








Modelling Financial Markets

Chapter 16: Digital Assets and DeFi



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Which Measurements Must Change for Digital Assets?

> *"A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution."*
>
> — *Nakamoto (2008)*

- Bitcoin started as electronic cash without an intermediary; today it is a financial asset
  - Bitcoin's market value: 1.63 trillion USD (US dollar) on 18 Sep 2026, i.e. 82.5% of the value of the five large coins
- Crypto returns come from a 7-day market, compared with markets that trade 5 days a week
  - Bitcoin's annualised volatility in 2018–2026: 64%, 3.3 times that of the S&P 500 index (19%; Standard & Poor's)
- The Bitcoin–equity link changed in 2020, not at the launch of spot ETFs in January 2024
  - ETF: exchange-traded fund, here one that holds the coin
  - break date at the minimum sum of squared residuals: 9 Mar 2020; sup-Wald statistic 20.12, above the 5% critical value 11.55
  - correlation of daily Bitcoin and S&P 500 returns: 0.01 before the break, 0.40 after
- A stablecoin's 1 USD peg depends on its reserves and on the settlement of redemptions
  - after Silicon Valley Bank was closed, USD Coin hit a daily low of 0.8774 USD on Saturday, 11 March 2023

Central chart: Is Bitcoin Moving with Shares?

Digital Assets: What Is Different and How to Measure It

Digital Assets, Pricing and Liquidity

- The question: do cryptos, stablecoins and DeFi pools behave like familiar assets?
  - which measurements must change?
- Part I: the crypto market, stylised facts of returns, crypto indices (CRIX), stablecoins
- Part II: DeFi and automated market makers, spot ETFs and crypto equities
  - DeFi: decentralised finance
  - spot ETFs: exchange-traded funds that hold the coin
  - Cryptos as an asset class, a case study, one pass of the research loop assisted by artificial intelligence (AI)
- Detailed examples and advanced topics are in Further Topics, after the Conclusions

What you should be able to do

- compute the statistics of a 7-day market on the right calendar
  - with intervals for its tail index, volatility and correlations
- compute a market-value index with a divisor
  - estimate a stablecoin peg as a threshold autoregression (threshold AR)
- derive the price of a swap in a constant-product pool and interpret providing liquidity as selling volatility
- interpret ETF tracking, crypto betas and asset-class claims
  - with tests that respect closing times and unknown break dates
- Further Topics and Appendix: reading for familiarity

Blockchains, Coins and Tokens



Bust of Satoshi Nakamoto, Graphisoft Park, Budapest

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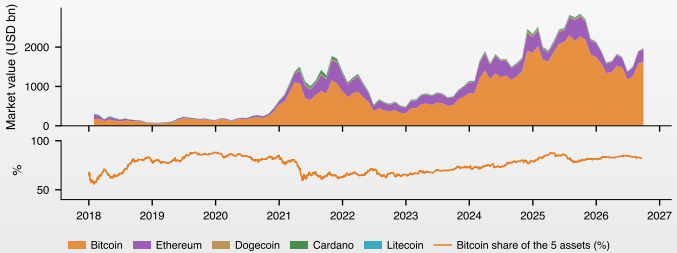
- **Blockchain** Nakamoto (2008): a shared ledger of blocks, each carrying the hash of the previous one
  - Secured by proof of work (Bitcoin) or proof of stake (Ethereum since 15 September 2022, ethereum.org: The Merge)
- Bitcoin issuance halves over time up to 21 million coins Bitcoin.org FAQ
  - 20.09 million existed on 18 Sep 2026
- **Coins** are native assets of a blockchain
  - **tokens** (stablecoins, governance tokens, tokenised funds) are issued by **smart contracts** Buterin (2014)
  - DeFi is built from such contracts Schär (2021)
- Surveys: technology and governance Böhme, Christin, Edelman & Moore (2015), Lin, Pele & Ren (2025); economics Härdle, Harvey & Reule (2020)

Seven-Day Crypto Data Next to Five-Day Markets

Series	Content	Period
Crypto prices	Bitcoin, Ethereum, Solana, Dogecoin, Cardano, Litecoin; daily, 7 days a week	2018–2026
Stablecoins	Tether (USDT), USD Coin (USDC), Dai (DAI) prices; supply by coin (DefiLlama)	2019–2026
Coin supply	Coins in circulation, daily (Coin Metrics Community Data)	2018–2026
DeFi	Total value locked, all chains and by chain (DefiLlama)	2020–2026
ETFs and shares	spot ETFs on Bitcoin (IBIT) and Ether (ETHA), Coinbase, Strategy; S&P 500, Nasdaq 100 (ETF QQQ), gold (XAU/USD, USD per ounce)	2018–2026
Asset universe	26 assets: cryptos, equity indices, currencies, commodities, bonds	2019–2026

- Daily closes up to 18 September 2026; crypto on its own 7-day calendar
  - analyses that combine assets use common days
  - prices aligned on common trading days first, then returns
  - some tests use longer histories and extra series

Market Value of Five Cryptocurrencies



- **Market value** of coin i at month end t: $M_{i,t} = P_{i,t} \times Q_{i,t}$
  - $P_{i,t}$: the price in USD
  - $Q_{i,t}$: the coins in circulation
  - Bitcoin: 1.63 trillion USD on 18 Sep 2026
- The five coins: 1 970 billion USD today, peak 3 135 billion (6 Oct 2025)
  - the value moves with prices, supply changes slowly
- Bitcoin's share: 82.5% today, between 55.8% (1 Feb 2018) and 88.5% (5 Sep 2019)

MFM_ch16_market_value ▶ Interactive

▸ Further Topics: market concentration

Stylised Facts: the Question and the Choices

- **The question**: do crypto returns share the stylised facts of equities, on a different scale?
  - heavy tails, volatility clustering, deep drawdowns
- Why it matters: VaR, ES and every confidence interval depend on how heavy the tail is
  - VaR: value at risk; ES: expected shortfall
  - If the fourth moment is infinite, the sample kurtosis estimates no parameter: we need the tail index α, which measures how heavy the tail is
- Comparison assets
  - the S&P 500, the reference risk asset
  - gold, the asset Bitcoin is compared with as "digital gold"
- Sample 2018–2026: one window for all assets
  - with the March 2020 crash, the 2022 crypto winter and the spot ETF launch
- Calendar: each asset's statistics on its own calendar
  - 7 days for crypto, 5 for shares
  - pairs of assets on common days

The Measures: Return, Volatility, Skewness, Drawdown, Clustering

- Daily log return in %: $r_t = 100\,(\ln P_t - \ln P_{t-1})$
  - P_t: the close on day t
  - each series on its own calendar
- Annualised volatility $\sigma_a = s\sqrt{A}$
  - s: sample standard deviation of r_t; A: return periods per year, about 365 for crypto and 252 for shares
- Skewness $S = \overline{(r - \bar{r})^3}/s^3$, a bar denoting the sample mean: $S < 0$ means a longer left tail
- Maximum drawdown $\mathrm{MDD} = \min_t(P_t/\max_{u \le t} P_u - 1)$
  - the deepest fall from the running peak, between -100% and 0
- Volatility clustering $\hat{\rho}_1(|r|)$: sample autocorrelation of $|r_t|$ at lag 1
  - Chapter 1
  - > 0: large moves tend to follow large moves

Historical VaR and ES

- Historical daily $\mathrm{VaR}_{1\%} = -q_{1\%}(r)$ and $\mathrm{ES}_{2.5\%} = -\frac{1}{0.025}\int_0^{0.025} q_u(r)\,du$
  - Chapter 7
  - $q_p(r)$: empirical p-quantile of the returns
  - historical ES: the integral of the empirical quantile function, with a fractional weight on the boundary observation
  - Why two levels: the Basel market-risk rules replaced VaR at 1% by ES at 2.5% (Basel Committee, MAR33, para. 33.3); under the Normal distribution both are close, 2.326σ and 2.338σ; $\mathrm{ES}_p \geq \mathrm{VaR}_p$ holds at the same p, not across levels

Estimating the Tail Index: the Hill Estimator

- Power-law tail of the daily loss $L = -r$: $P(L > x) \sim Cx^{-\alpha}$ for large x
  - $C > 0$: a scale constant
  - α: the **tail index**; smaller α = heavier tail; the moment $E|r|^q$ is finite only for $q < \alpha$
- Hill estimator Hill (1975) from the k largest losses: $\hat{\alpha}_H = \left(\frac{1}{k}\sum_{j=1}^{k} \ln(L_{(j)}/L_{(k+1)})\right)^{-1}$
  - $L_{(1)} \geq L_{(2)} \geq \ldots$: the losses sorted from the largest; $k = 5\%$ of the losses; $L_{(k+1)}$ acts as the threshold
  - Large ratios $L_{(j)}/L_{(k+1)}$ on average give a small $\hat{\alpha}_H$: a heavy tail
  - Standard error (SE) $\mathrm{SE} = \hat{\alpha}_H/\sqrt{k}$ for independent and identically distributed (i.i.d.) data; 95% confidence interval (CI) $\hat{\alpha}_H \pm 1.96\,\mathrm{SE}$ (conditions and dependence in the Appendix)

Crypto vs Shares and Gold, 2018–2026 (1/2)

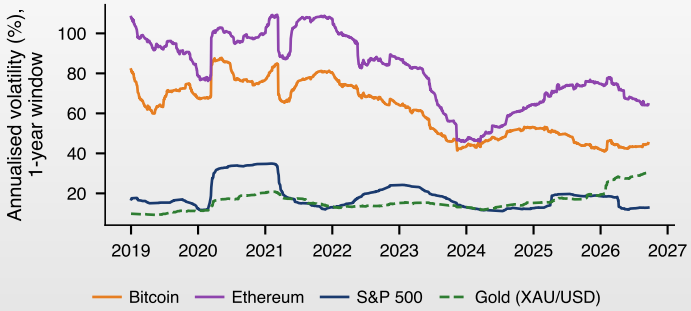
	n	σ_a (%)	S	MDD (%)	$\hat{\rho}_1(\lvert r\rvert)$	$\hat{\alpha}_H$	95% CI	$\mathrm{VaR}_{1\%}$	$\mathrm{ES}_{2.5\%}$
Bitcoin	3 182	64	−0.95	−82	0.17	2.73	[2.12, 3.33]	10.2	10.3
Ethereum	3 182	84	−0.82	−94	0.16	2.67	[2.08, 3.26]	13.5	13.7
Solana	2 351	117	−0.22	−96	0.28	2.85	[2.12, 3.58]	14.5	17.2
Dogecoin	3 182	124	4.88	−92	0.37	2.83	[2.22, 3.45]	13.8	16.4
S&P 500	2 189	19	−0.65	−34	0.36	2.87	[2.07, 3.67]	3.4	3.8
Gold	2 269	17	−0.55	−28	0.13	2.75	[1.99, 3.51]	3.0	3.2

- Daily log returns in %, Solana from April 2020; VaR and ES are losses of the daily log return
- Bitcoin: 3.3 times the S&P 500's volatility
  - daily VaR 1% 10.2% in log terms, a loss of 9.7% of the position
- Hill indices 2.67–2.87 for all six assets
  - crypto is more volatile, but the data give no strong evidence of different tail indices
  - tests in the Appendix

Crypto vs Shares and Gold, 2018–2026 (2/2)

- All six i.i.d. 95% CIs lie below 3.7: $\alpha < 4$, hence $E|r|^4 = \infty$, is plausible but not proven
- If $\alpha < 4$, the sample excess kurtosis estimates no finite parameter
  - $K = \overline{(r - \bar{r})^4}/s^4 - 3$
  - report $\hat{\alpha}_H$ with its CI
- Type of evidence: description (it shows what happened in the data, without testing a cause)

Volatility Over Time

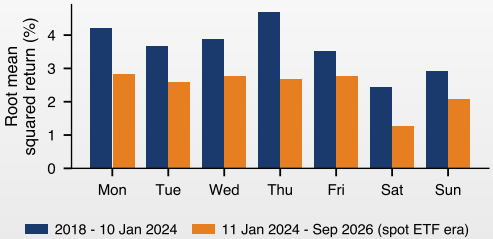


- Annualised standard deviation of daily log returns over a moving one-year window
- Bitcoin fell from 88% to 45% today, Ethereum from 109% to 64%
  - consistent with a maturing market, but the chart does not identify the cause
- Bitcoin is still 3.5 times as volatile as the S&P 500 (13%); gold reached 30% in 2026, a record for the sample

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▸ Further Topics: drawdowns

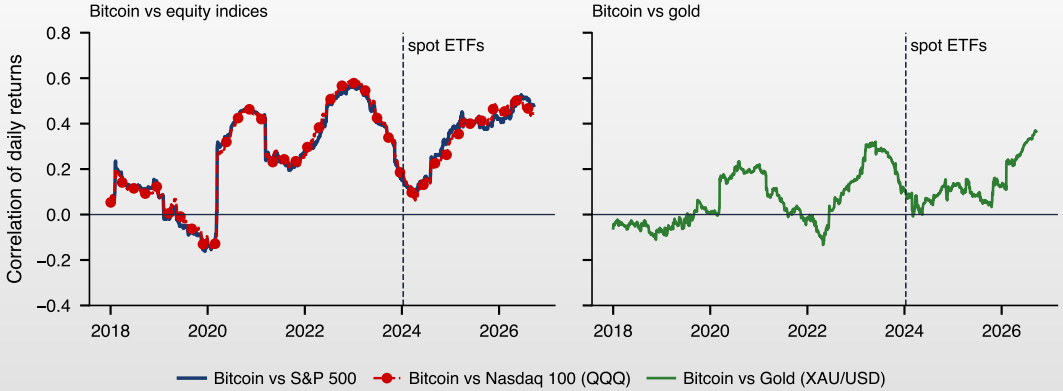
A 7-Day Market with a Weekend Lull



- Root mean squared daily Bitcoin return by day of the week
  - weekend variance / weekday variance: 0.45 before and 0.39 after 11 January 2024
- Bitcoin trades on Saturdays, but macro news, US (United States) equity trading and ETF flows do not
  - all days calmer after the ETF launch (weekday root mean square 4.01% $\rightarrow$ 2.73%)
- Annualise with 365 days, but do not treat all days as identical
  - did the ratio change? Seminar 16, B1

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Is Bitcoin Moving with Shares? (1/2)



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Is Bitcoin Moving with Shares? (2/2)

- Chart: correlation of daily log returns over a moving window of 250 common days
- Whole-period correlations, Bitcoin vs S&P 500
  - 0.02 (2017–2019, $n = 754$), 0.39 (2020–2023, $n = 1\,006$)
  - 0.38 since the ETFs ($n = 674$ days)
- Nasdaq 100 like the S&P 500 (0.38); gold -0.02, 0.12, 0.22
  - a smaller diversification benefit than before 2020 Bouri et al. (2017), Liu & Tsyvinski (2021)
- The chart suggests a break in early 2020, not in January 2024: tested next

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Bitcoin's Beta to the S&P 500: the CAPM

- CAPM (capital asset pricing model): $E[r_i] - r_f = \beta_i \, (E[r_m] - r_f)$, with $\beta_i = \mathrm{Cov}(r_i, r_m)/\mathrm{Var}(r_m)$
  - r_f: the risk-free rate; r_m: the market return (here the S&P 500); r_i: Bitcoin's return
- Why beta and not only the correlation: $\beta = \rho \, \sigma_{\mathrm{BTC}}/\sigma_{\mathrm{SPX}}$
  - The correlation says how tight the link is; beta says how much Bitcoin moves per 1% move of the S&P 500 (the hedge ratio)
  - On the common days of 2016–2026: $\rho = 0.259$ and $\sigma_{\mathrm{BTC}}/\sigma_{\mathrm{SPX}} = 3.68$ give $\beta = 0.95$
- Estimation: $\hat{b}$ from $r_t^{\mathrm{BTC}} = a + b \, r_t^{\mathrm{SPX}} + u_t$, daily log returns in %

Did the Bitcoin–Equity Link Break in 2020? (1/2): Model and sup-Wald

- The beta regression on 2 693 common days since January 2016
  - (a, b) allowed to shift at an unknown date τ: one pair before τ, another after
- $W(\tau)$: the Wald statistic of H_0 (the same (a, b) before and after τ)
  - with Newey–West covariances (8 lags) in each regime
- sup-Wald Andrews (1993): $\sup_{\tau \in [0.15T, 0.85T]} W(\tau)$, the largest W over the central 70% of the T days
  - Under H_0 the sup is not χ^2_2, because τ is chosen to make W large
  - 5% critical value 11.55, simulated from the limit distribution (Appendix)
- Several breaks at unknown dates: the sequential tests and CIs of Bai & Perron (1998)
  - the test of a second break, $\sup F(2 \mid 1) = 10.33$, $p = 0.16$: no further break detected at 5%

Did the Bitcoin–Equity Link Break in 2020? (2/2): the Result

- sup $W = 20.12$: above the 5% critical value 11.55
  - $p = 0.0008$, from 20 000 simulations of the limit distribution
  - so the no-break hypothesis is rejected
  - Newey–West covariances require finite moments of the score $x_t u_t$ and weak dependence, conditions the tails of Chapter 1 question
  - The maximum is reached on 14 Aug 2017, two trading days from the edge of the admissible range, far from the estimated break date
  - The rejection does not rest on that point: at the estimated date $W = 18.81$, above the 1% critical value 15.18
- Least-squares break date: 9 Mar 2020
  - the τ that minimises the sum of squared residuals (SSR) of the two regimes
  - 95% CI Bai (1997): 11 Feb 2019 – 29 Dec 2021, an interval that excludes January 2024
  - the interval is asymmetric: the S&P 500 variance rises from 0.75 to 1.61, the residual variance falls from 20.9 to 12.5
  - At the ETF launch $W = 2.31$, far below the critical value
- Correlation: 0.01 before, 0.40 after
  - difference 0.39, 95% CI $[0.24, 0.46]$ (block bootstrap, at the estimated date)
  - adjusted for volatility (Forbes–Rigobon, in Further Topics), the 2020–2023 correlation falls from 0.39 to 0.23, but the rise stays positive
- Type of evidence: description
  - the one-break model dates the change in the coefficients (a, b); it does not identify a mechanism
  - $\rho = b\,\sigma_{\mathrm{SPX}}/\sigma_{\mathrm{BTC}}$ also depends on the variances: a break in (a, b) is not a break in the correlation

Beta Before and After the Break, with HAC Errors

- The same OLS (ordinary least squares) regression in each regime, with Newey–West SE, robust to heteroskedasticity and autocorrelation (HAC): 1 051 common days before 9 Mar 2020, 1 642 after
- Before: $\hat{b} = 0.07$, HAC SE 0.22 (OLS SE 0.16): $t = 0.3$, beta not significantly different from 0
- After: $\hat{b} = 1.22$, HAC SE 0.14 (OLS SE 0.07): $t = 8.6$
- HAC SE are 1.4 times (before) and 2.1 times (after) the OLS SE: OLS intervals would be far too narrow
  - HAC SE handle dependence, but require finite moments of the score $x_t u_t$, a condition the tails of Chapter 1 question
- Since the ETFs (674 days): $\hat{b} = 1.20$, HAC SE 0.20
  - in 2020–2023 $\hat{b} = 1.15$, HAC SE 0.17
  - the launch shows no change in beta

Crypto Indices: the Question and the Choices

- **The question**: how do we build a benchmark for a market with many coins, dominated by one?
- Why it matters: the crypto market return is the return of such an index
  - in the Gibbons–Ross–Shanken (GRS) test of factor models (Section 7) and in the case study
- Market-value weights: the index is replicable
  - it holds the coins in the proportions the market holds them
- The divisor: keeps the level continuous when members or quantities change
- Number of constituents: chosen from the data by the CRIX rule, not fixed in advance
- Main threat: survivorship bias, if the universe of coins is chosen ex post, among the coins that exist today

A Crypto Index and Its Divisor



Humboldt-Universität zu Berlin, where CRIX was developed

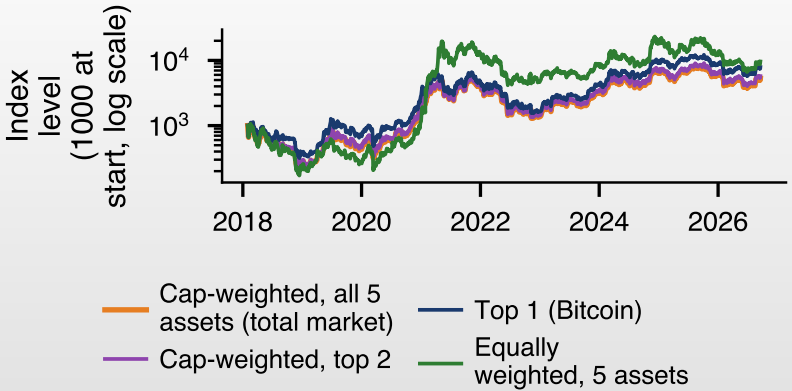
Photo: Diego Delso (2016); CC BY-SA 4.0; Wikimedia Commons

- **CRIX** Trimborn & Härdle (2018): a market-value weighted benchmark of the crypto market, published at thecrix.de
  - Review of crypto indices: Găman, Pele, Härdle & Będowska-Sójka (2026)
- Level on day t of reallocation period τ: $I_t = \sum_{i=1}^{k} P_{i,t}\, Q_{i,\tau} / D_\tau$
  - k: number of constituents (not the Hill k); $P_{i,t}$: price of coin i on day t; $Q_{i,\tau}$: its coins in circulation, fixed within the period; D_τ: the **divisor**
  - Index return = weighted average of the constituents' returns, weights = value shares at the start of the period; a Laspeyres-type index (quantities fixed within the period)
- At a reallocation on day t the divisor is reset so that the level does not jump
  - reallocation: new Q, new members
  - $D_{\tau+1} = \sum_i P_{i,t} Q_{i,\tau+1} / I_t$
  - Here: monthly reallocation, weights from the previous month end; worked examples: Seminar 16, A1, A4

How Many Constituents? The CRIX Rule and the AIC

- Candidate index CRIX(k, β), fitted to track the total market index TMI (all coins) Trimborn & Härdle (2018)
  - the k_1 largest coins enter with weight 1
  - the next $s = k - k_1$ with free weights $\beta = (\beta_1, \ldots, \beta_s)$, fitted to the data (here β are weights, not market betas)
- Tracking residual: $\hat{\varepsilon}_t(k, \beta) = \Delta \ln \mathrm{TMI}_t - \Delta \ln \mathrm{CRIX}_t(k, \beta)$, the daily gap between the two index log returns
- Akaike information criterion (AIC) $\mathrm{AIC}(k) = -2 \log L\{\hat{\varepsilon}(k, \hat{\beta})\} + 2s$
  - the rule picks the k with the smallest AIC
  - L: likelihood of the residuals; $2s$: penalty for the free weights
  - k reviewed quarterly on the last three months of data, members reallocated monthly by market value
  - Two stopping rules, ECRIX (first rise of the AIC) and EFCRIX (global minimum); likelihood details in the Appendix
- Here TMI holds only the five coins with public supply, so $k \leq 4$

Crypto Indices Since 2018 (1/2)



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Crypto Indices Since 2018 (2/2)

- Log scale; total-market index 21.4% a year over 8.6 years, volatility 65.7%
  - Bitcoin alone did better: the other coins lost weight and value
- Equal weights: 30.0% a year at 82.5% volatility
  - driven by 2021, when the equal-weighted index rose 998% and the value-weighted index 113%
  - a few extreme months, not a stable premium
- The five coins are chosen ex post, among today's survivors
  - the index returns are biased upwards (survivorship bias, in Further Topics; Będowska-Sójka, Wójcik & Pele (2026))
- Type of evidence: description
  - an ex post comparison on five survivors, not evidence that one weighting rule is better

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Stablecoins: the Question and the Choices

- **The question**: how tightly do stablecoins hold their 1 USD peg, and what mechanism holds it?
- Why it matters: they are the cash leg of the crypto market
  - 303 billion USD on 18 Sep 2026
  - a peg can be lost over one weekend
- The coins: Tether, USD Coin and Dai, with daily prices over the whole sample
  - Tether and USD Coin: the largest fiat-backed stablecoins; Dai: crypto-backed
  - Sky Dollar, larger today, exists only since 2024
- Sample 2021–2026: earlier quotes are thin and noisy
- The model: a threshold AR, not a linear AR(1)
  - arbitrage creates a band around the peg
  - so the speed of reversion depends on the size of the deviation

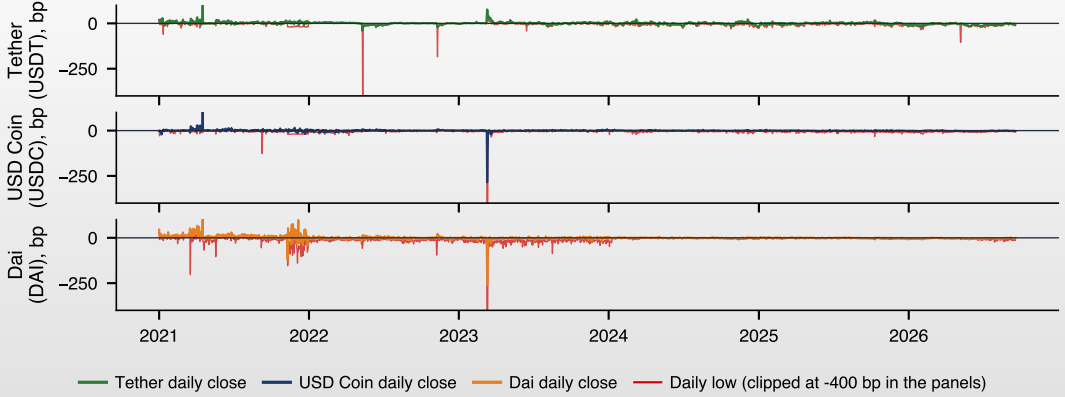
How a Stablecoin Holds 1 USD

Design	Backing	Examples (USD bn, 18 Sep 2026)
Fiat-backed	Bank deposits and short US Treasury bills held by the issuer (fiat: government-issued currency, here USD)	Tether (USDT) 183.33, USD Coin (USDC) 73.92, PayPal USD (PYUSD) 2.81
Crypto-backed	Over-collateralised crypto deposits or hedged positions, managed by smart contracts	Sky Dollar (USDS) 6.49, Ethena USDe 4.74, Dai (DAI) 4.77
Algorithmic	No external collateral: supply expands and contracts against a second token	TerraUSD (collapsed in May 2022)

- Authorised traders **mint** and **redeem** with the issuer
  - mint: pay 1 USD, receive 1 coin; redeem: return 1 coin, receive 1 USD
  - arbitrage keeps the market price P (USD) in the band $1 - c_r \leq P \leq 1 + c_m$
  - c_r, c_m: the redemption and minting costs per coin, in USD
  - Below the band: buy and redeem; above: mint and sell (evidence: Lyons & Viswanath-Natraj (2023); Tether issuance and Bitcoin prices: Griffin & Shams (2020))
- The band breaks when redemption stops or the backing is doubted
  - reserve and redemption rules: MiCA, Regulation (EU) 2023/1114, GENIUS Act, Public Law 119-27

▸ Further Topics: stablecoin supply

Deviations from 1 USD (1/2)



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Deviations from 1 USD (2/2)

- Daily close and daily low minus 1 USD, in basis points (1 bp = 0.01%), 2021–2026
  - Lows below −400 bp are cut at the panel edge
- Standard deviation of the close, 2021–2026: 7.3 bp (Tether), 7.5 bp (USD Coin), 10.8 bp (Dai)
  - Beyond 50 bp on less than 1% of the days
- Stress is rare but deep: USD Coin's low −1 226 bp (11 Mar 2023), Tether's −515 bp (12 May 2022)
- A stablecoin is a claim with a short put on the reserves
  - per coin the holder gets $\min(1, R) = 1 - \max(1 - R, 0)$
  - R: the reserve value per coin in USD
  - A sure 1 USD minus a put on R with strike 1, sold to the issuer: nothing happens most days, a large loss is possible on a few

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Arbitrage Bands Imply a Threshold AR (1/2): Deviation and Model

- Deviation from the peg in basis points: $d_t = 10^4(P_t - 1)$
  - P_t: the daily close in USD
  - $P_t = 0.997$ gives $d_t = -30$ bp
- Threshold autoregression (TAR), the equilibrium form of Balke & Fomby (1997):

$$d_t = \begin{cases} \phi_{\text{in}}\, d_{t-1} + \varepsilon_t, & |d_{t-1}| \le c \\ \phi_{\text{out}}\, d_{t-1} + \varepsilon_t, & |d_{t-1}| > c \end{cases}$$

  - c: the half-width of the band in bp; ε_t: a shock with mean 0
  - Inside the band arbitrage does not pay (weak pull); outside it does (fast reversion)
  - ϕ_{in}, ϕ_{out}: persistence inside and outside the band
  - Half-life $\ln 0.5 / \ln|\phi|$: the number of days in which a deviation halves

Arbitrage Bands Imply a Threshold AR (2/2): Threshold and Linearity Test

- $\hat{c}$ by least squares over a grid of $|d_{t-1}|$ values, keeping at least 15% of the days in each regime
  - $\hat{c}$ is an empirical regime boundary: it equals the cost, $10^4 c_r$ bp, only under a symmetric band ($c_r = c_m$) and no other frictions
- H_0: $\phi_{\text{in}} = \phi_{\text{out}}$: one linear AR(1), no band; H_1: the two persistences differ
  - For a given c: $W(c)$, the Wald statistic of H_0 with White standard errors White (1980) (robust to heteroskedasticity); the test uses $\sup_c W(c)$, the largest over the grid
- Under H_0 the threshold c does not enter the model: it is not identified
  - so $\sup_c W(c)$ is not χ^2_1 (the Davies problem)
  - χ^2_1 critical values are too small for a maximum over many c: the test would reject too often, so p comes from a bootstrap Hansen (1996)

Threshold AR Estimates for the Three Stablecoins

	$\hat{c}$ (bp)	$\hat{\phi}_{\text{in}}$ (SE)	$\hat{\phi}_{\text{out}}$ (SE)	sup W	p bootstrap	p (χ^2_1)	$\hat{\phi}$ linear
Tether	8.6	0.86 (0.02)	0.69 (0.11)	7.4	0.084	0.006	0.73
USD Coin	2.9	0.59 (0.04)	0.28 (0.03)	32.5	0.001	< 0.001	0.29
Dai	6.0	0.72 (0.08)	0.32 (0.07)	13.8	0.005	< 0.001	0.33

- USD Coin and Dai: linearity rejected (bootstrap in the AR(1), with independent multipliers)
  - reversion is much faster outside the band (half-life 0.5 vs 1.3 days for USD Coin)
  - USD Coin, $p = 0.001$: the smallest value possible with $B = 999$ replications, so the true p-value may be smaller
  - Without 9–16 March 2023, USD Coin gives $p = 0.29$ (bootstrap) but 0.019 (χ^2_1): one stress week carries the evidence, and χ^2 over-rejects (step by step: Seminar 16, B5)
- Tether: the widest band ($\hat{c} = 8.6$ bp) and slow reversion inside it
  - but linearity is not rejected at 5%
  - consistent with fewer direct redeemers Ma, Zeng & Zhang (2025)
- Type of evidence: description
  - $\hat{c}$ sits near the trimming limit (USD Coin, Tether): the half-lives depend on this imprecise estimate, the linearity test does not

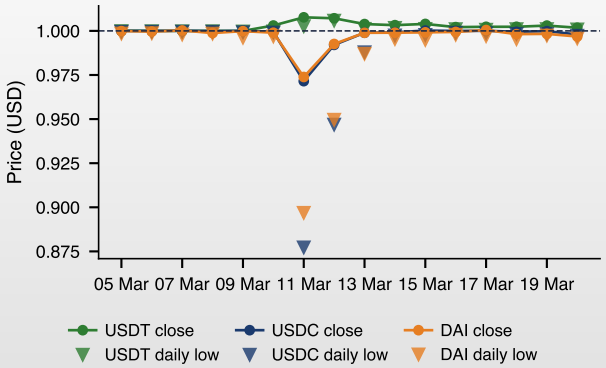
Threshold AR: Inference with Serial Dependence

- The p-values of the table are conditional on the AR(1) and on independent multipliers η_t
  - lag-1 autocorrelation of the score $d_{t-\mathbf{1}}\hat{e}_t$: -0.20 (Tether), -0.23 (USD Coin), -0.11 (Dai)
  - the AR(1) residuals stay autocorrelated: Ljung–Box with 10 lags, $p < 0.01$ for all three
- Two corrections, with the threshold $\hat{c}$ re-estimated each time
  - multipliers held constant on blocks of 13 consecutive days (dependent wild bootstrap, Shao (2010))
  - lagged differences Δd_{t-j} added until Ljung–Box no longer rejects at 5%

	p AR(1), independent	p AR(1), blocks	p with lags, blocks	lags	Holm
Tether	0.084	0.083	0.223	5	0.446
USD Coin	0.001	0.001	0.001	2	0.003
Dai	0.005	0.011	0.376	8	0.446

- Holm: p-values corrected for multiple testing, over the family of the three coins, on the column with lags and blocks
- Only the rejection for USD Coin survives both corrections; for Dai it depends on the AR(1) specification

March 2023: a Bank Failure Hits USD Coin



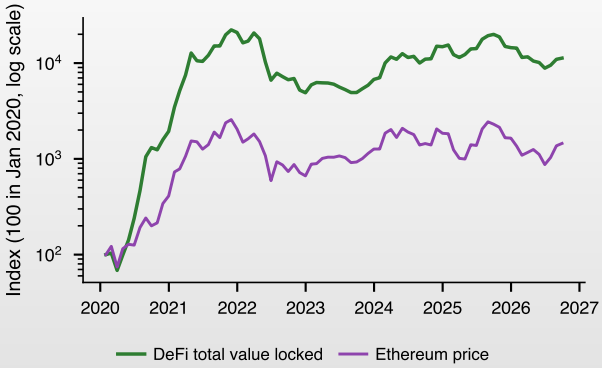
- Daily close (lines) and daily low (triangles), 5–20 March 2023
- Friday 10 March: Silicon Valley Bank (SVB) is closed
  - the FDIC (Federal Deposit Insurance Corporation) is appointed receiver FDIC (2023)
  - USD Coin held part of its reserves there Diop, Chevallier & Sanhaji (2024)
- Weekend: redemptions could not settle, the band disappeared
  - USD Coin's low 0.8774 USD on Saturday 11 March (close 0.9715)
  - Dai, crypto-backed, fell to 0.8970

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DeFi: the Question and the Choices

- **The question**: how does a liquidity pool set a price without an order book?
  - and who bears the cost of arbitrage?
- Why it matters: the pool is the counterparty to every swap
  - the liquidity provider holds a negative-gamma position: it loses, against its benchmark, on large price moves in either direction
- The model: the constant product $x \cdot y = k$ (Uniswap v2)
  - the base form of weighted pools and concentrated liquidity
- The pool studied: Ether against a stablecoin, a single risky asset, so losses read directly in USD
- The benchmark: LVR (loss versus rebalancing), not simple holding
  - it separates the cost of arbitrage from the direction of the price
- Market size: total value locked (TVL)

Total Value Locked in DeFi



- Total value locked (TVL), all chains (DefiLlama), and the Ether price
  - TVL: the value of tokens deposited in DeFi protocols
  - Month ends, 100 in January 2020, log scale
- TVL: 0.6 billion USD in January 2020, 177 billion at the peak (9 Nov 2021)
  - 88 billion today (50% below the peak)

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The Constant-Product Market Maker (1/2): the Pool and a Swap

- A pool holds reserves x of token X and y of token Y Adams, Zinsmeister & Robinson (2020)
  - e.g. X is Ether and Y is USD Coin
  - it accepts any trade that keeps $x \cdot y = k$
  - k: the invariant (not the k of the indices), set by the deposits; pool price (marginal): $p = y/x$ units of Y per X
- A trader sells Δx of X and receives Δy of Y
  - the pool keeps a fee f, so only $\gamma \Delta x$ enters the invariant, $\gamma = 1 - f$
  - $(x + \gamma \Delta x)(y - \Delta y) = xy \;\Rightarrow\; \Delta y = \dfrac{y \gamma \Delta x}{x + \gamma \Delta x}$
  - The fee $(1 - \gamma)\Delta x$ stays in the reserve, so the product of the reserves grows

The Constant-Product Market Maker (2/2): the Execution Price

- Execution price $\Delta y/\Delta x = p\,\dfrac{\gamma}{1+\gamma\delta}$, with $\delta = \Delta x/x$ the trade size relative to the reserve
  - **Shortfall** against the pool price: $1-\gamma/(1+\gamma\delta)$: fee plus price impact; it grows with f and with δ
- Arbitrageurs keep p in line with other exchanges: the pool price follows the market Angeris et al. (2019)

Impermanent Loss and Loss Versus Rebalancing (1/3): the Positions

- A **liquidity provider** (LP) deposits x_0 of X and $y_0 = p_0 x_0$ of Y
  - half and half in value
  - receives a share of the pool
  - p_0: the price at the deposit; p_1: the market price later; $r = p_1/p_0$: the price relative (written P_1/P_0 in the chart)
  - Without fees, arbitrage moves the pool to p_1: from $xy = k$ and $y/x = p_1$, $x_1 = \sqrt{k/p_1}$, $y_1 = \sqrt{k\,p_1}$
- Two values in units of Y
  - the pool position $V_{\mathrm{LP}} = p_1 x_1 + y_1 = 2\sqrt{k\,p_1}$
  - simply holding the deposit $V_{\mathrm{H}} = p_1 x_0 + y_0$

Impermanent Loss and Loss Versus Rebalancing (2/3): Impermanent Loss

- **Impermanent loss** (IL), the relative shortfall of the pool against holding, using $k = p_0 x_0^2$:

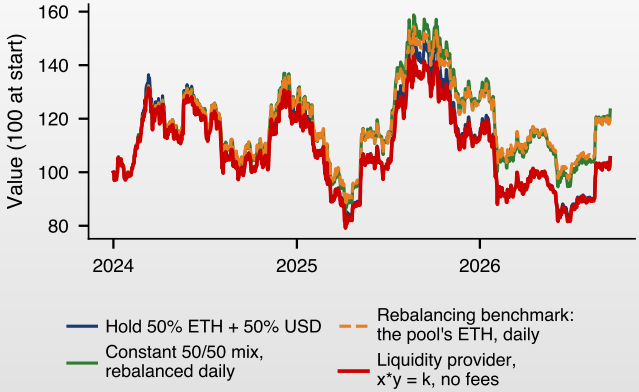
  $$\mathrm{IL}(r) = \frac{V_{\mathrm{LP}}}{V_{\mathrm{H}}} - 1 = \frac{2\sqrt{r}}{1+r} - 1 \leq 0$$

  - $\mathrm{IL} = 0$ only at $r = 1$; $r = 2$ or $r = 0.5$: -5.72%; $r = 4$ or 0.25: -20.00%
  - Symmetric in $\ln r$: a doubling and a halving of the price cost the same
  - Small moves cost little: IL grows roughly with $(\ln r)^2$
- "Impermanent": it vanishes if the price returns to p_0
  - it becomes permanent if the LP withdraws before the price returns

Impermanent Loss and Loss Versus Rebalancing (3/3): LVR

- A fairer benchmark than holding: a **rebalancing** portfolio Milionis et al. (2022)
  - it holds the pool's quantity of Ether at every instant and trades at market prices
  - The pool rebalances too, but at stale prices set by arbitrageurs: it always trades at a loss to them
- For $x \cdot y = k$, LVR grows at the instantaneous rate $d\mathrm{LVR}_t/(V_t\,dt) = \sigma_t^2/8$
  - loss versus rebalancing (LVR): the gap between the rebalancing portfolio and the pool
  - derivation by Itô in the Appendix
  - V: the pool value; σ: annualised volatility of the price; example: $\sigma = 60\%$ gives $0.36/8 = 4.5\%$ of the pool value a year
  - A weighted pool $x^w y^{1-w} = k$, with $w \in (0,1)$ the value weight of X ($w = 1/2$: constant product), loses at the rate $\sigma^2 w(1-w)/2$ (Seminar 16, A7)
- Fees must exceed $\sigma^2/8$ a year for a liquidity provider to beat this benchmark, in the frictionless model
  - not to earn a positive return
  - a volatility-selling business

A Liquidity Provider in an Ether Pool (1/3)



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A Liquidity Provider in an Ether Pool (2/3)

- Daily Ether prices from 1 January 2024
  - 100 invested half in Ether, half in USD
  - Pool with no fees; dashed: the LVR benchmark (the pool's Ether, rebalanced daily)
  - Constant 50/50 mix: weights reset daily (final value 123.3)
- Ether 2 352 $\to$ 2 611 USD ($r = 1.11$): impermanent loss only -0.14%
  - Final values: holding 105.5, pool 105.4, LVR benchmark 122.4

A Liquidity Provider in an Ether Pool (3/3)

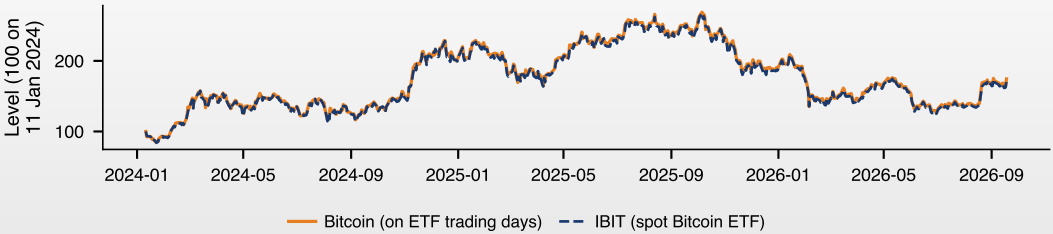
- Realised LVR 5.83% a year against $\sigma^2/8 = 5.79\%$ ($\sigma = 68.1\%$)
  - 95% block-bootstrap CI $[4.88, 6.98]\%$
  - the CI of a realised variance requires a finite fourth moment, which Chapter 1 questions
- Unlike impermanent loss, LVR accumulates with every move
  - in the frictionless model, fees would have to earn about 5.79% a year to match the benchmark
  - with fees, arbitrage happens only outside the band, and the loss is smaller Milionis, Moallemi & Roughgarden (2025)
- Why the LVR benchmark beats holding here
  - R_T: the LVR benchmark, V_T^{LP}: the pool, V_T^{H}: holding
  - $R_T - V_T^{\mathrm{H}} = \mathrm{LVR}_T + (V_T^{\mathrm{LP}} - V_T^{\mathrm{H}})$
  - here $122.4 - 105.5 \approx 17.0 + (105.4 - 105.5)$
  - The price ended near its start ($r = 1.11$), so the impermanent-loss term is tiny; after a strong trend ($r = 4$: IL $= -20.00\%$ of V_T^{H}) it can outweigh LVR, and holding can come out ahead
- Type of evidence: comparison of procedures (it shows which method works better on the same data, not why)
  - three positions on the same Ether price path, without fees

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Spot ETFs: the Question and the Choices

- **The question**: does a spot ETF replicate the coin it holds?
  - and how much of a crypto equity's risk comes from Bitcoin?
- Why it matters: spot ETFs are how US stock-market investors hold Bitcoin and Ether
  - since January 2024
- The funds: IBIT (iShares Bitcoin Trust) and ETHA (iShares Ethereum Trust), the BlackRock spot ETFs on Bitcoin and Ether BlackRock (2026)
  - same issuer and same 0.25% annual fee
- The measures: beta to the coin, tracking error and fee drift, daily and weekly
  - because the daily closes are not simultaneous
- The equities: two different channels of Bitcoin exposure
  - Coinbase: revenue from trading volume
  - Strategy: Bitcoin on the balance sheet
- Inference: Newey–West SE Newey & West (1987), because the residuals show volatility clustering

IBIT Follows Bitcoin



- Bitcoin and IBIT on ETF trading days, both 100 on 11 Jan 2024
- A **spot ETF** holds the coins with a custodian and trades in US hours
- **Authorised participants** create or redeem shares against coins
  - this keeps the price within a cost band around the net asset value, as for a stablecoin
  - NAV: the fund's holdings at market prices, per share

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Measuring How Well an ETF Tracks (1/2): Regression and Tracking Error

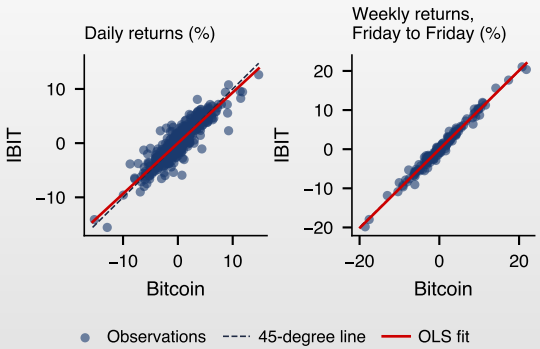
- Regression on the ETF's trading days: $r_t^{\mathrm{ETF}} = a + \beta\, r_t^{\mathrm{coin}} + u_t$
  - r_t^{ETF}, r_t^{coin}: log returns of the ETF and of its coin; a perfect tracker has $\beta = 1$, $R^2 = 1$
- **Tracking error** $\mathrm{TE} = \mathrm{sd}(r_t^{\mathrm{ETF}} - r_t^{\mathrm{coin}})\sqrt{A}$: annualised volatility of the return gap
  - sd: sample standard deviation; $A = 252$ daily, 52 weekly
- **Fee drift**: slope of $\ln(P_t^{\mathrm{ETF}}/P_t^{\mathrm{coin}})$ on time in years
  - the fund sells coins to pay its fee, so the ratio falls by about the fee each year

Measuring How Well an ETF Tracks (2/2): Results

	$\hat{\beta}$ daily	R^2 daily	$\hat{\beta}$ weekly	R^2 weekly	TE daily	TE weekly
IBIT / Bitcoin	0.93	0.82	1.01	0.98	20.8	6.2
ETHA / Ether	0.94	0.83	0.99	0.98	29.9	9.0

- TE (Tracking Error) in % a year; weekly data show a near-perfect tracker
  - fee drift -0.24% a year (IBIT) and -0.24% (ETHA), close to the 0.25% fee
- Daily data understate the fit: the coin and the ETF close at different times
- IBIT trades 2.2 billion USD a day on average
- Type of evidence: description

Daily vs Weekly Returns and the Dimson Beta



- IBIT against Bitcoin: daily returns on common days (left), Friday-to-Friday returns (right); R^2 0.82 vs 0.98
- The coin's daily close and the New York close are hours apart
  - Part of day t's move reaches the ETF on $t \pm 1$: the daily OLS beta is biased towards zero
  - Newey–West fixes the SE, not the bias; the Dimson beta, with lagged coin returns, fixes the bias

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Crypto Equities: Coinbase and Strategy

- **Coinbase**: a US crypto exchange, revenue tied to trading volume
  - **Strategy** (formerly MicroStrategy): a software company holding Bitcoin as its main treasury asset
- Weekly two-factor regression, Newey–West SE, April 2021 – September 2026
  - $r_t^{\text{stock}} = a + \beta_{\text{BTC}}\, r_t^{\text{BTC}} + \beta_{\text{SPX}}\, r_t^{\text{S\&P}} + u_t$
  - r_t^{stock}, r_t^{BTC}, $r_t^{\text{S\&P}}$: weekly log returns of the stock, Bitcoin and the S&P 500
  - β_{BTC}, β_{SPX}: the stock's sensitivity to each, holding the other fixed
  - each SE requires finite moments of the score $x_t u_t$, a condition the tails of Chapter 1 question

	$\hat{\beta}_{\text{BTC}}$	SE	$\hat{\beta}_{\text{SPX}}$	SE	R^2	σ_a (%)
Coinbase	0.79	0.09	2.05	0.22	0.52	84
Strategy	1.18	0.11	1.02	0.24	0.63	89

- Strategy: 1.18 times Bitcoin plus 1.02 times the S&P 500
  - a leveraged Bitcoin position with extra equity risk
- Coinbase: $\hat{\beta}_{\text{SPX}} = 2.05$: a high-beta US stock, not a Bitcoin proxy
- Type of evidence: association (the variables move together; this does not prove causation)

▸ Further Topics: rolling beta to Bitcoin

The Asset Class: the Question and the Choices

- **The question**: are cryptos a separate asset class?
  - does a crypto market factor explain their mean returns?
- Why it matters: a separate class needs its own allocation and benchmark
  - an incomplete factor model leaves nonzero alphas
- Two views: pricing and distribution
  - pricing: a factor model tested by GRS (zero alphas) and Fama–MacBeth (risk premium)
  - distribution: a map of return features, as in Pele et al. (2023)
- Test assets: eight large coins, weekly excess returns from February 2018, $T = 450$
  - Bitcoin, Ethereum, XRP (native token of the XRP Ledger), BNB (native token of BNB Chain, formerly Binance Coin), Cardano, Dogecoin, Litecoin, Chainlink
  - excess returns over the 4-week bill (DTB4WK)
- Market factor: the excess return of the index of Section 3
- Weekly frequency: avoids matching 7-day and 5-day calendars and follows the case study
- The map: one window before and one after the spot ETF launch
  - to see whether institutionalisation moved cryptos towards the other classes

Testing a Factor Model (1/2): the Time-Series Regression

- Time-series regression for each test asset $i = 1, \ldots, N$: $r_{it}^e = \alpha_i + \beta_i' f_t + \varepsilon_{it}$, $t = 1, \ldots, T$
  - r_{it}^e: excess return of asset i in week t (over the Treasury bill); f_t: vector of the K traded factor returns; β_i: the asset's loadings; ε_{it}: residual
  - α_i: the mean excess return the factors leave unexplained; a correct model has $H_0 \colon \alpha_1 = \cdots = \alpha_N = 0$

Testing a Factor Model (2/2): the GRS Test

- Gibbons, Ross & Shanken (1989): $\mathrm{GRS} = \dfrac{T-N-K}{N} \dfrac{\hat{\alpha}' \hat{\Sigma}^{-1} \hat{\alpha}}{1 + \bar{f}' \hat{\Omega}^{-1} \bar{f}} \sim F(N, T-N-K)$ under H_0
  - $\hat{\alpha}$: vector of the N estimated alphas; $\hat{\Sigma}$: residual covariance; $\bar{f}$, $\hat{\Omega}$: mean and covariance of the factors
  - Numerator: the alphas, weighted by the precision of the residuals; denominator: one plus the squared Sharpe ratio of the factors
  - A large GRS (small p-value from the F distribution) rejects zero alphas
- F is exact only for i.i.d. errors with the Normal distribution
  - with crypto tails, add a wild bootstrap under H_0 (▸ Appendix)

Worked Result: GRS and Fama–MacBeth on Eight Coins

- GRS $= 2.16$: zero alphas rejected at 5%
  - $p = 0.029$ from $F(8, 450 - 9)$, $p = 0.018$ from the wild bootstrap with independent multipliers (with serial dependence: next slide)
  - without normality, the test requires finite moments of ε_t and $f_t\varepsilon_t$ and weak dependence, conditions Chapter 1 questions
  - Largest alphas: Dogecoin +99% a year ($t = 1.23$), BNB +51% ($t = 1.97$, the only one at the 5% threshold, and only just)
  - With eight tests, Holm's correction needs $p < 0.05/8 \approx 0.006$ for the smallest p-value, and BNB has $p \approx 0.05$: the rejection is joint, not driven by one coin
- Fama–MacBeth factor risk premium: $\hat{\lambda} = 163\%$ a year, SE 147 (FM) and 156 (Shanken)
  - the mean excess return of the factor was 39% ($t = 1.81$)
  - Betas range only from 0.90 to 1.40: the premium is estimated very imprecisely in this cross-section
- Survivorship may contribute to the rejection; GRS does not identify its cause
  - coins chosen because they are large today
- Type of evidence: description

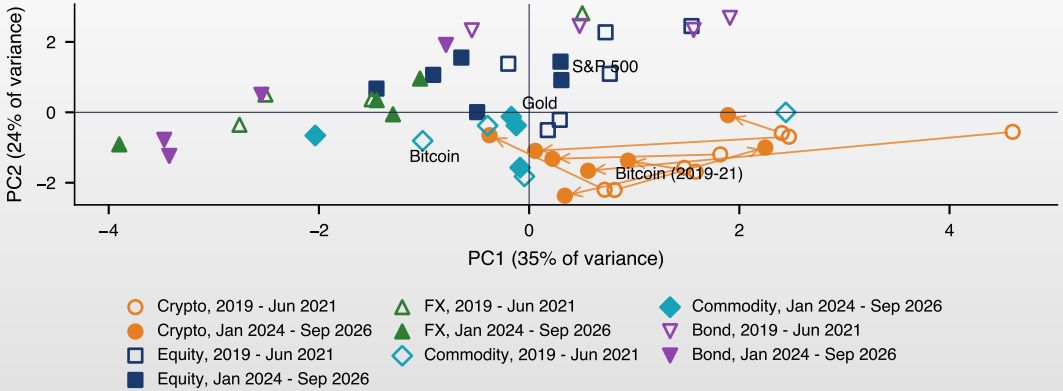
GRS and Fama–MacBeth: Inference with Serial Dependence

- The weekly residuals are not independent
  - Ljung–Box with 4 lags rejects no autocorrelation at 5% for 5 of the 8 coins
  - for the squared residuals: 6 of the 8 coins (volatility clustering)
- GRS with a dependent wild bootstrap (Shao (2010)): multipliers held constant on blocks of 4 weeks
  - $p = 0.038$, against 0.018 with independent multipliers: the rejection at 5% stands
- Fama–MacBeth, $\hat{\lambda} = 163\%$ a year
  - Newey–West SE of the weekly slopes (5 lags): 141
  - block bootstrap of 4 weeks, re-estimating both stages: SE 112, 95% CI $[-50, 381]\%$
  - the Shanken correction assumes i.i.d. errors and factors, which the diagnostics above do not support
- Type of evidence: comparison of procedures
  - zero alphas are rejected at 5% under every calibration; the premium stays very imprecisely estimated

A Separate Asset Class? The Evidence So Far

- Pele et al. (2023) compare cryptos with stocks, exchange rates, commodities, bonds and real-estate indices
  - Dimension reduction: most of the variation is explained by **tail**, **memory** and **moment** factors of log returns
  - Classification methods place cryptos in a separate class, mainly because of the tail factor
  - The tail factor (76% of the variance) loads on quantiles, conditional tail expectations, variance and the α-stable parameters (Pele et al. (2023), Sect. 3.2): it mixes scale with shape, not a difference in Hill indices
  - Over time cryptos became more alike and more distinct from other assets
- Hedge or safe haven? Baur & Lucey (2010)
  - a hedge is uncorrelated or negatively correlated on average
  - a safe haven on the worst market days
- Early evidence: Bitcoin a diversifier, a hedge only in some periods Bouri et al. (2017)
  - tested with post-ETF data in Seminar 16, C2

Cryptos Among Other Asset Classes (1/2)



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Cryptos Among Other Asset Classes (2/2)

- First two principal components, PC1 (first Principal Component) and PC2 (35% and 24% of the variance)
  - Hollow: W1 (2019–2021), filled: W2 (after the ETF launch); FX (Foreign eXchange) in the legend: currencies
  - Arrows: each crypto
- PC1 measures volatility, kurtosis, drawdown depth and tail heaviness (loadings in the Appendix)
  - In W1 the cryptos sit apart on PC1, as in Pele et al. (2023); in W2 some of them overlap with the commodities
  - kurtosis exists only with a finite fourth moment, and the usual inference on it needs a finite eighth moment; Chapter 1 questions both
- Point estimates move towards the others
  - distance to equities 3.03 $\rightarrow$ 2.63, to commodities 2.26 $\rightarrow$ 1.86
  - Bitcoin's volatility 77% $\rightarrow$ 47%
- Cryptos became more alike (spread around their centre 2.13 $\rightarrow$ 1.43)
  - Median Hill index of the cryptos 2.32 $\rightarrow$ 3.26
- Type of evidence: description
  - all 95% bootstrap CIs of the move contain 0 (Further Topics)

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Further Topics: significance of the move

Case Study: Three Factors for Crypto Returns

- Liu, Tsyvinski & Wu (2022), *Journal of Finance*: the equity factor-model toolkit applied to crypto
  - portfolio sorts, factor regressions, GRS
  - on the whole cross-section of coins
  - Question: do a few crypto-specific factors price the expected returns of all coins, as market, size and momentum do for stocks?
- It builds on Liu & Tsyvinski (2021): aggregate crypto returns barely load on stock, currency or metal factors
  - but are predicted by crypto momentum and attention
  - The step: from the time series of three coins to the cross-section of 1,827 coins
- Why it matters: a factor model is the benchmark for every claimed crypto anomaly
  - and for judging crypto funds
  - if coins are segmented from equities, stock factors are the wrong benchmark
- Link to this chapter
  - the index of Section 3, the GRS test of Section 7
  - data-quality threats in the Appendix

Case Study: Building Crypto Factors from 1,827 Coins

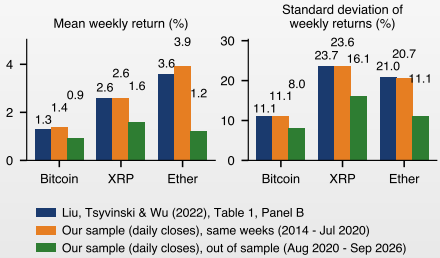
- Data: daily prices, volumes and market values of active and defunct coins (Liu, Tsyvinski & Wu (2022), Sect. 2)
  - from an aggregator of over 200 exchanges
  - market value of at least 1 million USD: 1,827 coins, first week of 2014 – July 2020
  - Weekly returns from daily closes: 52 weeks a year, the last one of eight days (nine in leap years); CMKT (Crypto Market factor, value-weighted market excess return): value-weighted return of all coins minus the one-month bill rate
- Anomalies: 24 characteristics from the equity literature (Table 2)
  - they need only price, volume and market value
  - each week, value-weighted quintile portfolios and a top-minus-bottom spread
  - CSMB: bottom 30% minus top 30% of coins by market value; CMOM (Crypto Momentum factor, past winners minus losers) $= \frac{1}{2}(\mathrm{SH} + \mathrm{BH}) - \frac{1}{2}(\mathrm{SL} + \mathrm{BL})$ from 2×3 sorts on size and past three-week return (S/B: small/big coins; H/L: high/low past three-week return; Liu, Tsyvinski & Wu (2022), Sect. 4.1, Eq. 1)
- Tests: the ten significant spreads regressed on one-, two- and three-factor models (Tables 8–9)
  - joint GRS tests on all 24 spreads (Table 11)
  - The one-factor model, CMKT alone, is the crypto CAPM
  - Multiple testing: p-values adjusted for the many strategies tried (the error rate used: ▸ Appendix)

Case Study: Size and Momentum Explain the Anomalies

Result	Numbers in the paper	Reading
Weekly returns (Table 1)	mean, s.d.: market 1.3%, 11.2%; Bitcoin 1.3%, 11.1%; XRP 2.6%, 23.7%; Ether 3.6%, 21.0%	the market behaves like Bitcoin
Size (Table 3)	top minus bottom quintile -5.8% a week ($t = -2.45$)	premium in the smallest coins
Momentum (Table 4)	three-week winners minus losers 3.1% a week ($t = 2.65$)	winners keep winning
Crypto CAPM (Table 8)	all ten alphas significant; momentum $\alpha = 3.0\%$ ($t = 2.55$), $R^2 = 0.001$	the market alone fails
Three factors (Table 9)	no alpha significant; momentum $\alpha = 1.1\%$ ($t = 1.41$), CMOM loading 1.12, $R^2 = 0.58$	size and momentum explain the ten anomalies
Joint test (Table 11)	GRS on 24 spreads: $p = 0.033$ (Fama–French six factors), $p = 0.194$ (crypto three)	stock factors are the wrong benchmark

- Momentum is a large-coin effect, the opposite of stocks
  - 3.2% a week above the median size, insignificant below (Table IA.10)
  - IA: the paper's Internet Appendix
- Type of evidence: comparison of procedures

Case Study: Table 1 Rebuilt on the Course Data



- Weekly simple returns on the paper's calendar
  - Blue: Table 1, Panel B of Liu, Tsyvinski & Wu (2022)
  - Orange: our sample (daily closes), same weeks
  - Green: August 2020 – 16 Sep 2026

- Largest gap from the paper, the Ether mean: 0.26 i.i.d. standard errors (SE 1.29 points)
- Out of sample (319 weeks): means 0.9, 1.6, 1.2%
  - S.d. 8.0, 16.1, 11.1%: Ether's volatility roughly halves

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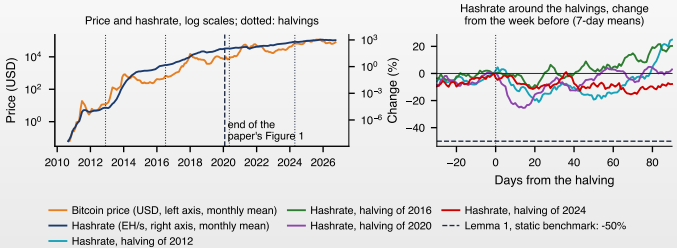
Limitations and Out-of-Sample Tests

- **Limits** stated by the authors (Section 6.2)
  - A short sample, 2014 – July 2020, while investors were still learning about the technology: only the split-half check (same signs)
  - Most coins cannot be shorted; for the top 20 coins, after trading costs, about 60% of the returns of long–short strategies that short Bitcoin remain (Table IA.16)
  - With prices from three reputable USD exchanges, the market-value size spread is no longer significant (Table IA.15)
- **Open extension**, out of sample, August 2020 – September 2026 (the 2022 collapses, the spot ETFs)
  - Does three-week momentum among large coins survive?
  - Is CMKT still segmented from equities?
  - Crypto-CAPM alphas; the segmentation regression of footnote 16 of Liu, Tsyvinski & Wu (2022) on the global five factors Fama & French (2015) and five exchange rates; a break test at an unknown date (Seminar 16, C3)

Case Study: The Bitcoin Price and Network Security

- Pagnotta (2022), *Review of Financial Studies*: an equilibrium model in which the Bitcoin price and the security of the network are determined jointly
  - users value coins taking into account the risk of an attack that would block transfers
  - miners invest computing power for the block reward, paid in bitcoin
- **Hashrate**: the total computing power of the miners, in hashes per second (1 EH/s $= 10^{18}$ hashes/s)
  - an attack succeeds only with computing power comparable to that of the honest miners
- The question: does Bitcoin's security depend on its price, not only on its technology?
- The paper's claims
  - the same technology admits multiple equilibria, with very different prices and security levels
  - the price–security feedback amplifies the volatility impact of fundamental shocks
  - it can produce boom–bust cycles not driven by fundamentals
  - the empirical implication: price and hashrate move together (Figure 1, Aug 2010 – Jan 2020)
- **Security budget**: the block reward plus the fees paid to miners
  - daily share of the block reward, Jul 2010 – Jan 2020: median 99.21%, mean 97.57% (the paper, Section 1.1, footnote 16, in the manuscript SSRN 3264448)
  - the same share in the Coin Metrics Community Data data: median 99.20%, mean 97.58%
  - every halving cuts the block reward in half

Case Study: Bitcoin Price and Hashrate, 2010–2026



- Left: monthly means of the price (course data) and of the hashrate (Coin Metrics Community Data), Aug 2010 – Sep 2026
  - hashrate: 952 EH/s, mean of the 7 days to 18 Sep 2026
- Right: change of the hashrate from the week before each of the four halvings
  - dashed line: −50%, the static benchmark of Lemma 1 with the expected price, the cost and the number of miners unchanged
  - largest fall in the first 90 days: −25% (2020, day 14)

MFM_ch16_pagnotta ▶ Interactive

Case Study: Lemma 1 and the Replication on the Course Data

- Lemma 1 of Pagnotta (2022): m identical miners; the lemma allows a convex cost, here the case of a linear cost c per unit of hashrate
  - equilibrium hashrate: $H^* = \frac{m-1}{m} \frac{\delta R \, \mathbb{E}_t[p_{t+\mathbf{1}}]}{c}$
  - R: the reward per block, in bitcoin; $p_{t+\mathbf{1}}$: the price at which the miner sells the reward; δ: the discount factor
  - Prediction 1: the elasticity of H^* to R is 1, with $\mathbb{E}_t[p_{t+\mathbf{1}}]$, c and m held constant
  - Prediction 2: with the expected price, the cost and the number of miners unchanged, a halving halves H^*
- Data: the Bitcoin price from the course data; hashrate, coins issued, fees and number of blocks from Coin Metrics Community Data
  - USD reward per block on day t: $B_t = p_t \times$ (subsidy + fees) / number of blocks
  - the miners' daily revenue also moves with the number of blocks found on day t, which also enters the hashrate estimate
  - the current price p_t stands in for the expected price $\mathbb{E}_t[p_{t+\mathbf{1}}]$
- The paper shows the series only (Figure 1); the replication adds three tests
  - regression of $\ln H$ on $\ln B$ and a linear trend, on monthly means: the trend absorbs the fall of the cost c as hardware becomes more efficient
  - the Engle & Granger (1987) test: the null hypothesis is no cointegration between $\ln H$ and $\ln B$
  - the four halvings: the subsidy per block halves at blocks 210,000, 420,000, 630,000 and 840,000; the dates are the UTC (Coordinated Universal Time, Temps Universel Coordonné) days of these blocks
- Windows: the paper's (Aug 2010 – Jan 2020) and out of sample (Feb 2020 – Aug 2026)

Case Study: At Halvings, Hashrate Falls Less than the Static 50% Benchmark

Test	Course data	Interpretation
Levels, Aug 2010 – Jan 2020	elasticity to B: 1.19 (HAC SE 0.21) with trend; 2.44 without trend ($R^2 = 0.91$)	association close to 1
Other revenues, with trend	subsidy per block 1.21; daily revenue 1.15	the same association
Levels, out of sample	elasticity: 0.03 (HAC SE 0.09) with trend; 0.82 without trend ($R^2 = 0.24$)	not significant next to the trend, which does not show that the link has gone
Engle–Granger	p-value: 0.79 (paper's window), 0.72 (out of sample)	no evidence of cointegration: the level regressions are descriptive
Halving, day 30: USD reward per block	2012: −45%; 2016: −56%; 2020: −48%; 2024: −52%	close to −50%
Halving, day 30: hashrate	2012: −10%; 2016: +4%; 2020: −5%; 2024: −8%	changes smaller than the static −50% benchmark

- Type of evidence: description
  - descriptive associations: they do not identify the effect of security on the price and do not test the multiple equilibria
- Lemma 1 is a static equilibrium condition: hardware already bought is a sunk cost, and miners stay active while the reward covers the cost of energy
- Limits: the hashrate is estimated from block times; miners anticipate the halvings; the expected price is not observed

An Open Question and the Discovery Loop (1/2)

- Did the US spot Bitcoin ETFs (since 11 January 2024) integrate Bitcoin with equities?
  - In the mean?
  - In the tails?
  - In price discovery?
- A live question
  - Lee & Lee (2025) study the shifted links after the approval
  - Kang & Lee (2026) title their answer "institutionalization without integration"
- Formal version: $H_0 : \rho_{\text{post}} = \rho_{\text{pre}}$ against $H_1 : \rho_{\text{post}} > \rho_{\text{pre}}$
  - ρ_{pre}, ρ_{post}: the correlations before and after the launch
  - the same for the lower-tail dependence λ_L of Chapter 6
  - lower-tail dependence: the probability of an extreme Bitcoin loss given an extreme equity loss
  - Falsified if the data choose a break far from January 2024, or if placebo dates give changes as large

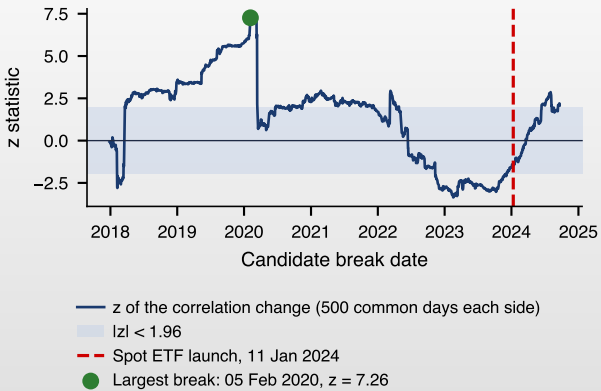
An Open Question and the Discovery Loop (2/2)

- Identification: the launch is an event, not an experiment
  - it overlaps with the rate cycle, the April 2024 halving and flows that respond to prices
- The discovery loop in one pass, the assistant drafts and the researcher verifies
  - literature (resolve every DOI, the digital object identifier of a paper), competing hypotheses, code that first replicates $\hat{\rho} = 0.39$ and 0.38, a pre-registered robustness plan, a referee-style critique; reusable prompts in the Appendix

What the Researcher Must Check

- **Calendars**: align prices on common trading days first, then compute returns
  - returns computed on each calendar and then matched pair Bitcoin's Sunday-to-Monday move with the S&P 500's Friday-to-Monday move
- **Closing times and volatility** Forbes & Rigobon (2002)
  - non-synchronous closes bias daily correlations towards zero, high volatility raises them
  - check weekly returns, the lead–lag correlation and the adjusted ρ^*
- **Survivorship and supply**: use the universe as it was on each date and the circulating supply, as in the CRIX rule
- **Invented references and numbers**
  - every DOI resolves on Crossref with the same title
  - every number is regenerated by code in the repository

Mini-Case: Did the ETFs Break the Correlation? (1/2)



- Assistant's claim: "the spot ETFs integrated Bitcoin with equities"
  - $\hat{\rho} = 0.39$ in 2020–2023 ($n = 1\,006$), 0.38 since 11 January 2024 ($n = 674$)
- Difference -0.01, 95% CI $[-0.12, 0.13]$ (circular block bootstrap, blocks of 20 days)
  - the CI of a correlation requires a finite fourth moment, which Chapter 1 questions

ai_discovery_case.py ▶ Interactive

Mini-Case: Did the ETFs Break the Correlation? (2/2)

- Chart: $z(\tau) = [\operatorname{atanh} \hat{\rho}_{\text{after}} - \operatorname{atanh} \hat{\rho}_{\text{before}}] / \sqrt{2/(h-3)}$
  - for each candidate date τ
  - correlations over $h = 500$ days on each side of τ
  - atanh (Fisher) makes each estimate roughly Normal with variance $1/(h-3)$, so $z \approx N(0,1)$ without a change
- Largest $z(\tau)$ on 5 Feb 2020 ($z = 7.26$); at the ETF date $z = -1.48$
  - 82% of candidate dates give a larger $|z|$
  - The scan points to early 2020, near the date that minimises the SSR in the break regression (9 Mar 2020)
- The ± 1.96 band of the chart is pointwise
  - the maximum over more than a thousand overlapping dates needs its own critical value, as for sup W
- With heavy tails and volatility clustering the variance of $\operatorname{atanh} \hat{\rho}$ is typically larger than $1/(h-3)$
- So the scan is descriptive; the inference is the sup-Wald test
- Type of evidence: description
  - the ETF launch is an event, not an experiment

Your Turn: a Project Seed

- **Question**: did institutionalisation change Bitcoin's **tail** dependence and **price discovery** with US equities?
- **Replicate first**: $\hat{\rho} = 0.39$ and 0.38 from this chapter
  - then one headline estimate of Lee & Lee (2025) or Kang & Lee (2026) on their own sample
  - Extend: lower-tail dependence from copulas (Chapter 6), weekly returns, IBIT–Bitcoin lead–lag in US trading hours
- **Deliverables**: a pre-registered plan committed before the results
  - a notebook that regenerates every number
  - effect sizes with CIs and the full list of specifications
- **AI log** (templates)
  - prompts that shaped the design in `AI_USE.md`
  - each wrong claim (an invented DOI, returns on mismatched calendars) and the test that caught it in `AI_ERRORS.md`

Key Takeaways

- Crypto returns: 3–7 times the volatility of shares, drawdowns above 80%
  - calmer weekends, heavy tails close to those of shares and gold
  - the one-break model dates the change in the coefficients to March 2020
  - the chosen comparison detects no rise in the correlation after the ETFs
  - beta to the S&P 500 rose from near 0 to above 1
- Indices: a Laspeyres-type index with a divisor
  - CRIX picks its size by an AIC on tracking residuals
  - value weights make crypto indices mostly Bitcoin
- Stablecoins: the peg is an arbitrage band that holds only while redemption works
  - a threshold AR fits it, tested with a bootstrap, not with χ^2
- DeFi: $xy = k$ prices a swap as fee plus price impact
  - providing liquidity is selling volatility, at $\sigma^2/8$ a year
- ETFs track one for one once closing times are handled
  - cryptos sat apart in distribution in 2019–2021 and moved closer to other classes after the ETFs (shift not significant); a risk asset in co-movement

Check Your Understanding, and What Comes Next

Retrieval questions

- Why is Bitcoin's sample kurtosis not reported?
- What replaces it?
- Why can a threshold or a break date not be tested with χ^2 critical values?
- When does a stablecoin's arbitrage band stop holding the peg?
- Why does a liquidity provider fall behind the rebalancing benchmark even if the price returns to p_0?
- Why does a daily regression understate how well IBIT tracks Bitcoin?

Next chapters

- Chapter 17, bubbles and crashes: explosive-root tests and log-periodic power laws, crypto booms as a main case
- Chapter 18, systemic risk: spillovers between crypto, banks and equity markets
- Revisit
  - VaR and ES for crypto (Chapter 7, Pele & Mazurencu-Marinescu-Pele (2019b))
  - Arbitrage, segmentation and liquidity across crypto exchanges (Chapter 10, Makarov & Schoar (2020), Agakishiev, Härdle & Pele (2025))
- Reading
  - Härdle, Harvey & Reule (2020), Franke, Härdle & Hafner (2019), Schär (2021)

The Practical Decision

- **What I choose**
  - the Hill tail index with its CI instead of the sample kurtosis: all six 95% CIs lie below 3.7
  - beta and correlation with equities from the regime after the 9 Mar 2020 break ($\hat{b} = 1.22$), on common days
  - weekly returns to judge ETF tracking ($\hat{\beta}$ of 1.01 and 0.99)
  - for a stablecoin peg, a threshold AR tested by a bootstrap that keeps the dependence, not with χ^2
- **Under which conditions**
  - redemption at the issuer works: otherwise the arbitrage band disappears (USD Coin, 11 Mar 2023)
  - for a liquidity provider: fees above $\sigma^2/8$, i.e. 5.79% a year in the Ether pool, the frictionless benchmark
- **What result would make me drop it**
  - a later break, confirmed by a new sup-Wald test and dated by the SSR minimum, after which beta returns towards 0
  - dependence-robust Hill CIs lying entirely above 4, for both tails: only then would the kurtosis have a finite parameter to estimate
  - on weekly data, an ETF beta far from 1 or a fee drift well above the 0.25% fee

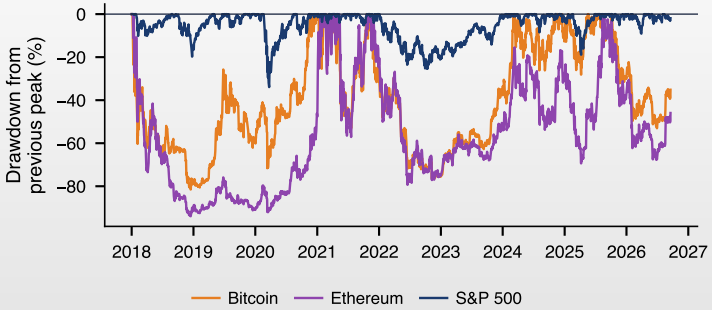
Longer Histories and Extra Series

Series	Period	Used for
Bitcoin and S&P 500, daily	from January 2016	2017–2019 correlations, the Forbes–Rigobon base period, the break test (2 693 common days)
Bitcoin and XRP; Ether	from January 2014; from 6 August 2015 (week 32)	the case study: Table 1 of Liu, Tsyvinski & Wu (2022) rebuilt, 52-week volatility
XRP, BNB, Chainlink, daily	weekly returns from 9 Feb 2018	the GRS test on eight coins (Section 7), the case-study replication
4-week Treasury bill, DTB4WK (FRED)	the same weeks as the GRS test	excess returns in the GRS test
One-month Treasury bill rate (Kenneth French Data Library)	the case-study weeks	the case study, definition of CMKT
Spot ETFs IBIT and ETHA	from 11 Jan 2024 and 23 Jul 2024	ETF tracking and the Dimson beta
Coinbase and Strategy shares	weekly, April 2021 – September 2026	the two-factor regressions of crypto equities

Market Value of Five Cryptocurrencies: Concentration

- Concentration: **Herfindahl–Hirschman index** $\mathrm{HHI} = \sum_i w_i^2$, $w_i = M_{i,t} / \sum_j M_{j,t}$ the value share of coin i
  - From $1/5$ (equal shares) to 1 (one coin)
  - Today 0.71, an effective number of coins $1/\mathrm{HHI} = 1.42$
- The five coins: Bitcoin, Ethereum, Dogecoin, Cardano and Litecoin
  - Solana has no public circulating-supply series, hence no market value (Section 3)
- XRP and Chainlink are left out: their reported supply includes issuer-held tokens
  - XRP: the native token of the XRP Ledger
  - free float matters

Drawdowns: How Deep and How Long



- Fall from the previous maximum, in %: Bitcoin was more than 50% below its peak on 36% of the days since 2018
  - Ethereum on 61%; the S&P 500 at most 34% (March 2020)
- Today: Bitcoin 35%, Ethereum 46% below their peaks
  - long "crypto winters" (2018–2020, 2022–2023) follow each boom (bubbles: Chapter 17)

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Bitcoin's Beta to the S&P 500: HAC Standard Errors

- Newey–West SE Newey & West (1987): $\widehat{\mathrm{Var}}(\hat{a}, \hat{b}) = (X'X)^{-1}\,\hat{S}\,(X'X)^{-1}$
  - $\hat{S} = \hat{\Gamma}_0 + \sum_{l=1}^{L}(1 - \frac{l}{L+1})(\hat{\Gamma}_l + \hat{\Gamma}_l')$, with $\hat{\Gamma}_l = \sum_t \hat{u}_t \hat{u}_{t-l}\, x_t x_{t-l}'$ and $x_t = (1, r_t^{\mathrm{SPX}})'$, row t of the matrix X
  - The Bartlett weights $1 - l/(L+1)$ keep $\hat{S}$ positive semi-definite
  - $L = \lfloor 4(T/100)^{2/9} \rfloor$ lags: 8 for $T = 2\,693$ days
- Why HAC (heteroskedasticity and autocorrelation consistent)?
  - with volatility clustering the variance of u_t rises on turbulent market days
  - OLS SE assume a constant variance and uncorrelated errors
- Whole sample: $\hat{b} = 0.95$, HAC SE 0.13, against an OLS SE of 0.07
  - One beta for 2016–2026 assumes a stable link: first test for a break at an unknown date

Correlation or Volatility? The Forbes–Rigobon Adjustment (1/2)

- Pearson correlation rises mechanically when the volatility of the driving market rises Forbes & Rigobon (2002)
  - with the same underlying link
- Adjusted correlation: the correlation the volatile period would show at the volatility of the calm period
  - $\rho^* = \dfrac{\rho}{\sqrt{1+\delta(1-\rho^2)}}, \qquad \delta = \dfrac{\sigma^2_{\mathrm{SPX,high}}}{\sigma^2_{\mathrm{SPX,low}}} - 1$
  - ρ: correlation of Bitcoin and S&P 500 returns in the volatile period; ρ^*: the adjusted correlation
  - $\sigma^2_{\mathrm{SPX,high}}$, $\sigma^2_{\mathrm{SPX,low}}$: variance of S&P 500 returns in the volatile period and in the calm base period (2017–2019)
  - δ: relative rise of that variance; $\delta = 0$ gives $\rho^* = \rho$, $\delta > 0$ gives $\rho^* < \rho$
- Caveat: the adjustment assumes no endogeneity and no omitted common shocks
  - a pandemic hardly satisfies these assumptions

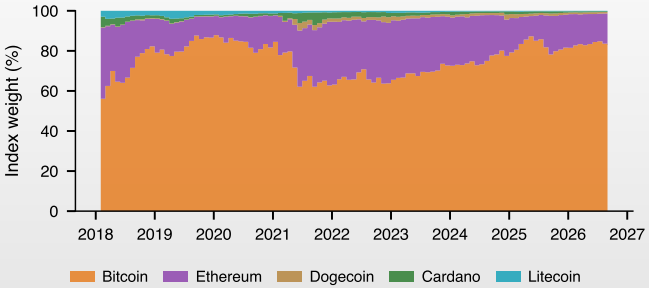
Correlation or Volatility? The Forbes–Rigobon Adjustment (2/2)

- 2020–2023: S&P 500 volatility 23.1% vs 12.8% ($\delta = 2.24$); $\hat{\rho} = 0.39$ becomes $\rho^* = 0.23$
  - Since the ETFs: $\hat{\rho} = 0.38$, $\rho^* = 0.33$ ($\delta = 0.42$)
- Change against 2017–2019 ($\hat{\rho} = 0.02$): $\rho^*_{\text{2020–23}} - \hat{\rho}_{\text{2017–19}} = 0.21$
  - 95% block-bootstrap CI $[0.09, 0.33]$
  - The CI excludes 0: the adjusted correlation did rise
  - Under the adjustment's assumptions, 44% of the raw rise is attributed to volatility; the adjusted increase remains positive: stronger co-movement, not an identified mechanism

Survivorship Bias: Indices of Survivors

- The universe at the start: coins that will survive ($S_i = 1$) and coins that will die ($S_i = 0$)
  - $\pi = P(S_i = 1)$
- Population mean: $E[\bar{r}_i] = \pi\, E[\bar{r}_i \mid S_i = 1] + (1 - \pi)\, E[\bar{r}_i \mid S_i = 0]$
  - $\bar{r}_i$: the mean return of coin i
- Bias of a survivors-only sample: $E[\bar{r}_i \mid S_i = 1] - E[\bar{r}_i] = (1 - \pi)\big(E[\bar{r}_i \mid S_i = 1] - E[\bar{r}_i \mid S_i = 0]\big)$
  - Positive when survival depends on returns: coins that crash are delisted Brown, Goetzmann, Ibbotson & Ross (1992)
  - Grows with the share of dead coins, $1 - \pi$, large in crypto
- Our indices use five coins chosen today
  - their returns are probably higher than those of a strategy one could have run in 2018
- Remedy: the point-in-time universe, dead coins included, as in Liu, Tsyvinski & Wu (2022)

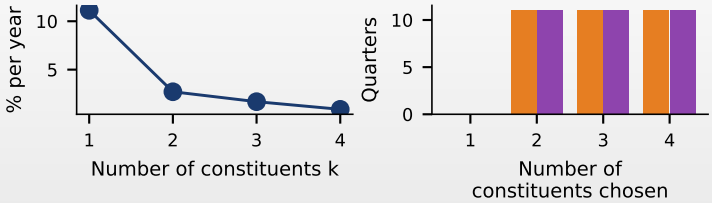
Index Weights Over Time



- Value weights at each monthly reallocation, %: Bitcoin 75.7% on average, never below 56.1%, today 82.9%
  - Ethereum today 15.8%; the other three together today below 2%
- 82.9% against the 82.5% market-value share: index weights are fixed at the last reallocation
- Altcoins (coins other than Bitcoin) gain weight late in booms (2021) and lose it in busts
  - a value-weighted crypto index is mostly Bitcoin

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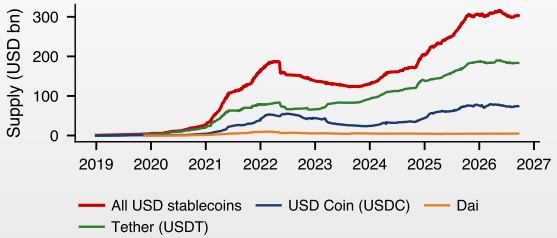
Tracking Error and the CRIX Rule



- Left: tracking error of the top-k index (weights 1) against the five-coin index
  - 11.1% a year for Bitcoin alone, 2.7% for two coins, 1.7% for three, 0.9% for four
- Right: the rule picks 2 coins in 11, 3 in 11 and 4 in 11 of 33 quarters (2 in the last)
  - ECRIX and EFCRIX agree in every quarter

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The Rise of Stablecoins



- Supply in circulation (DefiLlama): from 4.2 billion USD in January 2020 to 303 billion on 18 Sep 2026
  - about 73 times: the cash leg of crypto trading and DeFi
- Tether 183 billion (61%), USD Coin 74 billion (24%)
  - for the 42 coins above 100 million USD at extraction (21 Oct 2026), valued on 18 Sep 2026, a universe chosen retrospectively: fiat-backed 278, crypto-backed 22.2, algorithmic 0.15 billion
- 2022–2023 dip: 187 billion (2 Apr 2022) to 123 billion (19 Aug 2023)
  - TerraUSD vanished, USD Coin fell from 55 to 23 billion

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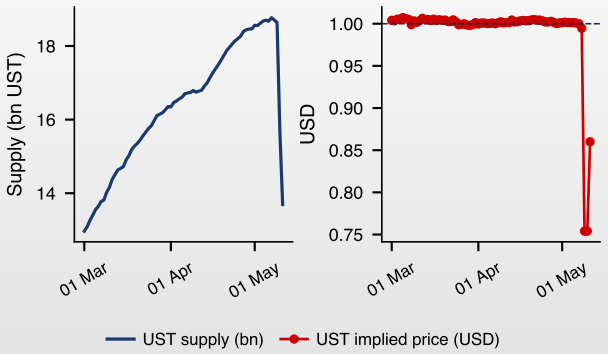
Arbitrage Bands Imply a Threshold AR: the Hansen Bootstrap

- p-value from the fixed-regressor bootstrap of Hansen (1996), valid with heteroskedasticity
  - Artificial series $d_t^* = \hat{\varepsilon}_t \eta_t$: $\hat{\varepsilon}_t$ the residuals of the linear AR(1) (the model under H_0), $\eta_t \sim N(0,1)$ independent draws
  - Valid if the scores $d_{t-1}\hat{\varepsilon}_t$ are serially uncorrelated; otherwise η_t is held constant on blocks of consecutive days (Shao (2010))
  - The regressors d_{t-1} are held fixed; $\sup_c W^*(c)$ is recomputed on each artificial series; p = share of replications with $\sup_c W^* \geq \sup_c W$

The Band Width: an Imprecise Estimate

- $\hat{c}$ for USD Coin (2.87 bp) is close to the largest threshold the trimming admits (2.93 bp)
  - the estimate is imprecise
  - Tether: $\hat{c} = 8.57$ bp, largest threshold 8.77 bp; Dai, with 19.6% of the days outside the band, is farther from the boundary
- The half-lives depend on $\hat{c}$
- The linearity test does not: $\sup_c W(c)$ and its bootstrap search the whole grid

TerraUSD: an Algorithmic Stablecoin Collapses



- Supply and implied price (market value / supply), March–May 2022 (DefiLlama)
- TerraUSD (UST) held its peg by exchange with the token LUNA (Luna, native token of the Terra blockchain), not with reserves
  - Demand came from the high yields of the Anchor lending protocol Liu, Makarov & Schoar (2023)
- Supply 18.8 billion UST on 7 May 2022, 13.7 billion four days later (27% redeemed)
  - Implied price 0.754 USD on 10 May 2022
  - The series ends on 11 May 2022; UST fell much further afterwards

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Two Stress Episodes: Doubted Backing, Failed Design

USD Coin, March 2023: the backing

- Tether rose to 1.0077 (high 1.0296): a flight to the other large stablecoin
- Sunday 12 March: all SVB depositors protected Treasury, Federal Reserve & FDIC (2023)
  - The peg returned within two days: the event tested the backing, not the design

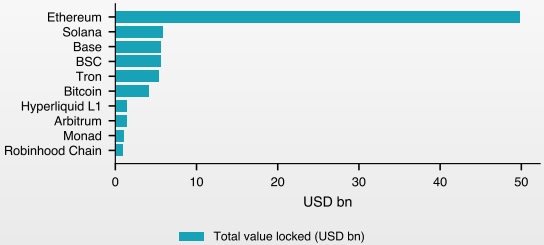
TerraUSD, May 2022: the design

- Redemption minted more LUNA and pushed its price down: a death spiral
  - within days, UST and LUNA lost almost all their value
  - Sophisticated holders ran first and lost less Liu, Makarov & Schoar (2023)
- Without external collateral the peg rests on confidence alone: a run is self-fulfilling

Total Value Locked in DeFi

- Monthly $\Delta \ln \mathrm{TVL}_t = a + b\,\Delta \ln P_t^{\mathrm{ETH}} + u_t$, Newey–West SE
  - b: the % change in TVL per 1% change in the Ether price P_t^{ETH}: $\hat{b} = 0.79$ (SE 0.07), $R^2 = 0.62$, $n = 79$ complete months (Feb 2020 – Aug 2026)
- TVL mixes token prices with net deposits
  - measure activity with fixed-price quantities, net flows or fees
  - DeFi risks: leverage, liquidity mismatch, interconnection Aramonte, Huang & Schrimpf (2021)

Where the Value Is Locked



- TVL by blockchain on 18 September 2026 (DefiLlama): the ten largest of the 15 largest at extraction (16 Oct 2026), so a retrospective ranking
  - Ethereum 49.8 billion USD, 57% of 88 billion
  - Solana 5.9, Base 5.6, BSC (BNB Smart Chain) 5.6, Tron 5.4
- Base and Arbitrum are layer-2 networks that settle on Ethereum
  - cheaper transactions, the same security anchor
- L1 in the chart (Hyperliquid L1): a layer-1 network, a blockchain that settles its own transactions

MFM_ch16_defi

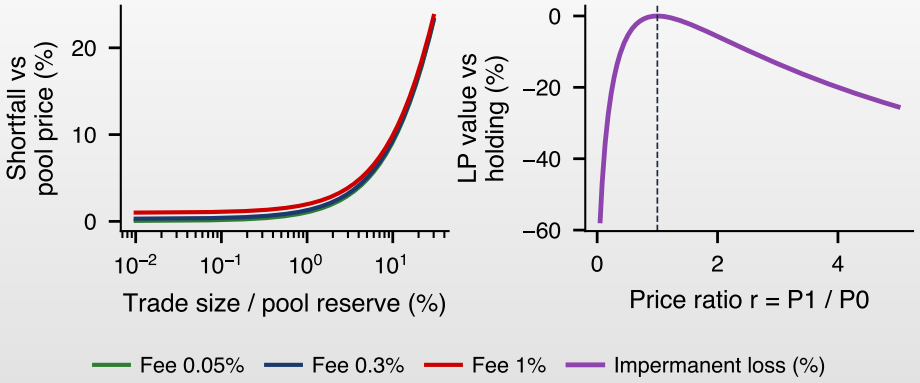
Worked Example: a Swap in an Ether Pool

- Pool: $x = 2\,000$ Ether, $y = 5\,000\,000$ USD Coin, so $p = 2\,500$ and $k = 10\,000 \times 10^6$; fee $f = 0.3\%$
  - Sell $\Delta x = 20$ Ether ($\delta = 1\%$); after the fee, $\gamma \Delta x = 19.94$
- $\Delta y = \dfrac{5\,000\,000 \times 19.94}{2\,000 + 19.94} = 49\,358$ USD Coin, an execution price of $2\,467.9$ per Ether
- Shortfall: $1 - 2\,467.9/2\,500 = 1.28\%$ (0.3% fee plus about 1% price impact)
- New pool price: $(5\,000\,000 - \Delta y)/2\,020 = 2\,450.8$, i.e. -1.97%
  - Why 2 020 and not 2 019.94: the fee of 0.06 Ether stays in the reserve (Uniswap v2); the product of the reserves rises to $2\,020 \times 4\,950\,642 = 10\,000.3 \times 10^6$, slightly above k: the invariant binds for the fee-adjusted input only, and the extra is the liquidity providers' income
  - An arbitrageur now buys Ether in the pool and sells it elsewhere until p is back inside the no-arbitrage band that the fee sets around the market price (Seminar 16, A2)

The Fee Sets a No-Arbitrage Band

- P: the Ether price on other exchanges; $p = y/x$: the pool price
- An arbitrageur can sell Ether to the pool or buy Ether from it
- Selling: one Ether brought to the pool receives, at the margin, γp USD Coin
  - the formula for Δy with $\Delta x \to 0$
  - it pays if $\gamma p > P$, i.e. $p > P/\gamma$
- Buying: one Ether taken from the pool costs, at the margin, p/γ USD Coin
  - it pays if $p/\gamma < P$, i.e. $p < \gamma P$
- No-arbitrage band: $\gamma P \leq p \leq P/\gamma$, of relative width $1/\gamma - \gamma \approx 2f$
  - Inside the band arbitrage does not cover the fee, and the pool price is not corrected
- In the previous example: $p = 2\,450.8 < \gamma P = 0.997 \times 2\,500 = 2\,492.5$
  - so arbitrageurs buy Ether until the effective pool price reaches $\gamma P = 2\,492.5$, not $2\,500$
  - the fee retained in the reserve takes the reserve ratio to $2\,492.56$, just inside the band

Slippage and Impermanent Loss (1/2)



MFM_ch16_amm

Slippage and Impermanent Loss (2/2)

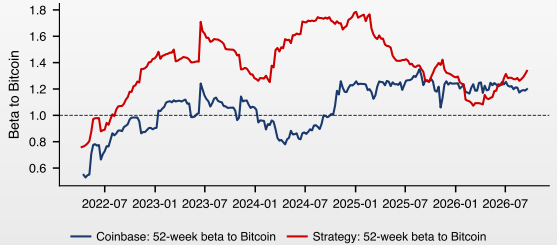
- Left: shortfall by trade size and fee
  - with a 0.3% fee 0.40% for 0.1% of the reserve, 1.28% for 1%, 9.34% for 10%
  - 0.05% fee: 1.04% at 1%
- Small trades pay the fee, large ones the price impact (about δ)
  - liquidity is the depth of the reserve, not an order book (Chapter 10)
  - concentrated liquidity deepens the pool near the price (Uniswap v3)
- Right: value of a liquidity provider (LP) position relative to holding the two tokens
  - against the price ratio $r = P_1/P_0$

MFM_ch16_amm

The Dimson Beta: Leads and Lags

- Dimson (1979): $r_t^{\text{ETF}} = a + \sum_{j=-1}^{1} b_j \, r_{t+j}^{\text{coin}} + u_t$
  - b_j: response to the coin's return on day $t + j$ ($j = -1$: the day before, $+1$: the day after)
  - Sum beta $\hat{\beta}_D = \hat{b}_{-1} + \hat{b}_0 + \hat{b}_1$ collects the whole response
- Which lag should carry the delay?
  - the coin's daily close comes after the 16:00 New York close
  - so the ETF's day-t return already contains the last hours of the coin's day $t - 1$
  - expect $b_{-1} > 0$ and $b_1 \approx 0$
  - IBIT: $\hat{b}_{-1} = 0.05$, $\hat{b}_0 = 0.94$, $\hat{b}_1 = 0.01$, the pattern of a coin close after New York
- IBIT: $\hat{\beta}_D = 1.00$ (SE 0.023) vs OLS 0.94 on the same 671 days
  - ETHA: 0.99 (SE 0.022) vs 0.94: within one SE of 1, as the weekly betas
  - Where prices form is a separate question Hasbrouck (1995)

Beta to Bitcoin Over Time



- Rolling 52-week beta of weekly returns on Bitcoin; Strategy's ranged from 0.76 to 1.79
  - both stocks are more volatile than Bitcoin (55% a year on the same weeks)
- When did the beta change? A break test at an unknown date: Seminar 16, B3, B4

MFM_ch16_equities

Testing a Factor Model: Fama–MacBeth

- Premium of a factor: Fama–MacBeth (FM), two passes
  - Pass 1: the betas $\hat{\beta}_i$ from the time-series regressions
  - Pass 2: each week, regress the N excess returns on the $\hat{\beta}_i$ across assets; the slope $\hat{\lambda}_t$ is that week's reward per unit of beta
  - $\hat{\lambda}$ = mean of the $\hat{\lambda}_t$: the factor premium; $\mathrm{SE}_{FM} = \mathrm{sd}(\hat{\lambda}_t)/\sqrt{T}$
- The FM standard error ignores that $\hat{\beta}_i$ is estimated: Shanken (1992) correction
  - Weak factors (small spread of β across assets) over-reject Kleibergen (2009)
- GRS judges the model, FM the premium
  - a priced factor in an incomplete model can fail GRS
  - formulas and robust versions: Appendix

A Small Replication: Features and Method

- 26 assets in two windows
  - 8 cryptos, 6 equity indices (including BET, the main index of the Bucharest Stock Exchange)
  - 4 exchange rates, 4 commodities, 4 bond ETFs
  - W1: January 2019 – June 2021; W2: 11 January 2024 – 18 September 2026 (after the spot ETFs)
- Eight features of daily log returns, each asset on its own calendar
  - Moments: $\ln \sigma_a$ (log annualised volatility), skewness S, $\ln(1+K)$ (K: excess kurtosis)
  - Memory: $\hat{\rho}_1(r)$, $\hat{\rho}_1(r^2)$, lag-1 autocorrelations of the returns and of the squared returns
  - Tails: Hill indices of losses and gains (5% largest); maximum drawdown
- **Principal component analysis** (PCA) of the standardised features
  - directions of maximum variance
  - Chapter 3
  - Distance between class centroids in the full 8-dimensional standardised space

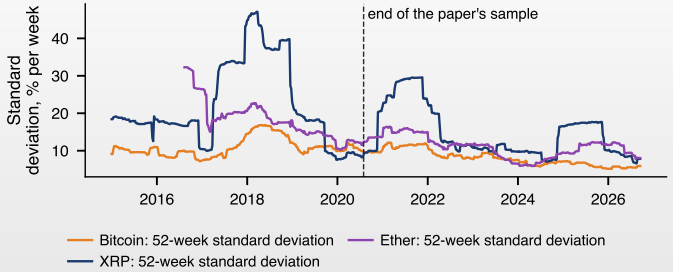
Is the Move Significant? (1/2): a Block Bootstrap of the Map

- In each window, draw the same calendar days for all 26 assets
  - moving blocks of 28 days: four weeks, 20 trading days
  - Each asset keeps its own returns on those days, so cross-asset dependence is preserved
  - Recompute the eight features, the common standardisation and the centroid distances
  - 500 replications
  - Statistic: change $\Delta = d_{W2} - d_{W1}$ in the distance between the crypto centroid and each class centroid

Is the Move Significant? (2/2): the Intervals

- Equities: $\Delta = -0.39$, 95% CI $[-1.04, 0.59]$; commodities: -0.41 $[-1.38, 0.47]$; both contain 0
  - Currencies: -0.89 $[-1.67, 0.18]$; bonds: -0.62 $[-1.55, 0.92]$
  - Spread of the cryptos around their centre: -0.70 $[-1.09, 0.31]$
- Reading: all five indicative intervals contain 0
  - the map shows cryptos apart
  - it does not establish a reliable post-ETF move towards the other classes
- Caveat: $\ln(1+K)$ and $\hat{\rho}_1(r^2)$ have population targets only if $Er^4 < \infty$
  - doubtful here: they enter as sample descriptors
  - so the map is descriptive and these intervals are indicative

Case Study: Volatility Before and After the Paper's Sample



- Standard deviation of weekly returns over the last 52 weeks
  - dashed: end of July 2020, the end of the paper's sample
  - peaks: XRP 47.1% (Mar 2018), Bitcoin 16.9% (May 2018) a week
- Last window: Bitcoin 5.9%, XRP 7.8%, Ether 8.0% a week; volatility moves in regimes
  - factor tests across 2020 need HAC errors and a break test at an unknown date
  - Seminar 16, C3

MFM_ch16_ltw

Replicating Crypto Factors

- **Replicate** with the course data
  - daily closes of the nine non-stable coins (Bitcoin, Ether, XRP, BNB, Cardano, Solana, Dogecoin, Litecoin, Chainlink)
  - one-month Treasury bill rate from the Kenneth French Data Library, the bill of the paper's CMKT
  - Data limit: CSMB and CMOM need breakpoints over hundreds of coins with market values, dead coins included; the nearest published design is the top-20 tercile of Table IA.13, but our nine coins are today's survivors

Tail Index, Inference Conditions (1/3)

- Regularly varying tail: $P(L > x) = x^{-\alpha}\ell(x)$, with ℓ slowly varying
  - slowly varying: $\ell(tx)/\ell(x) \to 1$ as $x \to \infty$, for every $t > 0$
  - the exact power law $Cx^{-\alpha}$ up to a slowly changing factor
  - n: number of losses; $k \to \infty$ and $k/n \to 0$: more tail observations, but a vanishing share of the sample
  - Second-order condition $\sqrt{k}\,A(n/k) \to 0$: $A(\cdot)$ measures how fast the tail approaches an exact power law; the condition makes the bias negligible
- Then $\sqrt{k}(\hat{\alpha}_H - \alpha) \Rightarrow N(0, \alpha^2)$, which gives $\mathrm{SE} = \hat{\alpha}_H/\sqrt{k}$
  - The 5% share is a finite-sample tuning choice, not implied by the theory

Back

Tail Index, Inference Conditions (2/3)

- With volatility clustering the variance depends on extremal dependence Drees (2000)
  - it is typically larger
  - a block bootstrap needs further tail and block-length conditions
  - Choose k from a Hill plot, $\hat{\alpha}_H$ against k, in a region where the curve is flat Drees, de Haan & Resnick (2000)
  - Bitcoin: $\hat{\alpha}_H = 2.73$ ($k = 78$), SE 0.31; with $k = 2.5\%$ or 10%: 3.48 and 2.59: the choice of k matters as much as the sampling error

Back

Tail Index, Inference Conditions (3/3)

- Block bootstrap intervals: up to 4.48 for the S&P 500
  - bootstrap CIs for the sample kurtosis are invalid when $\alpha < 4$
  - bootstrap failure with infinite variance, Athreya (1987)
  - Dogecoin: sample kurtosis 106.4, driven by a few social-media jumps, with $\hat{\alpha}_H = 2.83$: it estimates nothing stable
- Equal tails, H_0: $\alpha_1 = \alpha_2$: the difference $\hat{\alpha}_1 - \hat{\alpha}_2$, with a joint block bootstrap of 28 calendar days
  - both assets take their returns from the same days, each on its own calendar: common shocks are kept
  - Bitcoin vs S&P 500: -0.14, 95% CI $[-1.68, 0.71]$
  - Dogecoin vs S&P 500: -0.04, 95% CI $[-1.77, 0.85]$
  - Bitcoin vs gold: -0.02, 95% CI $[-1.13, 0.81]$
  - with k at 2.5% or 10% of the losses, all three intervals still contain 0
- Overlapping CIs are not a test of equality, and non-rejection is not equivalence
  - the intervals above admit differences above 1 in α: the closeness of the estimates stays a descriptive observation

Back

The sup-Wald Test, Null Limit and Critical Values

- Its limit is $\sup_r \|B(r) - rB(1)\|^2/[r(1-r)]$
  - $r = \tau/T$: the break fraction
  - B: a standard Brownian motion with one component per parameter
  - The sup runs over $r \in [0.15, 0.85]$, the trimming of the main slide; $B(r) - rB(1)$ is a Brownian bridge, zero at $r = 0$ and $r = 1$
  - Critical values simulated from this limit: 11.55 at 5%, 15.18 at 1%, independent of T
- Why the trimming: near $r = 0$ or $r = 1$ one regime has very few days
  - $r(1-r) \to 0$ and the supremum over all of $(0, 1)$ would be infinite
- Why the maximum of W does not date the break
  - $W(\tau)$ uses a separate HAC covariance in each regime
  - so its maximiser (here 14 Aug 2017) need not minimise the sum of squared residuals
  - The break is dated by the least-squares estimate (9 Mar 2020) and its interval

Back

The Published CRIX Rule in Detail

- Total market index TMI (all coins) and candidate indices CRIX(k, β) (Trimborn & Härdle (2018), Sect. 3)
  - the k_1 largest coins at weight 1
  - the next $s = k - k_1$ with free weights β
  - $\hat{\varepsilon}_t(k, \beta)$: the tracking residual of the main slide; $\hat{\beta}$ minimises $\|\hat{\varepsilon}(k, \beta)\|^2$, the sum of squared tracking residuals
- AIC(k) of the main slide
  - L evaluates the residuals under a kernel density estimated from the base residuals $\hat{\varepsilon}(k_1, 1)$
  - Epanechnikov kernel, a parabola $K(u) = \frac{3}{4\sqrt{5}}(1 - u^2/5)$ for $|u| \leq \sqrt{5}$ (variance 1); Sheather–Jones plug-in bandwidth, a data-driven choice of the smoothing width
  - Under Normal errors it reduces to $T \ln \hat{\sigma}_k^2 + 2s$, with T days and $\hat{\sigma}_k^2$ the residual variance (Trimborn & Härdle (2018, author version))
- ECRIX: $k_1 = 1$, steps of one coin, stop at the first rise of the AIC (Trimborn & Härdle (2018, author version), Sect. 5)
  - EFCRIX: global minimum
- The course implementation is a teaching adaptation of eqs. (19)–(20) of Trimborn & Härdle (2018, author version)
  - $\hat{\beta}$ by least squares, then L evaluated at $\hat{\beta}$: not necessarily the maximum of L
  - the search stops at 4 of the 5 coins of the universe
  - if the Sheather–Jones equation has no root, the bandwidth falls back to the Normal reference rule $1.06\, \hat{s}\, n^{-1/5}$

Back

Peg Deviations, Summary Statistics

	s.d. (bp)	$\lvert d_t \rvert > 50$ (%)	lowest close (bp)	lowest low (bp)
Tether	7.3	0.14	−41	−515
USD Coin	7.5	0.14	−285	−1 226
Dai	10.8	0.57	−261	−1 030

- Daily close, 1 January 2021 – 18 September 2026
  - $d_t = 10^4(P_t - 1)$ in basis points
  - lowest low from the daily low price

Back

Tokenisation of Real-World Assets

- **Tokenised yield-bearing instruments**: claims on Treasury bills and cash
  - issued as tokens on a blockchain; the legal form differs
  - Fund shares: BlackRock `BUIDL` 2.38 billion USD (18 Sep 2026, DefiLlama)
  - Circle `USYC` is another tokenised money-market fund share
  - A tokenised secured note issued by a special-purpose vehicle: Ondo `USDY`
- Unlike a payment stablecoin, they pay the Treasury yield to the holder
- Benefits and risks Aramonte, Huang & Schrimpf (2021)
  - benefits: settlement 24/7, use as collateral in DeFi
  - risks: smart-contract bugs, legal ownership, liquidity in stress
- Token economics and adoption: Cong, Li & Wang (2021)

Back

Decentralised Finance in Five Layers

- Schär (2021) describes DeFi as a stack of layers
  - **Settlement**: the blockchain and its native coin (Ether)
  - **Assets**: tokens issued on it (stablecoins, governance tokens)
  - **Protocols**: exchanges, lending, derivatives as smart contracts
  - **Applications** and **aggregators**: user interfaces that route trades across protocols
- Promise: open access, transparency, composability (protocols plug into each other)
- Risks: high leverage, liquidity mismatches, built-in interconnectedness, no shock absorber Aramonte, Huang & Schrimpf (2021)
  - "Decentralisation illusion": governance tends to concentrate power Aramonte, Huang & Schrimpf (2021)

Back

Deriving LVR $= \sigma^2/8$ (1/2)

- Pool value at market price P after arbitrage: $V(P) = Px + y = 2\sqrt{kP}$
  - concave: $V'(P) = \sqrt{k/P} = x$, $V''(P) = -\frac{1}{2}\sqrt{k}\,P^{-3/2}$
  - $V'(P)$ is the pool's Ether holding, $V'' < 0$ its negative gamma
  - Rebalancing portfolio, value R_t: holds the same $x_t = V'(P_t)$ Ether at every instant, so $dR_t = V'(P_t)\,dP_t$
- Price: geometric Brownian motion $dP_t = \mu P_t\,dt + \sigma P_t\,dW_t$
  - Itô's lemma: $dV_t = V'\,dP_t + \frac{1}{2}V''\,\sigma^2 P_t^2\,dt$
  - μ: drift; σ: volatility; W_t: standard Brownian motion; $V_t = V(P_t)$
  - Subtracting: $d(R_t - V_t) = -\frac{1}{2}V''(P_t)\,\sigma^2 P_t^2\,dt = \frac{1}{4}\sqrt{kP_t}\,\sigma^2\,dt = \dfrac{\sigma^2}{8}\,V_t\,dt$

Back

Deriving LVR $= \sigma^2/8$ (2/2): Reading and Assumptions

- LVR is the pool's negative gamma Milionis et al. (2022)
  - a short-volatility position that loses $\sigma^2/8$ of its value per unit of time, whatever the drift
- Assumptions: continuous geometric Brownian motion, frictionless arbitrage, no fees, continuous blocks
  - With fees and discrete blocks, arbitrageurs trade only when the price leaves the fee band, which lowers the loss Milionis, Moallemi & Roughgarden (2025)

Back

The Liquidity-Provider Benchmarks in Detail

- Realised LVR rate $\frac{1}{Y}\sum_t(\Delta R_t - \Delta V_t)/V_{t-1}$ per year, $Y = 2.7$ years; cumulative LVR 17.0 points over 2.7 years
  - ΔR_t, ΔV_t: daily changes in the values of the rebalancing portfolio and of the pool; V_{t-1}: the pool value the day before
- The match with theory is not a test
  - each day's loss is $g_t/2 - (\sqrt{1+g_t} - 1) \approx g_t^2/8$
  - g_t: the daily simple return of Ether
  - so the realised LVR is approximately the realised variance over 8: 5.83% against $\sum_t g_t^2/(8Y) = 5.88\%$
- 2.7 years pin the realised rate down only to about ± 1 point: CI $[4.88, 6.98]\%$
- The constant 50/50 mix (final value 123.3) is a different benchmark
  - same instantaneous rate: its log growth exceeds the pool's by $\sigma^2/8$, here 5.79% a year

Back

Crypto Factors, the Literature

- Value from on-chain adoption and segmentation from traditional markets Cong, Karolyi, Tang & Zhao (2026)
  - mispricing versus risk compensation Babiak & Bianchi (2025)
- Fundamentals: a generalized Metcalfe's law describes the long-run value Wheatley et al. (2019), Pele & Mazurencu-Marinescu-Pele (2019a)
  - network value grows with a power of the number of users
  - bubbles are deviations from it
  - adoption and token value reinforce each other Cong, Li & Wang (2021)
- Two different questions, tested separately on the next slides
  - Model adequacy: are all test-asset alphas zero for the chosen factors? (time series, GRS)
  - Risk premium: is a factor's premium nonzero? (a traded factor: its mean excess return; cross-section: Fama–MacBeth λ)

Back

Time-Series Test, GRS

- Model, H_0: $\alpha_1 = \cdots = \alpha_N = 0$ and the GRS statistic Gibbons, Ross & Shanken (1989): main slide *Testing a Factor Model (2/2)*
  - $\hat{\Sigma}$, $\hat{\Omega}$: residual and factor covariances, both divided by T
- The F distribution is exact only for i.i.d. errors with the Normal distribution
  - conditional on the factors
  - crypto returns have $\hat{\alpha}_H$ near 3 and volatility clustering
  - so these assumptions are doubtful for the residuals
  - Robust alternatives: a GMM (generalized method of moments) Wald test with HAC covariance, or a wild bootstrap under H_0: $r_t^{e*} = \hat{\beta}' f_t + \hat{\varepsilon}_t \eta_t$
  - r_t^{e*}: artificial excess returns with zero alphas; $\hat{\varepsilon}_t$: the residual vector of week t; $\eta_t = \pm 1$ with probability 1/2 (Rademacher), one draw for all assets (keeps the cross-correlation)
  - Valid if the residuals are serially uncorrelated (martingale differences), otherwise a block or dependent wild bootstrap
- $\hat{\Sigma}$ must be invertible and precise: N much smaller than T; weekly crypto data since 2018 give $T \approx 450$

Back

Cross-Section (1/2), Fama–MacBeth and Shanken

- Two passes: $\hat{\beta}_i$ from time series; then each week $r^e_{it} = \gamma_{\mathbf{0}t} + \hat{\beta}_i' \lambda_t + e_{it}$ across the N assets
  - $\gamma_{\mathbf{0}t}$: intercept, the excess return of a zero-beta asset; λ_t: the reward per unit of beta in week t; e_{it}: pricing error
  - $\hat{\lambda}$ and SE_{FM} as on the main slide; SE_{FM} is too small because $\hat{\beta}_i$ is estimated (errors in variables)
- Shanken (1992), for $\hat{\theta} = (\hat{\gamma}_{\mathbf{0}}, \hat{\lambda}')'$ and $X = [1, \hat{\beta}]$: $\widehat{\mathrm{Var}}(\hat{\theta}) = \frac{1}{T}\left[(X'X)^{-\mathbf{1}} X' \hat{\mathbf{\Sigma}} X (X'X)^{-\mathbf{1}} (1 + \hat{\lambda}' \hat{\mathbf{\Sigma}}_f^{-\mathbf{1}} \hat{\lambda}) + \hat{\mathbf{\Sigma}}_f^*\right]$
  - 1: a column of ones; $\hat{\beta}$: the $N \times K$ matrix of betas; $\hat{\mathbf{\Sigma}}$: residual covariance; $\hat{\mathbf{\Sigma}}_f$: factor covariance; $\hat{\mathbf{\Sigma}}_f^* = \begin{pmatrix} 0 & 0 \\ 0 & \hat{\mathbf{\Sigma}}_f \end{pmatrix}$
  - $\widehat{\mathrm{Var}}(\hat{\lambda})$ is its lower $K \times K$ block; the residual part is multiplied by $1 + \hat{\lambda}' \hat{\mathbf{\Sigma}}_f^{-\mathbf{1}} \hat{\lambda} > 1$, the factor part $\hat{\mathbf{\Sigma}}_f^*$ is added once
  - Assumptions: residuals serially uncorrelated and homoskedastic conditional on the factors

Back

Cross-Section (2/2), Weak Factors and Multiple Testing

- Weak or useless factors: λ is weakly identified and FM t-statistics over-reject
  - small spread of β, e.g. attention proxies
  - Use identification-robust tests of risk premia Kleibergen (2009)
- Multiple testing: dozens of crypto "anomalies" have been tried
  - a new factor should clear $|t| > 3$ Harvey, Liu & Zhu (2016), or control the false discovery rate
  - False discovery rate: the expected share of false findings among the rejected hypotheses
  - In the case study, Liu, Tsyvinski & Wu (2022): p-values adjusted under a generalised familywise error rate (k-FWER, probability of k or more false rejections, Lehmann & Romano (2005)), weaker than Holm's FWER (probability of at least one false rejection)

Back

Data-Quality Threats to Identification

- **Wash trading**: fake volume on unregulated exchanges inflates volume- and liquidity-based factors Cong, Li, Tang & Yang (2023)
  - Filter the exchanges and volumes using the rules of the cited paper before building any factor
- **Which price?** Large, recurrent gaps between exchanges, much larger across countries than within them Makarov & Schoar (2020)
- **Survivorship**: today's large coins are the winners Brown, Goetzmann, Ibbotson & Ross (1992), Będowska-Sójka, Wójcik & Pele (2026)
  - dead coins are missing from most samples
- Remedy: fix the investable universe ex ante (exchange list, volume filter)
  - keep delisted coins, report results by sub-universe

Back

The Asset-Class Map, PC1 Loadings

Feature (standardised)	PC1 loading
$\ln \sigma_a$, log annualised volatility	0.44
$\ln(1+K)$, kurtosis	0.52
Hill index of losses	−0.48
Hill index of gains	−0.34
Maximum drawdown (negative number)	−0.42
Skewness S	0.14
$\hat{\rho}_1(r^2)$	0.08
$\hat{\rho}_1(r)$	−0.05

- A loading is the weight of a standardised feature in PC1
  - the eight squared loadings sum to 1
  - the sign is chosen so that PC1 rises with volatility
  - High PC1: high volatility and kurtosis, low Hill indices (heavy tails), a very negative maximum drawdown (deep falls); skewness and the two autocorrelations hardly enter

Back

Common Mistakes

- Reporting sample kurtosis as a population estimate without discussing whether the fourth moment exists
  - if $\alpha < 4$, the population kurtosis does not exist and its bootstrap CI is invalid
  - $\hat{\alpha}_H < 4$ points to this case, it does not prove it: the interval for α decides
- Testing a threshold or a break date with χ^2 critical values: the nuisance parameter is not identified under H_0
- Comparing daily closes recorded at different times and concluding that an ETF tracks badly
- Reading TVL growth as adoption without separating the valuation of the tokens from net deposits
- Treating a stablecoin as risk-free cash: the peg is only as good as redemption and reserves
- Total instead of circulating supply in a market-value index
  - survivorship: studying only today's large coins ignores the many that failed Brown, Goetzmann, Ibbotson & Ross (1992), Będowska-Sójka, Wójcik & Pele (2026)

Back

The Discovery Loop, Reusable Prompts (1/2)

- **Literature**: map what is known, then verify every reference
  - "`List 2023-2026 papers that test for a change in Bitcoin-equity co-movement at the US spot ETF approval: DOI, sample, method, sign of the effect.`"
  - Resolve each DOI on Crossref, open the paper, find the number it is cited for; search with Semantic Scholar, Elicit or Google Scholar
- **Hypotheses**: competing mechanisms, each with an implication the others do not share
  - "`Give three competing mechanisms for the post-2020 Bitcoin-S&P 500 correlation and, for each, one testable implication that separates it from the others.`"
  - E.g. ETF flows predict an effect concentrated in US trading hours; a common monetary factor predicts co-movement with rate surprises already before 2024
- **Data and code**: on the course data, replicating a known number first
  - "`Take the daily closes of Bitcoin and the S&P 500 from the course data, align the prices on common trading days, then compute log returns and their 2020-2023 correlation.`"
  - Replicate first: this chapter reports 0.39 (2020–2023) and 0.38 (since the ETFs); no new result before the code hits both

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The Discovery Loop, Reusable Prompts (2/2)

- **Robustness**, fixed before the post-ETF results are seen
  - "`Draft a pre-registered robustness plan: break-date scan, placebo launch dates, weekly returns, tail dependence, block bootstrap; fix every tuning choice now.`"
  - The plan fixes windows, block length and the family of tests: the number of specifications is known and the FWER can be controlled
- **Critique**: ask the assistant to attack the result
  - "`Act as a referee: give the three strongest reasons this change in correlation could be spurious, and the test that would settle each.`"
  - Expect: non-synchronous closes, correlation that rises mechanically with volatility Forbes & Rigobon (2002), confounding events
- **Report**: effect size with a CI, every specification tried, what failed
  - The prompts that shaped the design go into `AI_USE.md`

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Further Study on Quantinar

Advanced courses on quantinar.com

- Are Cryptos becoming alternative Assets?
- Introduction to Blockchain and Cryptocurrencies
- Comparison of Cryptocurrency Indices
- Cryptocurrency as an Asset Class
- Smart Contracts
- Stabilising Onchain Stablecoin Risks
- Risk Premia in the Bitcoin Market

Code and data

- Quantlets: github.com/danpele/MFM/tree/main/Quantlets/Ch_16

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Acronyms used in this material (1/3)

- **AI**: Artificial Intelligence
- **AIC**: Akaike Information Criterion
- **AR**: AutoRegressive (model); in event studies: Abnormal Return
- **BET**: Bucharest Exchange Trading (index)
- **BNB**: BNB (native token of BNB Chain, formerly Binance Coin)
- **BSC**: BNB Smart Chain
- **BTC**: Bitcoin (ticker)
- **CAPM**: Capital Asset Pricing Model
- **CI**: Confidence Interval
- **CMKT**: Crypto Market factor (value-weighted market excess return)
- **CMOM**: Crypto Momentum factor (past winners minus losers)
- **COVID-19**: COronaVIrus Disease 2019
- **CRIX**: CRypto IndeX
- **CSMB**: Crypto Size factor (small minus big coins)
- **DAI**: Dai (stablecoin of the Maker/Sky protocol)
- **DeFi**: Decentralised Finance
- **DOI**: Digital Object Identifier
- **DTB4WK**: 4-Week Treasury Bill Secondary Market Rate (FRED series code)
- **ECRIX**: Exact CRIX (number of constituents chosen in steps of one)
- **EFCRIX**: Exact Full CRIX (global minimum of the AIC)
- **ES**: Expected Shortfall
- **ETF**: Exchange-Traded Fund

Acronyms used in this material (2/3)

- **ETH**: Ether (ticker of the native coin of Ethereum)
- **ETHA**: iShares Ethereum Trust ETF (ticker)
- **EU**: European Union
- **FAQ**: Frequently Asked Questions
- **FDIC**: Federal Deposit Insurance Corporation
- **FM**: Fama–MacBeth (regressions)
- **FRED**: Federal Reserve Economic Data
- **FWER**: Family-Wise Error Rate
- **FX**: Foreign eXchange
- **GENIUS**: Guiding and Establishing National Innovation for U.S. Stablecoins (Act)
- **GMM**: Generalized Method of Moments
- **GRS**: Gibbons–Ross–Shanken (test)
- **HAC**: Heteroskedasticity and Autocorrelation Consistent (standard errors)
- **HHI**: Herfindahl–Hirschman Index
- **IA**: Internet Appendix (of the cited paper)
- **IBIT**: iShares Bitcoin Trust ETF (ticker)
- **IL**: Impermanent Loss
- **L1**: Layer 1 (a blockchain that settles its own transactions)
- **LP**: Liquidity Provider
- **LUNA**: Luna (native token of the Terra blockchain)
- **LVR**: Loss-Versus-Rebalancing
- **MAR33**: Basel Framework, chapter MAR33 (internal models approach: capital requirements calculation)
- **MDD**: Maximum DrawDown

Acronyms used in this material (3/3)

- **MiCA**: Markets in Crypto-Assets (EU Regulation 2023/1114)
- **NAV**: Net Asset Value
- **OLS**: Ordinary Least Squares
- **PC1**: first Principal Component
- **PC2**: second Principal Component
- **PCA**: Principal Component Analysis
- **PYUSD**: PayPal USD (stablecoin)
- **QQQ**: Invesco QQQ Trust, an ETF on the Nasdaq-100 index (ticker)
- **SE**: Standard Error
- **SPX**: S&P 500 index (ticker)
- **SSR**: Sum of Squared Residuals
- **SVB**: Silicon Valley Bank
- **TAR**: Threshold AutoRegression
- **TE**: Tracking Error
- **TMI**: Total Market Index (all coins, CRIX methodology)
- **TVL**: Total Value Locked
- **US**: United States
- **USD**: US dollar, international currency code
- **USDC**: USD Coin (stablecoin issued by Circle)
- **USDS**: Sky Dollar (stablecoin, successor of DAI)
- **USDT**: Tether (stablecoin issued by Tether)
- **UST**: TerraUSD (algorithmic stablecoin of the Terra blockchain)
- **VaR**: Value-at-Risk
- **XAU**: gold, ISO 4217 code (one troy ounce)
- **XRP**: XRP (the native token of the XRP Ledger)

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