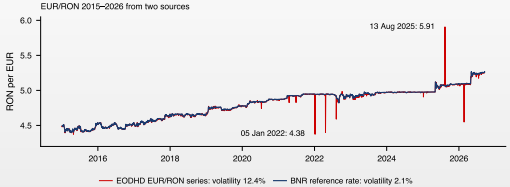


IBM Quantum System One, Ehningen (Germany), 2021

Photo: IBM Research (2021); CC BY 2.0; Wikimedia Commons

- **Not yet practical** (Chakrabarti et al. (2021); Appendix)
  - today's qubits are noisy
  - for the products and algorithms studied (an autocallable and a TARF, target accrual redemption forward), a useful pricing run needs about 8 000 logical qubits
- No wave of models replaced the earlier ones
  - VaR is still often computed with GARCH or by historical simulation
  - factor models still benchmark ML
- Timeline: Appendix; the stages and what made each possible: Appendix

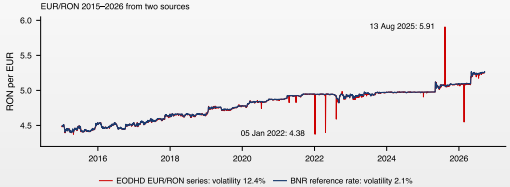
Data Errors: a Suspicious EUR/RON Move (1/2)



- The chart: EUR/RON (lei per euro)
  - red: the EODHD (EOD Historical Data, market data provider) series (aggregated interbank quotes, not an official rate)
  - blue: the BNR reference rate
- **Questions for the room**: on 13 Aug 2025 the EODHD series jumps from 5.0636 to 5.9065 RON (Romanian leu)
  - the jump is +15.4% as a log change, and the series falls back the next day
  - market event or data error?
  - how would you decide?

MFM_ch0_romania

Data Errors: a Suspicious EUR/RON Move (2/2)



- **Answer**: a **bad tick**
  - the jump reverses at once
  - the BNR reference rate, a second source, does not move
  - the same pattern: $\pm 12\%$ in early Jan 2022
  - annualised volatility 2015–2026: 12.4% with these ticks, 2.1% with the BNR rate
  - EUR/RON always from BNR
- Other errors in the course data: the Romanian 10-year yield
  - a 0.00% print on 24 Dec 2013
  - duplicated Sunday values in late 2022

MFM_ch0_romania

Sharpe Ratio: Precision and the Risk-Free Rate

- SPY, 2015–2026: $\widehat{\mathrm{SR}}_A = 0.82$, SE $= 0.29$
  - 95% confidence interval $[0.25,\ 1.39]$
- Like the mean, the SR is known only to about $\pm 2/\sqrt{Y}$
  - 11.7 years cannot separate 0.8 from 0.5
- With a risk-free rate $r_f > 0$ the Sharpe ratio is $(\mu_A - r_f)/\sigma_A$
  - every SR falls by r_f/σ_A, most for low-volatility assets
  - $r_f = 2\%$ lowers the SR by 0.20 at $\sigma_A = 10\%$, by only 0.05 at $\sigma_A = 40\%$; Chapter 0 sets $r_f = 0$
- The delta-method derivation and the effect of fat tails, dependence and selection on the SE: Appendix

Has the Stock–Bond Correlation Really Changed? (1/3)

- S&P 500 – TLT correlation computed once per subperiod (not the mean rolling correlation)
  - -0.47 in 2010–2020 ($n = 2\,769$), $+0.11$ in 2022–2026 ($n = 1\,182$)
  - estimated change $+0.58$
- **Fisher z test** of H_0: the two true correlations are equal

$$z = \frac{\operatorname{atanh} \hat{\rho}_2 - \operatorname{atanh} \hat{\rho}_1}{\sqrt{1/(n_1 - 3) + 1/(n_2 - 3)}}$$

  - $\hat{\rho}_k$, n_k: the sample correlation and the number of daily pairs in period k
  - for iid pairs from a bivariate Normal distribution, $\operatorname{atanh} \hat{\rho} = \frac{1}{2} \ln \frac{1+\hat{\rho}}{1-\hat{\rho}}$ is approximately Normal with variance $1/(n-3)$
  - under H_0, z follows the standard Normal distribution
- Result: $z = 17.9$, $p < 10^{-10}$
  - inference on correlations requires a finite fourth moment, which Chapter 1 questions

MFM_ch0_correlations

Has the Stock–Bond Correlation Really Changed? (2/3)

- The iid interval for the change, on the correlation scale

$$(\hat{\rho}_2 - \hat{\rho}_1) \pm 1.96 \sqrt{\frac{(1-\hat{\rho}_1^2)^2}{n_1 - 3} + \frac{(1-\hat{\rho}_2^2)^2}{n_2 - 3}}$$

  - $(1-\rho^2)^2/(n-3)$: the delta-method variance of a sample correlation, from $\mathrm{Var}(\operatorname{atanh}\hat{\rho}) \approx 1/(n-3)$
  - here $0.58 \pm 1.96 \times 0.032$, i.e. $[0.52,\ 0.65]$
- The iid interval is too narrow
  - volatility clustering makes pairs dependent
  - fat tails break normality
- **Stationary bootstrap**: blocks of consecutive days with random lengths (mean 20 days)
  - 95% interval $[0.48,\ 0.68]$, 1.59 times wider, still excluding 0

MFM_ch0_correlations

Has the Stock–Bond Correlation Really Changed? (3/3)

- Type of evidence: association (the variables move together; this does not prove causation)
  - the test establishes a change in association, not its cause
  - the 2022 inflation and rate shock needs its own test
  - rates and the yield curve: Appendix
- The windows are a choice
  - they leave out 2021, when US inflation began to rise, and the years before 2010
  - a conclusion that depends on the cut-off is fragile: report the change also with 2021 included or with an earlier start, with the alternative windows fixed in advance
- **Question for the room**: on the atanh scale the SE is $\sqrt{1/2\,766 + 1/1\,179} = 0.035$
  - which period contributes more to it, and why?

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- **Question for the room**: on the atanh scale the SE is $\sqrt{1/2\,766 + 1/1\,179} = 0.035$
  - which period contributes more to it, and why?
- **Answer**: 2022–2026, with $1/1\,179 = 0.00085$ against 0.00036
  - the shorter window dominates the uncertainty of a difference

MFM_ch0_correlations

Event Study: the December 2018 Bank Tax (1/4)

- **Question**: how much did Banca Transilvania (TLV) lose on the bank-tax announcement?
  - beyond the market's fall
- The tax was announced on the evening of 18 Dec 2018, so day 0 is 19 Dec 2018
- The **market model**, estimated by OLS on the estimation window $[-250, -11]$, $T = 240$ days

$$r_{i,t} = a + b\, r_{m,t} + \varepsilon_t$$

  - OLS: ordinary least squares
  - $r_{i,t}$, $r_{m,t}$: the daily log returns of TLV (adjusted close) and of the BET
  - the estimation window stops at day -11 so that the event cannot affect $\hat{a}$ and $\hat{b}$

Event Study: the December 2018 Bank Tax (2/4)

- The **abnormal return** (AR) and the **cumulative abnormal return** (CAR) over the event window $[0, +5]$

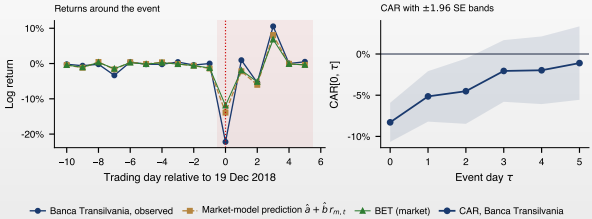
$$AR_\tau = r_{i,\tau} - (\hat{a} + \hat{b}\, r_{m,\tau}), \qquad CAR[0,+5] = \sum_{\tau=0}^{5} AR_\tau$$

- The test of H_0: the announcement has no effect on day 0

$$t = \frac{AR_0}{\mathrm{SE}(AR_0)}, \qquad \mathrm{SE}(AR_0) = \hat{\sigma}_\varepsilon \sqrt{1 + \frac{1}{T} + \frac{(r_{m,0} - \bar{r}_m)^2}{S_{mm}}}$$

  - $\hat{\sigma}_\varepsilon$: the residual standard deviation; $S_{mm} = \sum_t (r_{m,t} - \bar{r}_m)^2$ over the estimation window
  - the SE also includes the estimation error of $\hat{a}$ and $\hat{b}$, not only the day's noise (MacKinlay (1997))
  - with iid Normal errors, t follows a Student distribution with $T - 2$ degrees of freedom under H_0
  - the CAR sums $L = 6$ days, and its SE is computed in the same way

Event Study: the December 2018 Bank Tax (3/4)



- Left: the daily log returns of TLV, of the BET and the model prediction, on days $-10 \ldots +5$
  - red area: the event window
- Right: $CAR[0, \tau]$, cumulated up to day τ, with a ± 1.96 SE band
- Day 0: TLV -22.2%, model prediction -13.9%
  - so $AR_0 = -8.3\%$, $t = -6.85$, $p < 10^{-10}$
- $CAR[0, +5] = -1.1\%$, SE 2.27%, $t = -0.48$, $p = 0.63$
  - the band contains 0 from day 3 on

MFM_ch0_seminar

Event Study: the December 2018 Bank Tax (4/4)

- Market-model estimates: $\hat{b} = 1.18$, $\hat{\sigma}_\varepsilon = 0.83\%$ a day
- The day-0 abnormal reaction is clear, but days 1–3 recover most of the shock
  - over a week, the cumulative effect cannot be told apart from noise
- Type of evidence: causal identification of the day-0 reaction
  - if no other news arrived on day 0; the market-model prediction stands in for the counterfactual, with the limitations below
- Limitation: the BET holds TLV and BRD with large weights
  - so the prediction absorbs part of the banks' shock and the AR understates it
- Limitation: the announcement hit every bank on the same evening
  - so the AR of one bank is not independent of the others'
- Exercises: Seminar 0, B7 (TLV) and B8 (BRD)

Worked Example: Arbitrage Keeps the ETF Price Near NAV

- Illustrative numbers: NAV $= 100$, the ETF's market price $P = 100.50$
  - so the **premium** $(P - \text{NAV})/\text{NAV} = 0.5\%$
- **Questions for the room**
  - which trade does an authorised participant make?
  - what happens to the premium?

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- Illustrative numbers: NAV $= 100$, the ETF's market price $P = 100.50$
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- **Questions for the room**
  - which trade does an authorised participant make?
  - what happens to the premium?
- **Answer**: the AP buys the basket for 100 and delivers it to the issuer
  - it receives one new ETF share and sells it for 100.50, a profit of 0.50 before costs
  - the extra ETF shares push the price down until the premium disappears
  - at a **discount** $(P < \text{NAV})$ the AP buys ETF shares and redeems them for the basket
- Premiums persist where the NAV is stale or the basket is illiquid
  - the speed of decay is measured by the half-life of an AR(1), a first-order autoregressive model (Appendix)

Case Study: Where Eq. (5) and the Pass-Through Come From

- Assumptions behind eq. (5) (Section V.A)
  - every active investor has the same own elasticity $\bar{\mathcal{E}}$
  - those who turn passive are representative of active investors
- Step 1: put eq. (4) into the holdings-weighted average (eq. 2)

$$\mathcal{E}_{agg} = \alpha\,(\bar{\mathcal{E}} - \chi\,\mathcal{E}_{agg}) \quad \Rightarrow \quad \mathcal{E}_{agg} = \frac{\alpha\,\bar{\mathcal{E}}}{1+\chi\alpha}$$

  - active holdings have weight α, passive ones contribute 0
- Step 2: divide by the all-active value $\bar{\mathcal{E}}/(1+\chi)$, at $\alpha = 1$

$$\frac{\alpha(1+\chi)}{1+\chi\alpha} = \alpha + (1-\alpha)\,\frac{\chi\alpha}{1+\chi\alpha}$$

  - the result is eq. (5) written as one fraction
- Step 3: the elasticity of $\mathcal{E}_{agg}$ with respect to α is the pass-through of eq. (28)

$$\frac{d\ln\mathcal{E}_{agg}}{d\ln\alpha} = 1 - \frac{\chi\alpha}{1+\chi\alpha} = \frac{1}{1+\chi\alpha}$$

- Example, $\chi = 1$ and $\alpha = 0.5$
  - eq. (5) gives $0.5 \times 2/1.5 = 0.5 + 0.5 \times 0.5/1.5 = 0.67$
  - the pass-through is $1/1.5 = 0.67$, so a 1% fall in α lowers $\mathcal{E}_{agg}$ by about 0.67%
- If the most elastic investors turn passive first, the representativeness assumption fails (Seminar 0, C3)

Your Turn: A Project Seed, the Workflow

- **Replicate first**: SPY $\times 12.7$, RSP $\times 11.9$ and the mini-case t-statistics
  - with the same definitions (log returns of adjusted closes, joined on common days)
- **Pre-register**: H_0, H_1, test, sample, the HAC lag L (Appendix) and corrections, committed before the results
- **Report**: estimate, standard error, MDE and every specification tried
- **Publish** the code as a Quantlet
- **Log**
  - prompts that shaped decisions in `AI_USE.md`
  - every caught error in `AI_ERRORS.md`

Course Map, 20 Chapters

- **0** Financial Markets in 2026
- **1** Stylised Facts and Returns
- **2** Market Efficiency: From EMH to Adaptive Markets
- **3** Factor Models and Asset Pricing
- **4** Portfolio Optimisation
- **5** Conditional Volatility: GARCH Models
- **6** Multivariate Volatility and Dependence
- **7** Value-at-Risk and Expected Shortfall
- **8** Backtesting and Evaluating Risk Forecasts
- **9** Realised Volatility
- **10** Market Microstructure
- **11** Continuous-Time Models
- **12** Options and the Volatility Surface
- **13** Machine Learning in Finance
- **14** Deep Learning and Time-Series Foundation Models
- **15** LLMs and Sentiment Analysis
- **16** Digital Assets and DeFi
- **17** Bubbles and Crashes
- **18** Systemic Risk and Networks
- **19** Review and Project Presentations

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Core Reading and Further Study

- Franke, Härdle & Hafner (2019), *Statistics of Financial Markets*, 5th ed.: the main statistical reference
- Campbell, Lo & MacKinlay (1997), *The Econometrics of Financial Markets*
- Brandimarte (2018), *An Introduction to Financial Markets: A Quantitative Approach*
- Härdle & Simar (2019), *Applied Multivariate Statistical Analysis*
- Härdle, Chen & Overbeck (2017) (Eds.), *Applied Quantitative Finance*, 3rd ed.
- Härdle, Harvey & Reule (2020), "Understanding cryptocurrencies", *Journal of Financial Econometrics*
- Advanced courses on quantinar.com
  - Statistics of Financial Markets; Digital Transformation on Finance
  - Cryptos, NFTs, Digital Assets; Introduction to Blockchain and Cryptocurrencies

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Asset-Class Terms

- Money market
  - **T-bills**: Treasury bills, government debt with maturity below one year
  - **repo**: repurchase agreement, a short-term loan backed by securities
- Derivatives
  - **futures**: a contract to buy or sell later at a price fixed today
  - **options**: the right, not the obligation, to buy or sell at a fixed price
  - **swaps**: exchanges of cash-flow streams
- Risks
  - **idiosyncratic risk**: specific to one company
  - **counterparty risk**: the other side of a contract may not pay
- Infrastructure
  - **multilateral trading facility (MTF)**: a trading venue run outside a regulated exchange
  - **central counterparty (CCP)**: becomes the buyer to every seller and the seller to every buyer
  - **custodian**: keeps the securities

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Where Trading Happens

Venue	How trades are matched	What is visible	Examples
Exchange	central limit order book, auctions	quotes and trades	NYSE, Nasdaq, BVB
OTC dealer	bilateral, with a dealer	little, after the trade	bonds, FX, swaps
Dark pool	at a price taken from public markets	nothing before the trade	large equity orders
Centralised crypto exchange	order book run by a company, 24/7	quotes and trades	spot Bitcoin, Ether
AMM (decentralised)	a pool of two tokens, price set by a formula	the pool on the blockchain	Chapter 16

- **OTC**: negotiated between two parties without an exchange
- **AMM**: automated market maker, a smart contract that trades without an operator
- Makarov & Schoar (2020): large and persistent price gaps across the hundreds of crypto venues
  - kept open by capital controls and costs

Back

Auctions versus Continuous Trading

Call auctions

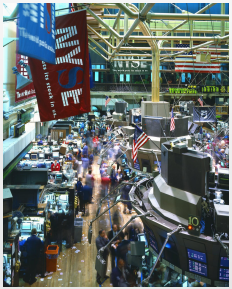
- Orders are collected, then matched at one price, the one that maximises traded volume
- Used at the open and the close of most exchanges
- They concentrate liquidity
  - the closing auction sets the daily close used in this course

Continuous trading

- Each incoming order is matched at once if possible
  - best price first, then the earliest order
- Speed is rewarded: **high-frequency trading (HFT)** operates in microseconds
- Budish, Cramton & Shim (2015): frequent batch auctions, to end the speed race

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From the Trading Floor to Screens



NYSE trading floor, 2011

Photo: Carol M. Highsmith (2011); public domain; Wikimedia Commons



Electronic displays, Chicago Board of Trade, 1992

Photo: Lars Plougmann (1992); CC BY-SA 2.0; Wikimedia Commons

- **Open outcry**: traders met on the floor
  - they matched orders by voice and hand signals
- Electronic order books gradually replaced floor trading
- Today orders are matched by computers (market microstructure: Chapter 10)

Back

The Roll Model of Bid–Ask Bounce

- Model: $p_t = m_t + (s/2)\, q_t$
  - m_t: the efficient price (a random walk); s: a constant spread in price units
  - $q_t = \pm 1$: independent, equally likely trade signs, unrelated to Δm_t
- Then $\Delta p_t = \Delta m_t + (s/2)(q_t - q_{t-1})$ and only the shared q_{t-1} links consecutive changes:

$$\operatorname{Cov}(\Delta p_t, \Delta p_{t-1}) = -\frac{s^2}{4}\operatorname{Var}(q_{t-1}) = -\frac{s^2}{4}, \qquad \hat{s} = 2\sqrt{-\hat{\gamma}_1}$$

- $\hat{\gamma}_1$: the sample autocovariance of consecutive price changes
  - $\hat{s}$ solves $\gamma_1 = -s^2/4$
- Limits (Roll (1984); Chapter 10)
  - if $\hat{\gamma}_1 > 0$ the estimator is undefined ($\hat{\gamma}_1 = 0$ gives $\hat{s} = 0$)
  - other dependence also moves $\hat{\gamma}_1$: not every negative autocovariance is bounce

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A Historic Crash, Black Tuesday, 1929



Crowd outside the NYSE, 29 October 1929

Photo: U.S. government (1929); public domain; Wikimedia Commons

- 29 October 1929 (**Black Tuesday**)
  - crowds gather outside the New York Stock Exchange during the crash
- The crash deepened a US recession that had begun in August 1929 (NBER business-cycle dates)
  - the initial contraction of the Great Depression lasted until 1933
- Bubbles and crashes: Chapter 17

Back

Tulips and the South Sea Bubble, Revisited



Jan Brueghel the Younger, *Satire on Tulip Mania*, c. 1640

Painting: Jan Brueghel the Younger; public domain; Wikimedia Commons



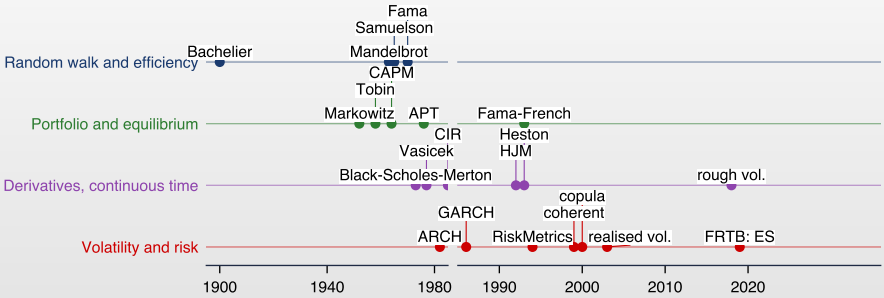
William Hogarth, *The South Sea Scheme*, 1721

Print: William Hogarth (1721); CC0; National Gallery of Art, Wikimedia Commons

- The popular story of tulip mania comes largely from moralising pamphlets written after the crash
- Garber (1989)
  - rare bulbs lost value the way new varieties do in later flower markets
  - common bulbs: a few winter weeks, tavern trading, contracts rarely enforced
- Goldgar (2007): few bankruptcies, little damage to the Dutch economy
- Frehen, Goetzmann & Rouwenhorst (2013): in 1720 the boom spread across Paris, London and Amsterdam
  - the largest rises for firms tied to Atlantic trade and to insurance
- Lesson (Chapter 17): a bubble rests on a story about a real change
  - dating it requires a model of fundamental value

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Financial Models, 1900–2026: an Overview

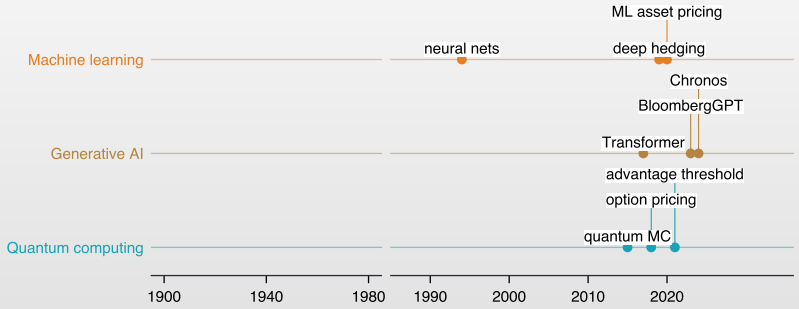


Abbreviations in the chart: CAPM, capital asset pricing model; APT, arbitrage pricing theory; Vasicek, CIR (Cox–Ingersoll–Ross) and HJM (Heath–Jarrow–Morton), interest-rate models; ARCH and GARCH, conditional-volatility models; FRTB: ES, the fundamental review of the trading book, which adopts expected shortfall; quantum MC, Monte Carlo simulation on quantum computers

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Machine Learning, Generative AI and Quantum Computing



ML, machine learning; AI, artificial intelligence; quantum MC, Monte Carlo simulation on quantum computers. The time axis marks the dates of the named modelling milestones.

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Origins: Random Walks and Efficient Markets



Louis Bachelier

Photo: unknown author; public domain; Wikimedia Commons

- Bachelier (1900), *Théorie de la spéculation*
  - prices of French government bonds follow a Brownian motion, five years before Einstein
  - the first option-pricing formula
- Mandelbrot (1963): cotton price changes have far heavier tails than the Normal distribution allows
- Samuelson (1965): geometric Brownian motion
  - keeps prices positive and makes log returns Normal
- Fama (1965); Fama (1970): the EMH in three forms
  - weak (past prices), semi-strong (public information) and strong (all information)
- In this course
  - stylised facts (Chapter 1)
  - efficiency tests (Chapter 2)
  - continuous-time models (Chapter 11)

Back

Portfolios and Equilibrium Pricing



William F. Sharpe, 2007

Photo: Larry D. Moore (2007); CC BY 4.0; Wikimedia Commons

- Tobin (1958): two-fund separation
  - with a riskless asset, every investor holds the same risky portfolio
- CAPM: $\mathbb{E}[R_i] - r_f = \beta_i \left(\mathbb{E}[R_m] - r_f\right)$ (Sharpe (1964); Lintner (1965); Mossin (1966))
  - only market risk is priced
  - R_i, R_m: returns of asset i and of the market portfolio; r_f: risk-free rate; $\beta_i = \mathrm{Cov}(R_i, R_m)/\mathrm{Var}(R_m)$
  - assumptions: mean–variance investors with common beliefs, borrowing and lending at r_f, no frictions
- APT (arbitrage pricing theory, Ross (1976))
  - no arbitrage with K common factors gives a linear factor model
- Three factors (Fama & French (1993))
  - market, size (SMB, small minus big) and value (HML, high minus low book-to-market)
- In this course
  - factor models (Chapter 3)
  - portfolio optimisation (Chapter 4)

Back

Derivatives and Continuous Time



Fischer Black, Myron Scholes, Robert C. Merton

Black: Dalmatine; public domain; Scholes: Nobellaureatesphotographer (2008); CC BY-SA 3.0; Merton: Digarnick (2006); public domain; Wikimedia Commons

- Black & Scholes (1973); Merton (1973): option prices by dynamic replication
- Heston (1993): stochastic volatility with a closed-form option price
- Gatheral, Jaisson & Rosenbaum (2018): rough volatility
  - the log-volatility follows a fractional Brownian motion with a Hurst exponent near 0.1
  - fractional Brownian motion B^H: Gaussian, with $\mathrm{Var}(B^H_{t+h} - B^H_t) = |h|^{2H}$, time step h and Hurst exponent $H \in (0, 1)$
  - $H = 1/2$ is Brownian motion, $H < 1/2$ gives rougher paths
- In this course: Chapters 11 and 12

Back

Interest-Rate Models

- Vasicek (1977): $dr_t = \kappa(\theta - r_t)\,dt + \sigma\,dW_t$
  - the short rate reverts to the level θ but can become negative
- CIR (Cox, Ingersoll & Ross (1985)): $dr_t = \kappa(\theta - r_t)\,dt + \sigma\sqrt{r_t}\,dW_t$
  - the rate's volatility shrinks as the rate approaches zero, so the rate does not become negative
- HJM (Heath, Jarrow & Morton (1992)): models the whole forward-rate curve directly
  - no arbitrage fixes each rate's drift as a function of its volatilities
- Notation
  - r_t: the short rate at time t (here a rate, not a log return); dr_t: its change over a short interval dt
  - $\kappa > 0$: the speed of mean reversion; θ: the long-run level
  - σ: the volatility of the rate; dW_t: the increment of a Brownian motion, Normal with mean 0 and variance dt
  - the drift $\kappa(\theta - r_t)$ is positive when $r_t < \theta$ and negative when $r_t > \theta$, so the rate is pulled towards θ

Back

Volatility Models



Robert F. Engle, 2022

Photo: Jon Demske (2022); CC BY-SA 4.0; Wikimedia Commons

- r_t: daily return (demeaned), σ_t^2: its variance conditional on the past
- ARCH (Engle (1982)): $\sigma_t^2 = \omega + \alpha r_{t-1}^2$
  - today's variance depends on yesterday's squared shock
- GARCH (Bollerslev (1986)): $\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$ adds the lagged variance
  - $\omega > 0$: a constant; $\alpha \geq 0$: the reaction to yesterday's shock; $\beta \geq 0$: the persistence of variance
  - $\alpha + \beta < 1$: the variance reverts to its long-run level $\omega/(1 - \alpha - \beta)$
- RiskMetrics (J.P. Morgan, 1994; J.P. Morgan/Reuters (1996)): $\sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1}^2$
  - an EWMA (exponentially weighted moving average) of squared returns; $\lambda = 0.94$ for daily data
  - the decay factor $\lambda \in (0, 1)$: a larger λ means a longer memory
  - the return k days back gets weight $(1 - \lambda)\lambda^{k-1}$: a GARCH(1,1) with $\omega = 0$ and $\alpha + \beta = 1$
- Realised volatility from intraday returns (Andersen et al. (2003))
- In this course: Chapters 5–9

Back

Risk Measurement and Regulation (1/2)

- VaR (value at risk) of a return X at tail probability α: the loss exceeded with probability α

$$\mathrm{VaR}_\alpha(X) = -q_\alpha(X), \qquad \mathrm{ES}_\alpha(X) = -\frac{1}{\alpha}\int_0^\alpha q_u(X)\,du$$

  - $q_\alpha(X)$: the α-quantile of X, with $P(X \le q_\alpha) = \alpha$
  - "VaR 1%" means $\alpha = 1\%$ (a tail probability here; in the case study α is the active share)
- ES (expected shortfall): the mean loss in the worst α of cases
  - the average of the quantiles q_u with $u \le \alpha$
  - for continuous distributions, $\mathrm{ES}_\alpha(X) = -\mathbb{E}[X \mid X \le q_\alpha(X)]$
  - $\mathrm{ES}_\alpha \ge \mathrm{VaR}_\alpha$: ES also looks at how bad the losses beyond the VaR are
- Basel: since 1996 the BCBS allows internal models of the 10-day VaR 1% (BCBS (1996))
  - BCBS: Basel Committee on Banking Supervision
  - the FRTB (fundamental review of the trading book) replaces it with ES 2.5% (BCBS (2019))

Back

Risk Measurement and Regulation (2/2)

- **Coherent** risk measure (Artzner et al. (1999)): $\rho(X)$ with four properties
  - $\rho(X)$: the capital needed for position X
  - monotone: a position that always pays less is riskier
  - translation invariant: adding cash c lowers ρ by c
  - positively homogeneous: doubling the position doubles ρ
  - subadditive: $\rho(X+Y) \leq \rho(X) + \rho(Y)$, diversification never adds risk
- VaR can violate subadditivity and so penalise diversification
- ES is coherent (Acerbi & Tasche (2002))
- EVT (extreme value theory) models the tails beyond most of the data (Embrechts, Klüppelberg & Mikosch (1997); McNeil & Frey (2000))

Back

The Gaussian Copula and 2008 (1/2)

- Li (2000): joint default times from marginal default probabilities F_i and a Normal dependence structure

$$\tau_i = F_i^{-1}(\Phi(Z_i)), \qquad Z_i = \sqrt{\rho}\, M + \sqrt{1-\rho}\, \varepsilon_i$$

  - τ_i: the default time of borrower i; $F_i(t)$: the probability that i defaults before t
  - Φ: the distribution function of the standard Normal distribution, which maps Z_i into $(0,1)$
  - Z_i: a latent credit variable; M: the common factor; ε_i: the borrower's own shock; $M, \varepsilon_i \sim \mathcal{N}(0,1)$, independent
  - $0 \leq \rho < 1$: the correlation between any two Z_i; a larger ρ makes defaults happen together, $\rho = 0$ makes them independent
- One parameter ρ priced every tranche of a CDO
  - the CDO is paid from the pool of loans in order of seniority

Back

The Gaussian Copula and 2008 (2/2)

- Drawbacks discussed after 2007–2008 (Donnelly & Embrechts (2010))
  - asymptotic tail independence: as the threshold becomes more extreme, the conditional probability of a joint default goes to zero
  - at finite thresholds the dependence can still be large
  - so default clustering in an extreme crisis is modelled poorly
  - this limit alone does not prove mispricing at the thresholds actually priced
  - the correlation implied by market prices differed across tranches, so one ρ could not price them all consistently
  - no economic factors in the model, so stress tests are hard to build
- Lesson: model risk
  - dependence in the tails (Chapter 6)
  - systemic risk (Chapter 18)

Back

Machine Learning in Finance

- Hutchinson, Lo & Poggio (1994): neural networks learn option prices and hedge ratios from data
  - without the Black–Scholes formula
- Gu, Kelly & Xiu (2020): US stocks, 1957–2016; trees and neural networks beat linear models out of sample
  - through nonlinear interactions
  - $R^2_{oos} = 1 - \sum(r_{i,t+1} - \hat{r}_{i,t+1})^2 / \sum r^2_{i,t+1}$ over the test months, with $r_{i,t+1}$ the excess return of stock i in month $t+1$ and $\hat{r}_{i,t+1}$ its forecast
  - the benchmark forecasts a zero excess return, so the denominator is not demeaned; $R^2_{oos} > 0$: the model beats it
- Buehler et al. (2019), deep hedging: a neural network chooses hedge positions under transaction costs
  - to minimise a convex risk measure, with no pricing model
- In this course: Chapters 13 and 14

Back

Generative AI and Foundation Models

- The Transformer (Vaswani et al. (2017)): attention layers read a whole sequence in parallel
- It underlies most current LLMs and GPT (generative pre-trained Transformer) models
- BloombergGPT (Wu et al. (2023)): a 50-billion-parameter LLM
  - a corpus of 363 billion tokens of financial text and 345 billion general tokens, of which 569 billion were used in training
- Foundation models for time series: Chronos forecasts new series zero-shot (Ansari et al. (2024))
  - it scales and quantises values into tokens
  - it trains a language model on many series
- Open problems besides look-ahead
  - cost, reproducibility and the decay of a signal once many traders use it
- In this course: Chapters 14 and 15

Back

Quantum Computing: Tested Applications



IBM Quantum System One, Ehningen (Germany), 2021

Photo: IBM Research (2021); CC BY 2.0; Wikimedia Commons

- The typical target: a mean (Orús, Mugel & Lizaso (2019); Herman et al. (2023))
  - e.g. an option price as a discounted expected payoff
- A VaR is a quantile, not a mean (Woerner & Egger (2019))
  - QAE (quantum amplitude estimation) estimates $P(X \leq x)$
  - a bisection search over x finds q_α
- **Demonstrated** on a few qubits
  - option pricing (Rebentrost, Gupt & Bromley (2018); Stamatopoulos et al. (2020))
  - VaR and ES (Woerner & Egger (2019))

Back

Classical Monte Carlo versus Quantum Amplitude Estimation (1/2)

- Target: $\mu = \mathbb{E}[f(X)]$, the mean discounted payoff f of a random market scenario X
- Classical MC (Monte Carlo) averages N simulated scenarios $X_1, \ldots, X_N$

$$\hat{\mu}_N = \frac{1}{N}\sum_{j=1}^{N} f(X_j), \qquad \mathrm{SE}(\hat{\mu}_N) = \frac{\sigma}{\sqrt{N}}$$

  - σ: the standard deviation of $f(X)$; accuracy ε needs $N = O(\sigma^2/\varepsilon^2)$ samples
- QAE prepares $\sqrt{1-\mu}\,|\psi_0\rangle|0\rangle + \sqrt{\mu}\,|\psi_1\rangle|1\rangle$ (for f scaled to $[0,1]$) (Woerner & Egger (2019))
  - μ is the probability of measuring 1 on the objective qubit, entangled with the register that holds X
  - $|0\rangle$, $|1\rangle$: the states of the objective qubit; $|\psi_0\rangle$, $|\psi_1\rangle$: the register states paired with them
- A Grover-type operator applied m times makes the error fall as $O(1/m)$
  - $O(1/\varepsilon)$ calls of the pricing circuit (Montanaro (2015); Rebentrost, Gupt & Bromley (2018))

Back

Classical Monte Carlo versus Quantum Amplitude Estimation (2/2)

- *Quadratic* means: where classical MC takes N steps, QAE takes about $\sqrt{N}$ (Montanaro (2015))
  - an exponential gain would mean about $\log N$
- In the oracle-query model no quantum algorithm does better in the worst case
  - up to logarithmic factors
- Why this is not yet an advantage
  - each call must load the distribution of X and run without error, which needs error correction
  - error-corrected operations are far slower than classical ones
  - classical MC parallelises easily across many processors

Back

Quantum Computing: Why It Is Not Yet Practical

- Preskill (2018) called the hardware of 2018 NISQ (Noisy Intermediate-Scale Quantum)
  - tens to a few hundred noisy qubits without error correction
- Physical qubit counts have grown since (IBM Research (2024))
  - IBM's Condor chip had 1 121 qubits in 2023
  - error mitigation: useful output from circuits with more than 100 qubits and 2 000 operations
- The real limit: the number of **logical qubits**
  - error-corrected, each built from many physical ones
- Usable circuit depth before noise dominates
  - it depends on error rates, connectivity, error mitigation and error correction
- Loading a return distribution into qubits is itself costly
- Chakrabarti et al. (2021): about 8 000 logical qubits for an autocallable and a TARF
  - a T-depth of 54 million; advantage would require running this program in about a second
  - T-depth: sequential layers of the slowest error-corrected gate
  - autocallable: a note redeemed early when the underlying crosses a barrier; TARF: target accrual redemption forward
  - an upper bound for these products and algorithms, not a universal threshold; still far beyond current machines

Back

What Changed Each Time, and Where It Appears in This Course

Wave	What made it possible	Chapters
Random walk, efficiency	probability theory, long price series	1, 2, 11
Portfolio, equilibrium	utility theory, the first stock-return databases	3, 4
Derivatives, continuous time	stochastic calculus, listed options (1973)	11, 12
Volatility and risk	daily and intraday data, bank regulation	5–9
Machine learning	large panels, cheap computing	13
Generative AI	text at scale, GPU (graphics processing unit) hardware, the Transformer	14, 15
Quantum computing	new hardware, not yet practical	outlook

- No wave replaced the earlier ones
  - implied volatility is still quoted through the Black–Scholes formula

Back

Dependence and Rates, Notation

- **Correlation**: Pearson correlation of returns, never of prices
- **Yield to maturity**: the internal rate of return of a bond's promised cash flows
  - equals the realised return only if the bond is held to maturity, pays in full and coupons are reinvested at the same yield
- **Yield-curve spread**: $y^{10y} - y^{2y}$
  - the 10-year government bond yield minus the 2-year yield
  - in percentage points or basis points (bp), 1 bp $= 0.01$ pp: $4.3\% - 3.8\% = 0.5$ pp $= 50$ bp
  - **inverted curve**: a negative spread, i.e. the 2-year rate exceeds the 10-year rate
  - in the US it has usually preceded recessions (Estrella & Mishkin (1998))

Back

$\sqrt{h}$ Scaling and the Variance Ratio

- Variance of an h-day return (the sum of h daily returns r_{t+j}), for stationary returns with variance σ^2:

$$\operatorname{Var}\Big(\sum_{j=1}^{h} r_{t+j}\Big) = h\sigma^2\Big[1 + 2\sum_{j=1}^{h-1}\Big(1 - \frac{j}{h}\Big)\rho_j\Big]$$

  - ρ_j: the correlation of returns j days apart; the weight $1 - j/h$: the share of such pairs
  - $\rho_j > 0$ raises the h-day variance above $h\sigma^2$, $\rho_j < 0$ lowers it
- The $\sqrt{h}$ rule is exact when returns are **uncorrelated** with **constant variance**
  - independence is not needed
- **Variance ratio** $\mathrm{VR}(h) = \text{Var}(h\text{-day return})/(h\sigma^2)$
  - 1 for uncorrelated returns, below 1 under mean reversion, above 1 under momentum
  - for an AR(1) with lag-1 autocorrelation ρ, $\mathrm{VR}(h) \to (1+\rho)/(1-\rho)$ as h grows
- S&P 500, 2000–2026: $\hat{\rho}_1 = -0.10$ and $\widehat{\mathrm{VR}}(20) = 0.75$
  - scaling daily volatility by $\sqrt{20}$ overstates the 20-day (monthly) risk
  - use the whole autocorrelation function or aggregate the returns directly (tests in Chapter 2)

Back

Variance Ratios and Variance Estimates, More Examples

- EUR/RON, BNR reference rate, 2005–2026: $\hat{\rho}_1 = 0.17$
  - a hypothetical AR(1) would give the factor $\sqrt{1.17/0.83} = 1.19$: the volatility annualised with $\sqrt{252}$, 4.4%, would become 5.3%
  - but $\hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4 < 0$ offset lag 1: monthly returns still give 4.4%
- EUR/RON series of the market-data provider: bad ticks give $\hat{\rho}_1 = -0.57$
  - they inflate the volatility
  - they are errors to remove, not an AR(1)
- Variance is estimated differently from the mean: for iid returns with a finite fourth moment

$$\mathrm{Var}(\hat{\sigma}^2) \approx \sigma^4(\kappa - 1)/T, \qquad \kappa = \mathbb{E}[(r_t - \mu)^4]/\sigma^4 \text{ (the kurtosis)}$$

  - precision improves slowly when κ is large (SPY daily: $\kappa = 17.1$, Chapter 1)
  - with volatility clustering, $\sigma^4(\kappa - 1)$ becomes the long-run variance of $(r_t - \mu)^2$; with an infinite fourth moment the approximation fails

Back

The Sharpe Ratio, Delta Method, Fat Tails, Dependence and Selection (1/2)

- **Delta method**: if $\sqrt{T}(\hat{\theta} - \theta) \xrightarrow{d} N(0, \sigma^2)$ and g is differentiable
  - then $\sqrt{T}\left(g(\hat{\theta}) - g(\theta)\right) \xrightarrow{d} N\left(0, g'(\theta)^2\sigma^2\right)$
  - θ: a parameter, $\hat{\theta}$: its estimator from T observations; $\xrightarrow{d}$: convergence in distribution as $T \to \infty$
  - a first-order Taylor expansion, $g(\hat{\theta}) \approx g(\theta) + g'(\theta)(\hat{\theta} - \theta)$, carries the standard error from $\hat{\theta}$ to $g(\hat{\theta})$
- Applied to $\mathrm{SR}_d = \mu_d/\sigma_d$, for iid returns following the Normal distribution (Lo (2002)):

$$\sqrt{T}\left(\widehat{\mathrm{SR}}_d - \mathrm{SR}_d\right) \xrightarrow{d} N\left(0,\ 1 + \mathrm{SR}_d^2/2\right) \quad \Rightarrow \quad \mathsf{SE}(\widehat{\mathrm{SR}}_A) = \sqrt{\frac{1 + \mathrm{SR}_A^2/(2A)}{Y}} \approx \frac{1}{\sqrt{Y}}$$

  - the 1 comes from estimating the mean, $\mathrm{SR}_d^2/2$ from estimating the standard deviation

Back

The Sharpe Ratio, Delta Method, Fat Tails, Dependence and Selection (2/2)

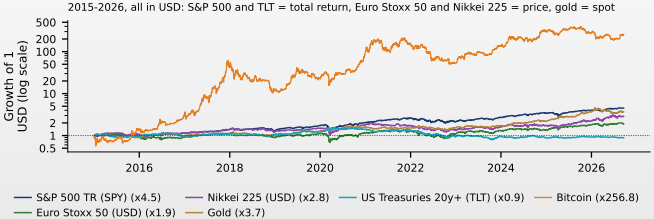
- Non-Normal iid returns with skewness γ_3 and kurtosis γ_4 (Opdyke (2007)): the asymptotic variance of $\sqrt{T}\,(\widehat{\mathrm{SR}}_d - \mathrm{SR}_d)$ is

$$V = 1 - \mathrm{SR}_d\gamma_3 + \mathrm{SR}_d^2(\gamma_4 - 1)/4, \qquad \mathrm{SE}(\widehat{\mathrm{SR}}_A) = \sqrt{AV/T}$$

  - $\gamma_3 = \mathbb{E}[(R_t - \mu_d)^3]/\sigma_d^3$, $\gamma_4 = \mathbb{E}[(R_t - \mu_d)^4]/\sigma_d^4$; the Normal distribution ($\gamma_3 = 0$, $\gamma_4 = 3$) gives back $1 + \mathrm{SR}_d^2/2$
  - for $\mathrm{SR}_d > 0$, negative skewness and fat tails raise V and widen the interval
  - for SPY the SE becomes 0.30 instead of 0.29, because the daily SR is small
- Dependent returns: HAC or studentised bootstrap inference (Ledoit & Wolf (2008))
- Picking the best of many assets or strategies ex post inflates the SR
  - the deflated SR corrects for it (Bailey & López de Prado (2014), Chapter 13)

Back

Asset Classes since 2015



- Value of 1 USD invested on 2 Jan 2015, log scale
- SPY and TLT are total return
  - Euro Stoxx 50 and Nikkei 225 are price indices, without dividends
  - so the comparison with SPY is not one between total returns of equivalent investments
  - converted to USD with daily FRED exchange rates
- Long Treasuries ended below their 2015 level ($\times 0.9$)

MFM_ch0_cross_asset

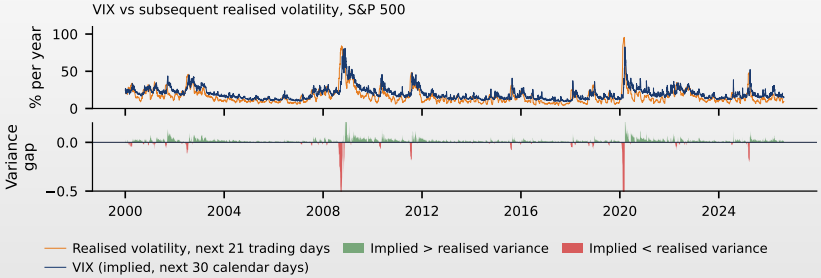
Back

Selection, Survivorship and Horizon Effects

- **Selection**: Bitcoin's $\times 257$ is chosen ex post among thousands of coins
  - some of which disappeared (survivorship bias)
  - a best-of-many Sharpe ratio needs deflating
- **Horizon**: the depth of the maximum drawdown grows with the sample length
  - if the log price is a driftless Brownian motion with annual volatility σ, the largest log fall from a peak, $D = \max_{s \leq t \leq T}(\ln P_s - \ln P_t)$, has $E[D] = \sqrt{\pi/2}\,\sigma\sqrt{T}$, with T in **years** (Magdon-Ismail et al. (2004))
  - $\sigma = 17.6\%$: $E[D] = 0.70$ over 10 years and 1.40 over 40, i.e. -50% and -75% as price declines, $e^{-E[D]} - 1$
  - the expected percentage MDD, $E[e^{-D}] - 1$, is smaller in magnitude (Jensen): about -47% and -71% in simulated paths
  - with a positive drift, as for the S&P 500, the expected MDD is smaller still: these numbers show the $\sqrt{T}$ scaling, not a forecast
- BET: -82.5% over 29 years
  - so it cannot be compared with the S&P 500 MDD over 11.7 years

Back

Implied versus Realised Variance (1/2)



- Upper panel: the VIX and the realised volatility, in % per year
  - VIX: implied, next 30 calendar days
  - realised: over the next 21 trading days
- Lower panel: $(\mathrm{VIX}/100)^2$ minus realised variance
  - green when implied variance exceeds realised, red otherwise

MFM_ch0_drawdowns_vix

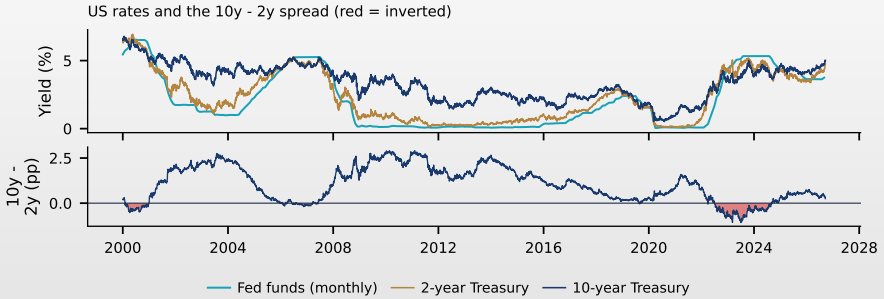
Back

Implied versus Realised Variance (2/2)

- $(\mathrm{VIX}/100)^2$: the annualised S&P 500 variance over 30 days
  - expected under the **risk-neutral** probability
- **Variance risk premium**: risk-neutral minus real-world expected variance
  - positive on average (Carr & Wu (2009))
- Realised volatility: $\sqrt{(252/21)\sum r^2}$ over the next 21 daily returns r
- 2000–2026: mean VIX 19.8 against 16.2 realised
- Ex post the gap is premium plus forecast error
- Bollerslev, Tauchen & Zhou (2009): the premium predicts quarterly returns

MFM_ch0_drawdowns_vix

Interest Rates and the Yield Curve (1/2)



- Upper panel: the 2- and 10-year US Treasury yields (daily) and the fed funds rate (monthly)
  - in %
- Lower panel: the 10-year − 2-year spread, in pp; red: an inverted curve

MFM_ch0_rates

Back

Interest Rates and the Yield Curve (2/2)

- Source: FRED
  - **fed funds**: the overnight rate targeted by the Federal Reserve
- Spread 10y − 2y
  - inverted from 6 Jul 2022 to 26 Aug 2024: 537 trading days, the longest spell
  - minimum −1.08 pp (3 Jul 2023)
- 18 Sep 2026: 10-year 5.01%, 2-year 4.76%, spread 25 bp
  - fed funds 3.63% (Aug 2026)
- A yield is not a realised return: TLT lost value in 2015–2026

MFM_ch0_rates

Asynchronous Closes

- Bitcoin (24:00 UTC) and the S&P 500 (20:00 or 21:00 UTC) close at different times
  - news arriving between the two times enters the two series on different days
- With a common factor, the same-day correlation is attenuated
  - the rest appears in lead/lag correlations
  - if only the common factor is split across days and daily variances are unchanged, the lead, same-day and lag correlations sum to the true one (Scholes & Williams (1977); Lo & MacKinlay (1990); Seminar 0, A12)
  - otherwise use longer returns (Burns, Engle & Mezrich (1998))
- Bitcoin – S&P 500, 2022–2026, common days
  - the correlations at lag 0, $+1$ and -1 sum to $0.42 + 0.03 - 0.05 = 0.40$
- The sum is a diagnostic, not a proof
  - the model allows no negative lead or lag correlation
  - but -0.05 is within two iid standard errors ($1/\sqrt{n} \approx 0.03$ for about 1 180 common days)
  - so it is compatible with sampling noise

Back

Correlation in Turbulent Periods, the Volatility Bias

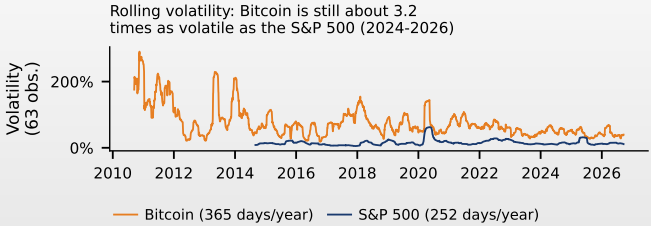
- Stable link $y = \beta x + \varepsilon$ with correlation ρ (Forbes & Rigobon (2002))

$$\rho^* = \rho \sqrt{\frac{1+\delta}{1+\delta\rho^2}}, \qquad |\rho^*| > |\rho| \quad \text{for } \delta > 0,\ 0 < |\rho| < 1$$

  - β and $\mathrm{Var}\,\varepsilon$ fixed
  - estimated only on days when $\mathrm{Var}(x)$ is $(1+\delta)$ times higher, it gives ρ^*
  - x: the return of the source market (S&P 500); y: the return of the other market; ε: the part of y unrelated to x
  - ρ^*: the correlation measured on the turbulent days; δ: the relative increase in $\mathrm{Var}(x)$
- Bitcoin – S&P 500, 2022–2026, days split by the VIX (top quartile: VIX > 21.8)
  - 0.52 on high-VIX days and 0.32 on the others
  - S&P 500 variance on high-VIX days over that on the other days: $1 + \delta = 4.64$
  - adjusted $\hat{\rho} = \rho^* / \sqrt{1 + \delta(1 - \rho^{*2})} = 0.27$, the first formula solved for ρ
  - reading: the adjusted 0.27 is not above the calm-day 0.32; the higher variance alone can account for the jump to 0.52, without a stronger link between the markets
- The adjustment assumes no omitted common shock and no feedback between the two markets

Back

Bitcoin Remains Several Times as Volatile as Equities

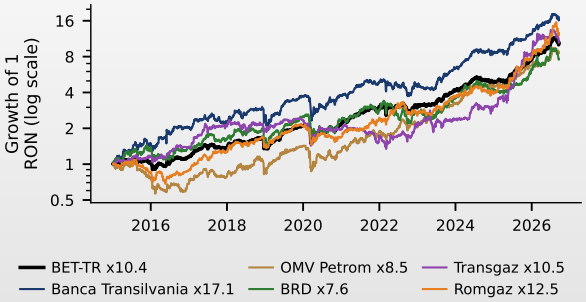


- Rolling annualised volatility over the last 63 observations
  - about 3 months for the S&P 500 (annualised with $\sqrt{252}$)
  - about 2 months for Bitcoin (annualised with $\sqrt{365}$)
- Bitcoin: 65% on average in 2015–2017, 46% in 2024–2026
  - S&P 500 in 2024–2026: 14%, so Bitcoin is still about **3.2 times** as volatile
- Volatility changes over time for both
  - a constant-volatility model is a poor description

MFM_ch0_correlations

Back

Romanian Equities since 2015

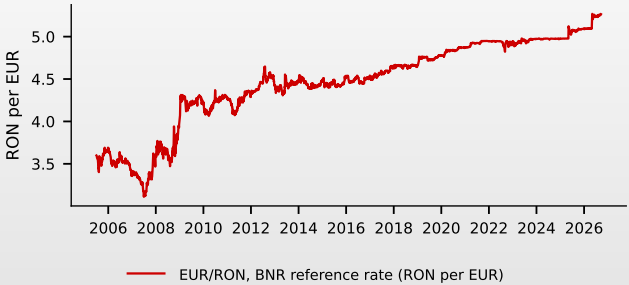


- Growth of 1 RON invested in January 2015, dividend-adjusted (adjusted close), log scale
- BET-TR $\times 10.4$
  - Banca Transilvania ($\times 17.1$) and Romgaz ($\times 12.5$) beat the index
  - BRD ($\times 7.6$) and OMV Petrom ($\times 8.5$) lagged it

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The Leu, the EUR/RON Reference Rate

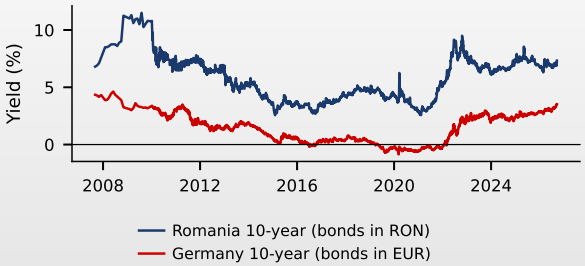


- EUR/RON (BNR reference rate) from 3.60 (4 Jul 2005) to 5.26
  - 4 Jul 2005: after the redenomination 1 RON = 10 000 ROL (Romanian old leu, ISO 4217 code, until 2005)
- A managed currency: annualised volatility of 2.1% in 2015–2026
  - long calm spells and occasional steps

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Back

Romanian and German 10-Year Government Yields



- 18 Sep 2026: Romania 7.33% (bonds in RON), Germany 3.53% (in EUR)
  - a cross-currency differential of 3.8 pp
- The differential is **not** a pure sovereign-credit spread; it mixes
  - expected leu depreciation, inflation and policy-rate gaps
  - credit, liquidity and term premia

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Back

The Bucharest Stock Exchange, 1882–1948

- **1 Dec 1882**: the securities and commodities exchange opens (BVB (history))
  - under a royal decree of King Carol I
- Early shocks (BVB press release (2017))
  - the 1907 peasants' revolt and the Balkan wars of 1912–1913
  - closed during the First World War
- After reopening (BVB press release (2017))
  - about 7 years of strong growth
  - then about 7 years of accelerated decline, overlapping the Great Depression of 1929–1933
- **1929**: a new law on exchanges Lege asupra burselor (1929)
  - with the explanatory memorandum of minister Virgil Madgearu
- **1948**: trading stops (BVB (history))
  - with the communist regime and the nationalisation of the economy



Share of UDR (Uzinele și Domeniile Reșița), June 1926. Scan: EDHAC e.V.; public domain; Wikimedia Commons

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Palatul Bursei, the Exchange's Historic Home



Photo: Nicolae Ionescu (1928); public domain; Wikimedia Commons



Photo: Chainwit. (2023); CC BY 4.0; Wikimedia Commons

- Built 1906–1912 Palatul Bursei
  - architect Ștefan Burcuș
  - Str. Ion Ghica 4
- Seat of the Exchange and the Chamber of Commerce
- From 1955: the State Central Library
- Left: stockbrokers, 1928
- Right: the building in 2023

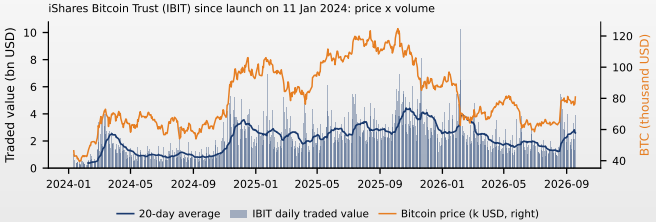
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Interruption and Rebirth, 1948–2023

- **1948–1995**: almost 50 years without a stock exchange (BVB press release (2017))
- **20 Nov 1995**: first session of the re-established exchange (BVB press release (2017))
  - 905 shares of 6 listed companies
- **1997**: BET starts with 10 companies (BVB (indices))
  - the basket may hold 10–20 and today has 20
  - **BET-FI**: the first sector index, for the financial investment companies (SIFs, listed in 1999)
- **2010**: BVB lists its own shares on its **regulated market** (BVB (history))
  - regulated market: an EU-authorised (European Union) trading venue
- **2020**: FTSE Russell reclassifies Romania from Frontier to Emerging (BVB (history))
- **2023**: the Hidroelectrica IPO (BVB (history))
- Largest daily BET move: 7 Jan 2009, -12.3% (-13.1% log)
  - after $+4.8\%$ the day before (official BVB daily archive BVB (indices))

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Spot Bitcoin ETFs



- **Spot** ETF: holds the asset itself, not futures
  - **ETP** (exchange-traded product) is the wider category
- Spot Bitcoin ETPs were approved in the US on 10 Jan 2024 (SEC (2024))
- IBIT (iShares Bitcoin Trust), trading since 11 Jan 2024
  - median daily traded value 1.97 bn USD, peak 10.3 bn USD (5 Feb 2026)
- Right axis: BTC, the ticker of Bitcoin, in thousand USD

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Stablecoin Supply



- **Supply**: the USD value of USD-pegged stablecoins in circulation (DefiLlama)
  - 4.2 bn (1 Jan 2020), peak 187.4 bn (2 Apr 2022)
  - trough 122.7 bn (19 Aug 2023), 303.0 bn (18 Sep 2026)
  - Seminar 0, B10
- The 2022–2023 fall followed the TerraUSD collapse and the crypto crash

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Tokenisation and Regulation

Tokenised real-world assets

- **RWA** (real-world assets): bonds, fund shares, deposits, commodities
- Token: a claim on such an asset, on a shared ledger
  - programmable, near-instant transfer and settlement
- Examples: tokenised money-market funds (BIS (2023); BIS (2025))

Regulation, key dates

- EU (European Union): MiCA, the Markets in Crypto-Assets regulation
- US: spot Bitcoin ETPs (Jan 2024) and the GENIUS (Guiding and Establishing National Innovation for U.S. Stablecoins, Act) Act on payment stablecoins
- The dates rules enter into force can serve as structural breaks in event studies

- Framework and supervisors: next slide
- Statistical question: do tokenised assets inherit the properties of the underlying?
  - or those of crypto markets?

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Regulation of Digital Assets

	European Union	United States
Framework	MiCA, Regulation (EU) 2023/1114	GENIUS Act, Public Law 119-27 for payment stablecoins
Crypto ETPs	exchange-traded notes on crypto	spot Bitcoin ETPs approved in January 2024 (SEC (2024))
Supervisors	ESMA, EBA, national authorities	SEC, CFTC, banking regulators

- ESMA: European Securities and Markets Authority
- EBA: European Banking Authority
- SEC: Securities and Exchange Commission
- CFTC: Commodity Futures Trading Commission
- BIS: the Bank for International Settlements, the bank of central banks
  - BIS (2023): a "unified ledger" combining central bank money, deposits and assets
  - BIS (2025): stablecoins fall short of the three tests for money: **singleness** (accepted at par by all), **elasticity** (supply follows payment needs), **integrity** (resists financial crime)

Back

ETF Premium Dynamics

- Statistical view of the arbitrage: an AR(1) premium

$$\pi_t = \phi\,\pi_{t-1} + u_t, \qquad \text{half-life} = \frac{\ln 0.5}{\ln \phi}, \quad 0 < \phi < 1$$

- π_t: the premium on day t
  - ϕ: its persistence; u_t: a new mispricing shock
- Half-life: the number of days for a premium to halve, solving $\phi^h = 0.5$
- ϕ close to 1: premiums decay slowly (Petajisto (2017))
  - as for ETFs on foreign stocks or illiquid bonds
- In the US, in-kind redemptions also let the fund avoid distributing capital gains
  - a US tax rule, not a general feature

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Case Study, Demand System and Estimation (1/2)

- **Data**: Form 13F filings (Section III.B)
  - managers with discretion over at least USD 100 million in 13(f) securities
  - characteristics X_k: log book equity (and its square), profitability, investment, dividend yield, each transformed towards the Normal distribution
  - universe $\mathcal{N}_i$: the stocks investor i held in the past three years
- **Demand** (eq. 13–14):

$$\log(w_{ik}/w_{i\mathbf{0}}) - p_k = \bar{d}_{\mathbf{0}i} + \bar{d}'_{\mathbf{1}i} X_k - \mathcal{E}_{ik}\, p_k + \xi\, \mathcal{E}_{agg,k} + \zeta' \mathcal{E}_{agg,k} X_k + \epsilon_{ik}$$

  - w_{ik}: weight of stock k in the portfolio of investor i, $w_{i\mathbf{0}}$: the outside asset
  - p_k: log market capitalisation, $P_k = e^{p_k}$ (shares normalised to 1)
  - $\bar{d}_{\mathbf{0}i}$, $\bar{d}_{\mathbf{1}i}$: demand intercept and characteristic loadings; ϵ_{ik}: demand shock

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Case Study, Demand System and Estimation (2/2)

- ξ, ζ: loadings on $\mathcal{E}_{agg,k}$ and on its interaction with X_k
  - both from the reference price
- Active investors: $\mathcal{E}_{ik} = \bar{\mathcal{E}}_{\mathbf{0}i} + \bar{\mathcal{E}}'_{\mathbf{1}i} X_k - \chi\, \mathcal{E}_{agg,k}$; passive: $\mathcal{E}_{ik} = 0$
  - baseline intercept $\bar{\mathcal{E}}_{\mathbf{0}i}$, loadings $\bar{\mathcal{E}}_{\mathbf{1}i}$ and strategic response χ
- Baseline estimates (Section III.B)
  - $\bar{\mathcal{E}}_{\mathbf{1}i}$ and ζ restricted to 0 on every characteristic except book equity
- **Equilibrium** (eq. 17–18): $\sum_i w_{ik} A_i = P_k$ and $\sum_i (w_{ik} A_i / P_k)\, \mathcal{E}_{ik} = \mathcal{E}_{agg,k}$
  - A_i: the investor's assets (not the annualisation factor A)
- **Estimator**: GMM with a nested fixed point solved by Newton's method (Section III.C)
  - with $\chi = 0$ it is the demand system of Koijen & Yogo (2019)
  - investors with fewer than 1 000 holdings pooled by AUM (assets under management) into groups of about 2 000 holdings (Table 2)
- **Price instrument** (eq. 23): $\hat{p}_{k,i} = \log \sum_{j \neq i} A_j \, 1\{k \in \mathcal{N}_j\} / |\mathcal{N}_j|$
  - built only from the other investors $j \neq i$, each spreading its assets A_j equally over its universe; $|\mathcal{N}_j|$: the number of stocks in it
- **Elasticity instrument** (eq. 24)
  - the elasticity condition solved with the same equal-weighted portfolios

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Case Study, Complete Results

Topic	Number in the paper	Where	Reading
Strategic response	$\chi = 2.97$ (SE 0.47); $\zeta = -0.48$ (SE 0.14)	Table 2, row 1	between 0 and ∞
Robustness	χ from 2.30 to 3.27 in rows 2–14; 0.04 without the elasticity instrument, 0.63 without instruments	Table 2	endogeneity hides the response
Elasticity level	mean $\mathcal{E}_{agg}$ 0.438 (fixed elasticities 0.39); residual cross-sectional SD 0.0395 (fixed 0.0842)	Table 3	reactions halve the dispersion
Rise of passive	median active share 81.4% (2001), 59.5% (2020)	Figure 5	passive 19% to 41%
Reduced form	slope of $\Delta \log \mathcal{E}_{agg}$ on $\Delta \log$ active share 0.385 (SE 0.022); 0.336 (SE 0.055) with Russell 1000/2000 switches	Table 4, col. 2, 5	consistent with one-third
Prices	per SD of elasticity, instrumented: volatility -0.383 (SE 0.123), illiquidity -0.396 (SE 0.131)	Table 5, col. 2, 8	inelastic: more volatile
Evolution	mean $\mathcal{E}_{agg}$ 0.41 to 0.27; with $\chi = 0$ the fall about twice as large	Figures 6, 8	response damped the fall

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Case Study, Limits and an Open Question

- Limits stated by the authors
  - one χ for all investors; no strategic entry, exit or fund flows (Section I.B)
  - one 13F form per institution: its active and passive funds cannot be separated (footnote 51)
  - the elasticity is for single stocks, not for the market as a whole (footnote 37)
  - those who turn passive are assumed representative of active investors (Section V.A)
- Passive ownership is endogenous
  - so identifying its effect (Sammon (2025)) relies on index reconstitutions and flow shocks
- What the course data can check: next slide

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Case Study, What the Course Data Can Reproduce

- Without the 13F holdings χ cannot be re-estimated
  - the arithmetic of Section V.A can be checked
  - 0.90 from eq. 5 and 0.33 from eq. 28, on the main slides
- The pass-through changes with α
  - integrating eq. (28) from $\alpha_0 = 0.814$ to $\alpha_1 = 0.595$ with $\chi = 2.97$ gives $\ln \frac{\alpha_1(1+\chi\alpha_0)}{\alpha_0(1+\chi\alpha_1)} = -0.313 + 0.211 = -0.102$
  - about 10%, the same order as the headline
- Data limitation: the paper's passive-ownership measure cannot be rebuilt from public aggregates
  - the Financial Accounts of the United States do not separate index from active mutual funds, and their ETF series mixes passive and active funds
  - so the paper's passive-share path, and the elasticity path implied by it, are not reproduced here
- The authors also point to bank regulation, fire sales and large sovereign investors (Section VI)

Back

Mini-Case, Estimation and Power

- **Minimum detectable effect (MDE)**: the smallest true effect detected with 80% power

$$\mathrm{MDE} = (z_{0.975} + z_{0.80}) \times \mathrm{SE} \approx 2.8 \times \mathrm{SE}$$

  - by a two-sided test at the 5% level (Normal approximation)
  - z_p: the p-quantile of the standard Normal distribution
  - SE: the HAC standard error of the annualised mean spread $252\,\bar{d}$, in pp a year
  - full sample: $\mathrm{SE} = 0.29/0.27 \approx 1.1$ pp, so $\mathrm{MDE} = 2.8 \times 1.1 = 3.1$ pp a year
  - 2015–2026: 4.9% a year, larger than the estimate itself (+2.85%)
- Causality: SPY − RSP is an outcome, not a measure of passive ownership
  - a causal claim needs an exogenous shifter

MFM_ch0_ai_discovery

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Mini-Case, the HAC Standard Error

- Daily spreads d_t are serially correlated and their variance changes over time
  - so the iid formula $s_d/\sqrt{T}$ misstates SE($\bar{d}$)
- The **Newey–West (HAC)** estimate adds the autocovariances up to lag L

$$\widehat{\mathrm{Var}}(\bar{d}) = \frac{1}{T}\Big[\hat{\gamma}_0 + 2\sum_{l=1}^{L}\Big(1 - \frac{l}{L+1}\Big)\hat{\gamma}_l\Big]$$

  - with weights that decline linearly
  - $\hat{\gamma}_l$: the sample autocovariance of d_t at lag l ($\hat{\gamma}_0$: its variance); T: the number of daily spreads
  - the declining weights keep the estimate non-negative; $L = 0$ gives back the iid variance
  - positive autocorrelation raises the SE, negative autocorrelation lowers it
- The lag L is stated in the pre-registration
  - the common rule $L = \lfloor 4(T/100)^{2/9} \rfloor$ gives $L = 9$ for $T = 5\,884$
  - report also the results with $L/2$ and $2L$

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A Minimal Python Workflow

```
import numpy as np, pandas as pd

URL = ('https://raw.githubusercontent.com/'
       'danpele/MFM/main/data/market/')
def load(sym, col='close'):
    d = pd.read_csv(URL + sym + '.csv',
                    index_col='date', parse_dates=True)
    return d[col].rename(sym)

spx = load('GSPC.INDX')                  # index: close
spy = load('SPY.US', 'adjusted_close')  # ETF
btc = load('BTC-USD.CC')                 # 24/7
r_spx = np.log(spx).diff()      # one series
# joint analysis: join PRICES on common
# days first, then take returns
p = pd.concat([spx, btc], axis=1).dropna()
r = np.log(p).diff().dropna()
dd = spx / spx.cummax() - 1              # drawdown
```

- Joined first, the prices give returns that cover the same calendar-date interval
  - Friday → Monday
- Otherwise Bitcoin Sunday → Monday is paired with S&P 500 Friday → Monday

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Data Checklist

- Plot the raw series and list the largest returns
- Compare two sources when possible
- Check adjustments: dividends, splits, redenominations
- Remove spurious non-trading-day records
- Investigate isolated outliers
  - correct or remove only confirmed data errors
- Document every change
- For several series, join the prices on common days before computing returns

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The Discovery Loop, Reusable Prompts (1/2)

- **Literature**
  - `List peer-reviewed papers since 2018 on passive ownership and price informativeness. For each: DOI, identification strategy, main estimate. Flag any paper you are not sure exists.`
  - resolve every DOI on Crossref (title and authors must match) and read the abstract yourself
- **Hypothesis**
  - `Give three competing explanations for SPY beating RSP after 2015 (passive flows, sector composition, earnings growth of the largest firms). For each, one testable implication the others do not share.`
- **Data and code**
  - `Write Python that reads the daily adjusted (total-return) prices of SPY and RSP from the course data, joins prices on common days, and tests H0: mean daily log-return spread = 0 with a Newey-West t-statistic.`

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The Discovery Loop, Reusable Prompts (2/2)

- **Robustness**
  - `Write a pre-registered robustness plan: subsamples, HAC bandwidths, simple vs log returns, a sector-neutral version. State in advance which outcome would falsify the hypothesis.`
- **Critique**
  - `Act as a hostile referee. List the three strongest threats to identification in this design and the test that addresses each.`
- **Report**: estimate, standard error, sample
  - every specification tried, also the failed ones
- What the researcher must check
  - invented references: resolve each DOI
  - unadjusted instead of adjusted close: reproduce the total-return multiples first
  - returns before the join: count the common days
  - window shopping: fix the windows in advance; low power: report the MDE

Back

References (1/7)

- Acerbi, C., Tasche, D. (2002). On the coherence of expected shortfall. *Journal of Banking & Finance*, 26(7), 1487–1503.
- Andersen, T.G., Bollerslev, T., Diebold, F.X., Labys, P. (2003). Modeling and forecasting realized volatility. *Econometrica*, 71(2), 579–625.
- Ansari, A.F. et al. (2024). Chronos: learning the language of time series. *arXiv:2403.07815*.
- Artzner, P., Delbaen, F., Eber, J.-M., Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9(3), 203–228.
- Bachelier, L. (1900). Théorie de la spéculation. *Annales scientifiques de l'École Normale Supérieure*, 17, 21–86.
- Bailey, D.H., López de Prado, M. (2014). The deflated Sharpe ratio: correcting for selection bias, backtest overfitting, and non-normality. *Journal of Portfolio Management*, 40(5), 94–107.
- BCBS (1996). Amendment to the capital accord to incorporate market risks. Basel Committee on Banking Supervision.
- BCBS (2019). Minimum capital requirements for market risk. Basel Committee on Banking Supervision.
- Ben-David, I., Franzoni, F., Moussawi, R. (2018). Do ETFs increase volatility? *Journal of Finance*, 73(6), 2471–2535.
- BIS (2023). Blueprint for the future monetary system: improving the old, enabling the new. Annual Economic Report, Chapter III.
- BIS (2025). The next-generation monetary and financial system. Annual Economic Report, Chapter III.
- Black, F., Scholes, M. (1973). The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3), 637–654.
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3), 307–327.
- Bollerslev, T., Tauchen, G., Zhou, H. (2009). Expected stock returns and variance risk premia. *Review of Financial Studies*, 22(11), 4463–4492.
- Brandimarte, P. (2018). *An Introduction to Financial Markets: A Quantitative Approach*. Wiley.
- Brown, S.J., Goetzmann, W., Ibbotson, R.G., Ross, S.A. (1992). Survivorship bias in performance studies. *Review of Financial Studies*, 5(4), 553–580.

References (2/7)

- Budish, E., Cramton, P., Shim, J. (2015). The high-frequency trading arms race: frequent batch auctions as a market design response. *Quarterly Journal of Economics*, 130(4), 1547–1621.
- Buehler, H., Gonon, L., Teichmann, J., Wood, B. (2019). Deep hedging. *Quantitative Finance*, 19(8), 1271–1291.
- Burns, P., Engle, R., Mezrich, J. (1998). Correlations and volatilities of asynchronous data. *Journal of Derivatives*, 5(4), 7–18.
- BVB (2017). 22 de ani de la prima tranzacție la Bursa de Valori București. Press release, 20 November 2017.
- BVB. BVB indices: BET, launched 19 September 1997. Bucharest Stock Exchange.
- BVB. Istoric (History). Bursa de Valori București (the Bucharest Stock Exchange).
- Campbell, J.Y., Lo, A.W., MacKinlay, A.C. (1997). *The Econometrics of Financial Markets*. Princeton University Press.
- Campbell, J.Y., Pflueger, C., Viceira, L.M. (2020). Macroeconomic drivers of bond and equity risks. *Journal of Political Economy*, 128(8), 3148–3185.
- Carr, P., Wu, L. (2009). Variance risk premiums. *Review of Financial Studies*, 22(3), 1311–1341.
- Chakrabarti, S., Krishnakumar, R., Mazzola, G., Stamatopoulos, N., Woerner, S., Zeng, W.J. (2021). A threshold for quantum advantage in derivative pricing. *Quantum*, 5, 463.
- Cox, J.C., Ingersoll, J.E., Ross, S.A. (1985). A theory of the term structure of interest rates. *Econometrica*, 53(2), 385–407.
- Donnelly, C., Embrechts, P. (2010). The devil is in the tails: actuarial mathematics and the subprime mortgage crisis. *ASTIN Bulletin*, 40(1), 1–33.
- Embrechts, P., Klüppelberg, C., Mikosch, T. (1997). *Modelling Extremal Events for Insurance and Finance*. Springer.
- Engle, R.F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50(4), 987–1007.
- Estrella, A., Mishkin, F.S. (1998). Predicting U.S. recessions: financial variables as leading indicators. *Review of Economics and Statistics*, 80(1), 45–61.
- ESMA. Markets in Crypto-Assets Regulation (MiCA), Regulation (EU) 2023/1114.
- Fama, E.F. (1965). The behavior of stock-market prices. *Journal of Business*, 38(1), 34–105.

References (3/7)

- Fama, E.F. (1970). Efficient capital markets: a review of theory and empirical work. *Journal of Finance*, 25(2), 383–417.
- Fama, E.F., French, K.R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), 3–56.
- Forbes, K.J., Rigobon, R. (2002). No contagion, only interdependence: measuring stock market comovements. *Journal of Finance*, 57(5), 2223–2261.
- Franke, J., Härdle, W.K., Hafner, C.M. (2019). *Statistics of Financial Markets: An Introduction* (5th ed.). Springer.
- Frehen, R.G.P., Goetzmann, W.N., Rouwenhorst, K.G. (2013). New evidence on the first financial bubble. *Journal of Financial Economics*, 108(3), 585–607.
- FTSE Russell (2026). FTSE Equity Country Classification, Interim Announcement, April 2026.
- Garber, P.M. (1989). Tulipmania. *Journal of Political Economy*, 97(3), 535–560.
- Gatheral, J., Jaisson, T., Rosenbaum, M. (2018). Volatility is rough. *Quantitative Finance*, 18(6), 933–949.
- Gelderblom, O. (2013). *Cities of Commerce: The Institutional Foundations of International Trade in the Low Countries, 1250–1650*. Princeton University Press.
- Gelderblom, O., Jonker, J. (2004). Completing a financial revolution: the finance of the Dutch East India trade and the rise of the Amsterdam capital market, 1595–1612. *Journal of Economic History*, 64(3), 641–672.
- Goldgar, A. (2007). *Tulipmania: Money, Honor, and Knowledge in the Dutch Golden Age*. University of Chicago Press.
- Gu, S., Kelly, B., Xiu, D. (2020). Empirical asset pricing via machine learning. *Review of Financial Studies*, 33(5), 2223–2273.
- Haddad, V., Huebner, P., Loualiche, E. (2025). How competitive is the stock market? Theory, evidence from portfolios, and implications for the rise of passive investing. *American Economic Review*, 115(3), 975–1018.
- Härdle, W.K., Chen, C.Y.-H., Overbeck, L. (Eds.) (2017). *Applied Quantitative Finance* (3rd ed.). Springer.
- Härdle, W.K., Harvey, C.R., Reule, R.C.G. (2020). Understanding cryptocurrencies. *Journal of Financial Econometrics*, 18(2), 181–208.
- Härdle, W.K., Simar, L. (2019). *Applied Multivariate Statistical Analysis*. Springer.

References (4/7)

- Heath, D., Jarrow, R., Morton, A. (1992). Bond pricing and the term structure of interest rates: a new methodology for contingent claims valuation. *Econometrica*, 60(1), 77–105.
- Herman, D., Googin, C., Liu, X., Sun, Y., Galda, A., Safro, I., Pistoia, M., Alexeev, Y. (2023). Quantum computing for finance. *Nature Reviews Physics*, 5(8), 450–465.
- Heston, S.L. (1993). A closed-form solution for options with stochastic volatility with applications to bond and currency options. *Review of Financial Studies*, 6(2), 327–343.
- Hutchinson, J.M., Lo, A.W., Poggio, T. (1994). A nonparametric approach to pricing and hedging derivative securities via learning networks. *Journal of Finance*, 49(3), 851–889.
- IBM Research (2024). The 2023 IBM Research annual letter. IBM Research Blog, 2 February 2024.
- J.P. Morgan/Reuters (1996). *RiskMetrics: Technical Document* (4th ed.). New York.
- Koijen, R.S.J., Yogo, M. (2019). A demand system approach to asset pricing. *Journal of Political Economy*, 127(4), 1475–1515.
- Ledoit, O., Wolf, M. (2008). Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance*, 15(5), 850–859.
- Lege asupra burselor (1929). *Lege asupra burselor* (Law on exchanges). Bucharest; digitised by Biblioteca Centrală Universitară Cluj.
- Li, D.X. (2000). On default correlation: a copula function approach. *Journal of Fixed Income*, 9(4), 43–54.
- Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics*, 47(1), 13–37.
- Liu, Y., Tsyvinski, A. (2021). Risks and returns of cryptocurrency. *Review of Financial Studies*, 34(6), 2689–2727.
- Lo, A.W. (2002). The statistics of Sharpe ratios. *Financial Analysts Journal*, 58(4), 36–52.
- Lo, A.W., MacKinlay, A.C. (1990). An econometric analysis of nonsynchronous trading. *Journal of Econometrics*, 45(1–2), 181–211.
- Lopez-Lira, A., Tang, Y. (2026). Can ChatGPT forecast stock price movements? Return predictability and large language models. *Journal of Financial Economics*, 184, 104335.
- MacKinlay, A.C. (1997). Event studies in economics and finance. *Journal of Economic Literature*, 35(1), 13–39.

References (5/7)

- Magdon-Ismail, M., Atiya, A.F., Pratap, A., Abu-Mostafa, Y.S. (2004). On the maximum drawdown of a Brownian motion. *Journal of Applied Probability*, 41(1), 147–161.
- Makarov, I., Schoar, A. (2020). Trading and arbitrage in cryptocurrency markets. *Journal of Financial Economics*, 135(2), 293–319.
- Mandelbrot, B. (1963). The variation of certain speculative prices. *Journal of Business*, 36(4), 394–419.
- Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, 7(1), 77–91.
- McNeil, A.J., Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal of Empirical Finance*, 7(3–4), 271–300.
- Merton, R.C. (1973). Theory of rational option pricing. *Bell Journal of Economics and Management Science*, 4(1), 141–183.
- Merton, R.C. (1980). On estimating the expected return on the market. *Journal of Financial Economics*, 8(4), 323–361.
- Montanaro, A. (2015). Quantum speedup of Monte Carlo methods. *Proceedings of the Royal Society A*, 471(2181), 20150301.
- Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica*, 34(4), 768–783.
- Nakamoto, S. (2008). Bitcoin: A peer-to-peer electronic cash system. White paper.
- NBER. US business cycle expansions and contractions. National Bureau of Economic Research.
- Neal, L. (1990). *The Rise of Financial Capitalism: International Capital Markets in the Age of Reason*. Cambridge University Press.
- Opdyke, J.D. (2007). Comparing Sharpe ratios: so where are the p-values? *Journal of Asset Management*, 8(5), 308–336.
- Orús, R., Mugel, S., Lizaso, E. (2019). Quantum computing for finance: overview and prospects. *Reviews in Physics*, 4, 100028.
- Palatul Bursei. Palatul Bursei (The Stock Exchange Palace, Bucharest). Wikipedia (Romanian), accessed 2026.
- Pele, D.T., Wesselhöfft, N., Härdle, W.K., Kolossiatis, M., Yatracos, Y.G. (2023). Are cryptos becoming alternative assets?. *The European Journal of Finance*, 29(10), 1064–1105.
- Petajisto, A. (2017). Inefficiencies in the pricing of exchange-traded funds. *Financial Analysts Journal*, 73(1), 24–54.

References (6/7)

- Petram, L.O. (2014). *The World's First Stock Exchange*. Columbia University Press.
- Pflueger, C. (2025). Back to the 1980s or not? The drivers of inflation and real risks in Treasury bonds. *Journal of Financial Economics*, 167, 104027.
- Preskill, J. (2018). Quantum computing in the NISQ era and beyond. *Quantum*, 2, 79.
- Rebentrost, P., Gupt, B., Bromley, T.R. (2018). Quantum computational finance: Monte Carlo pricing of financial derivatives. *Physical Review A*, 98(2), 022321.
- Roll, R. (1984). A simple implicit measure of the effective bid-ask spread in an efficient market. *Journal of Finance*, 39(4), 1127–1139.
- Roll, R. (1988). The international crash of October 1987. *Financial Analysts Journal*, 44(5), 19–35.
- Ross, S.A. (1976). The arbitrage theory of capital asset pricing. *Journal of Economic Theory*, 13(3), 341–360.
- Sammon, M. (2025). Passive ownership and price informativeness. *Management Science*, 71(6), 4582–4598.
- Samuelson, P.A. (1965). Rational theory of warrant pricing. *Industrial Management Review*, 6(2), 13–39. Reprinted in *Henry P. McKean Jr. Selecta* (2015), Springer, 195–232.
- Scholes, M., Williams, J. (1977). Estimating betas from nonsynchronous data. *Journal of Financial Economics*, 5(3), 309–327.
- SEC (2024). Statement on the Approval of Spot Bitcoin Exchange-Traded Products, 10 January 2024.
- Sharpe, W.F. (1964). Capital asset prices: a theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3), 425–442.
- Stamatopoulos, N., Egger, D.J., Sun, Y., Zoufal, C., Iten, R., Shen, N., Woerner, S. (2020). Option pricing using quantum computers. *Quantum*, 4, 291.
- Sylla, R., Wright, R.E., Cowen, D.J. (2009). Alexander Hamilton, central banker: crisis management during the U.S. financial panic of 1792. *Business History Review*, 83(1), 61–86.
- Tobin, J. (1958). Liquidity preference as behavior towards risk. *Review of Economic Studies*, 25(2), 65–86.
- U.S. Congress. Guiding and Establishing National Innovation for U.S. Stablecoins Act (GENIUS Act), Public Law 119-27.
- Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5(2), 177–188.

References (7/7)

- Vaswani, A. et al. (2017). Attention is all you need. *Advances in Neural Information Processing Systems*, 30.
- Woerner, S., Egger, D.J. (2019). Quantum risk analysis. *npj Quantum Information*, 5, 15.
- Wu, S. et al. (2023). BloombergGPT: a large language model for finance. *arXiv:2303.17564*.
- Code and data: github.com/danpele/MFM/tree/main/Quantlets/Ch_00

Acronyms used in this material (1/4)

- **AI**: Artificial Intelligence
- **AMM**: Automated Market Maker
- **AP**: Authorised Participant
- **APT**: Arbitrage Pricing Theory
- **AR**: AutoRegressive (model); in event studies: Abnormal Return
- **ARCH**: AutoRegressive Conditional Heteroskedasticity
- **ASE**: Academia de Studii Economice din București — *the Bucharest University of Economic Studies*
- **AUM**: Assets Under Management
- **BCBS**: Basel Committee on Banking Supervision
- **BET**: Bucharest Exchange Trading (index)
- **BET-FI**: BET Financial Investment companies index
- **BET-TR**: BET Total Return (index)
- **BIS**: Bank for International Settlements
- **BNR**: Banca Națională a României — *the National Bank of Romania*
- **BVB**: Bursa de Valori București — *the Bucharest Stock Exchange*
- **CAGR**: Compound Annual Growth Rate
- **CAPM**: Capital Asset Pricing Model
- **CAR**: Cumulative Abnormal Return
- **CBOE**: Chicago Board Options Exchange
- **CCP**: Central CounterParty
- **CDO**: Collateralised Debt Obligation
- **CFTC**: Commodity Futures Trading Commission

Acronyms used in this material (2/4)

- **CIR**: Cox–Ingersoll–Ross (model)
- **COVID-19**: COronaVIrus Disease 2019
- **CRSP**: Center for Research in Security Prices
- **DeFi**: Decentralised Finance
- **DJIA**: Dow Jones Industrial Average
- **EBA**: European Banking Authority
- **EMH**: Efficient Market Hypothesis
- **EOD**: End of Day
- **EODHD**: EOD Historical Data (market data provider)
- **ES**: Expected Shortfall
- **ESMA**: European Securities and Markets Authority
- **ETF**: Exchange-Traded Fund
- **ETP**: Exchange-Traded Product
- **EU**: European Union
- **EVT**: Extreme Value Theory
- **EWMA**: Exponentially Weighted Moving Average
- **FRED**: Federal Reserve Economic Data
- **FRTB**: Fundamental Review of the Trading Book
- **FTSE**: Financial Times Stock Exchange (FTSE Russell, index provider)
- **FX**: Foreign eXchange
- **GARCH**: Generalised AutoRegressive Conditional Heteroskedasticity
- **GENIUS**: Guiding and Establishing National Innovation for U.S. Stablecoins (Act)
- **GMM**: Generalized Method of Moments

Acronyms used in this material (3/4)

- **GPT**: Generative Pre-trained Transformer
- **GPU**: Graphics Processing Unit
- **HAC**: Heteroskedasticity and Autocorrelation Consistent (standard errors)
- **HFT**: High-Frequency Trading
- **HJM**: Heath–Jarrow–Morton (model)
- **HML**: High Minus Low (value factor)
- **IBM**: International Business Machines (Corporation)
- **IPO**: Initial Public Offering
- **LLM**: Large Language Model
- **MC**: Monte Carlo (simulation)
- **MDD**: Maximum DrawDown
- **MDE**: Minimum Detectable Effect
- **MiCA**: Markets in Crypto-Assets (EU Regulation 2023/1114)
- **ML**: Machine Learning
- **MTF**: Multilateral Trading Facility
- **NAV**: Net Asset Value
- **NBER**: National Bureau of Economic Research
- **NFT**: Non-Fungible Token
- **NISQ**: Noisy Intermediate-Scale Quantum
- **NYSE**: New York Stock Exchange
- **OLS**: Ordinary Least Squares
- **OMV**: Österreichische Mineralölverwaltung — *Austrian Mineral Oil Administration (OMV group)*
- **OTC**: Over-The-Counter

Acronyms used in this material (4/4)

- **QAE**: Quantum Amplitude Estimation
- **QR**: Quick Response (code)
- **ROL**: Romanian old leu (ISO 4217 code, until 2005)
- **RWA**: Real-World Assets
- **SD**: Standard Deviation
- **SE**: Standard Error
- **SEC**: Securities and Exchange Commission
- **SIF**: Societate de Investiții Financiare — *financial investment company*
- **SMB**: Small Minus Big (size factor)
- **SPDR**: Standard & Poor's Depositary Receipts
- **SR**: Sharpe Ratio
- **TARF**: Target Accrual Redemption Forward
- **TIPS**: Treasury Inflation-Protected Securities
- **TR**: Total Return
- **UDR**: Uzinele și Domeniile Reșița — *the Reșița Works and Domains*
- **UK**: United Kingdom
- **US**: United States
- **UTC**: Coordinated Universal Time (Temps Universel Coordonné)
- **VaR**: Value-at-Risk
- **VIX**: CBOE Volatility IndeX
- **VOC**: Vereenigde Oostindische Compagnie — *Dutch East India Company*