








Modelling Financial Markets

Chapter 17: Bubbles and Crashes



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Can Statistics Spot a Bubble Before It Bursts?

> *"The second theorem of the financial instability hypothesis is that over periods of prolonged prosperity, the economy transits from financial relations that make for a stable system to financial relations that make for an unstable system."*
>
> — *Minsky (1992)*

- Minsky: a long period of stability shifts borrowers towards speculative and Ponzi finance
  - Kindleberger's sequence (after Minsky): displacement, boom, euphoria, distress, panic Kindleberger & Aliber (2005)
- S&P (Standard & Poor's) Composite price-dividend ratio, 1871–2026: GSADF (Generalised Sup Augmented Dickey–Fuller) = 3.41 vs the 95% critical value 2.39
  - the full-sample ADF (Augmented Dickey–Fuller) test (-1.11) sees nothing
- After an industry price run-up above 100%, a 40% crash followed in 45% of cases Greenwood, Shleifer & You (2019)
  - against 11% for a random industry-month; 53 episodes in 27 industries
  - the mean return over the next 2 years is not significantly different from zero: run-ups raise the risk, not the predictable loss
- The credit behind the boom: after the R-zone (rapid growth of credit and asset prices), a household-sector crisis begins within 3 years in 34% of cases
  - against 4% in normal periods; only 25 crises in the data

▸ Central chart: S&P Composite (Shiller) Price-Dividend Ratio, 1871–2026

The Idea of the Chapter

Bubbles, Crashes and Their Detection

- Prices sometimes rise far faster than any plausible change in fundamentals
  - Then they lose most of their value
- Questions
  - can statistics detect such episodes **in real time**?
  - can it say **when** they end?
- Part I
  - What a bubble is, run-ups and crashes in the data
  - Tests for explosive prices and their date-stamping
- Part II: the LPPLS model (Log-Periodic Power Law Singularity)
  - Regime switching, crypto and artificial-intelligence (AI) stocks
  - Two case studies, one pass of the AI-assisted research loop

What you should be able to do

- derive the rational-bubble condition split price into fundamental value and bubble, say what is testable
- compute GSADF and the BSADF (Backward Sup ADF) date-stamping on rolling windows, with valid critical values
- compute an LPPLS fit and interpret the stability of its critical time and its alarms
- interpret booms and busts with a Markov-switching model and the test of the number of regimes
- Further Topics and Appendix: reading for familiarity

- Detailed examples, advanced topics: Further Topics, after the Conclusions
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Historical Bubble Episodes

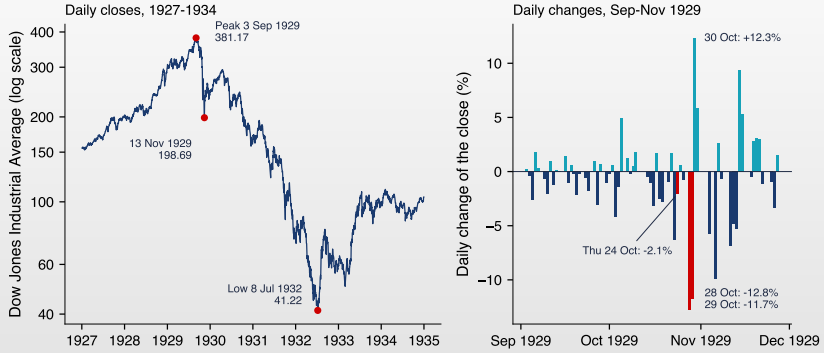


Hendrik Gerritsz Pot, *Flora's Wagon of Fools* (c. 1637), a satire of the tulip trade

Image: Hendrik Gerritsz Pot (c. 1637); public domain; Wikimedia Commons

- **Tulips, 1636–1637** Garber (1990)
  - Contracts on rare bulbs traded at very high prices until the market stopped in February 1637
  - Part of the path fits the fundamentals of a new luxury good
  - Lesson 1: calling something a bubble requires a model of the fundamental value
- **South Sea and Mississippi, 1720** Kindleberger & Aliber (2005)
  - Shares rose several-fold within months, then collapsed
  - Ingredients: a financial innovation, easy credit, a story about future profits
  - Lesson 2: bubbles come with credit expansion Kindleberger & Aliber (2005) and with high turnover Scheinkman & Xiong (2003)
- **1929** Kindleberger & Aliber (2005)
  - A boom partly financed with borrowed money ended in the October crash
  - Kindleberger's sequence (after Minsky): displacement, boom, euphoria, distress, panic
  - Lesson 3: we need tests that work on a single time series, in real time, before the panic

Case Study: Black Thursday, 24 October 1929 (1/2)



Source: S. H. Williamson, Daily Closing Value of the Dow Jones Average, MeasuringWorth (2026)

- Dow Jones Industrial Average, daily closes, log scale on the left; source: Williamson (2026)

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Case Study: Black Thursday, 24 October 1929 (2/2)



The crowd outside the New York Stock Exchange after the crash, October 1929

Photo: US (United States) government (Social Security Administration); public domain; Wikimedia Commons

- **Before**: the peak of 381.17 on 3 Sep 1929, after a rise of 99% in two years
  - By 23 Oct the index had already fallen by 20%, to 305.85; on that day alone −6.3%
- **Thursday, 24 Oct**: 12 894 650 shares traded on the New York Stock Exchange (NYSE), a record Time (1929)
  - Heads of the large New York banks, called together at J.P. Morgan & Co. by Thomas Lamont, organise a support pool
  - Richard Whitney, vice-president of the NYSE, bids 205 dollars a share for U.S. Steel, above the current price
  - Close: 299.47, that is −2.1%, a smaller fall than on 23 Oct
- **Monday 28 and Tuesday 29 Oct**: −12.8% and −11.7%, the largest daily falls of 1921–1955
  - 30 Oct: +12.3%; 13 Nov: 198.69, 48% below the peak
- **Afterwards**: a rebound of 48% to 17 Apr 1930, then the low of 41.22 on 8 Jul 1932 (−89%)
  - The nominal 1929 peak was regained only on 23 Nov 1954
- **The lesson**: the bankers' support stopped one day's panic, not the decline
  - The crash was a sequence of weeks and years, not a single day

The Financial Instability Hypothesis: Minsky

- Minsky classifies borrowers by whether their cash flows can service their debt Minsky (1992)
  - **Hedge** finance: cash flows cover both interest and principal
  - **Speculative** finance: they cover interest, but the principal must be rolled over
  - **Ponzi** finance: they do not even cover interest; debt grows and repayment relies on rising asset prices
- The fragility mechanism
  - A long calm period shifts borrowers towards speculative and Ponzi finance
  - Higher interest rates or falling prices turn speculative borrowers into Ponzi borrowers
  - Forced asset sales push prices down and drive other borrowers towards insolvency
- The link with this chapter
  - The credit that finances the boom: the second case study
  - Accelerating prices: the explosiveness tests

Fundamental Value and the Rational Bubble

- **Question**: which part of the price cannot be justified by dividends, and what can we test about it?
- No-arbitrage pricing: $P_t = \mathbb{E}_t[M_{t+1}(P_{t+1} + D_{t+1})]$
  - P_t: price at the end of period t; D_{t+1}: dividend paid in period $t+1$
  - M_{t+1}: stochastic discount factor; $\mathbb{E}_t$: expectation under the physical measure, given the information at t
- **Simplifying assumption**: $M_{t+1} \equiv 1/(1+r)$, so $P_t = \mathbb{E}_t[P_{t+1} + D_{t+1}]/(1+r)$
  - A constant risk premium does not make M_{t+1} constant
  - In general, the bubble satisfies $B_t = \mathbb{E}_t[M_{t+1} B_{t+1}]$
- Solving forward gives the **fundamental value** $F_t = \sum_{i \geq 1} \mathbb{E}_t[D_{t+i}]/(1+r)^i$
  - The present value of expected dividends
  - Gordon case, dividends growing at $g < r$: $F_t = D_{t+1}/(r-g)$
- But $P_t = F_t + B_t$ also solves the pricing equation, for any B_t with $\mathbb{E}_t[B_{t+1}] = (1+r)\,B_t$
  - B_t is the **rational bubble**: investors pay for it because they expect to resell at a higher price Blanchard & Watson (1982)

Why a Rational Bubble Must Explode

- From $\mathbb{E}_t[B_{t+1}] = (1+r)B_t$: $\mathbb{E}_t[B_{t+h}] = (1+r)^h B_t$ for any horizon of h periods
  - An **explosive conditional mean**
  - Individual paths can crash
  - The bubble grows faster than dividends ($r > g$), so its share of the price rises over time
- Rational bubbles are positive and exist from the first trade Diba & Grossman (1988)
  - A negative bubble would eventually push the expected price below zero
  - A bubble that is zero today must stay zero
- A bubble is the failure of the **transversality condition** $\lim_{h\to\infty} \mathbb{E}_t[P_{t+h}]/(1+r)^h = 0$
  - The price contains a value never paid out as dividends (derivation in the Appendix)
- **Testable implication**
  - If dividends are at most I(1) (stationary after one differencing), then without a bubble the price and the price-dividend ratio are **not explosive**

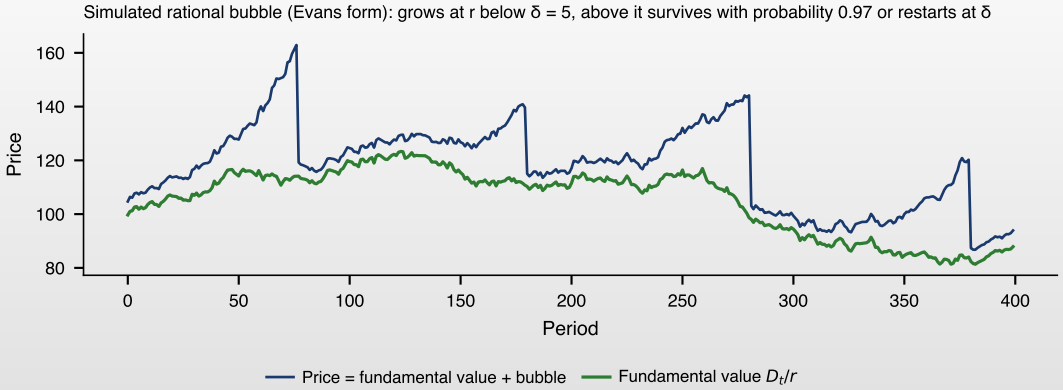
Bubbles That Burst: Blanchard and Watson

- Each period the bubble survives with probability π and collapses with probability $1-\pi$ Blanchard & Watson (1982)
  - $B_{t+1} = \frac{1+r}{\pi} B_t + \varepsilon_{t+1}$ with probability π, $B_{t+1} = \varepsilon_{t+1}$ otherwise
  - $\pi \in (0,1)$: survival probability; ε_{t+1}: a shock with mean zero given the information at t
- Expected growth is still r
  - While it survives the bubble grows faster, at $(1+r)/\pi - 1$, to pay for the crash risk
- Expected lifetime: $1/(1-\pi)$ periods, a geometric waiting time

The Evans Form: Bubbles That Restart

- The problem: after a Blanchard–Watson collapse, $B = \varepsilon$ can be negative, which Diba & Grossman (1988) rule out
- Evans form: the bubble starts and restarts from a value $\delta > 0$ Evans (1991)
  - Survival (probability π): $B_{t+1} = \left[\delta + \frac{1+r}{\pi}\left(B_t - \frac{\delta}{1+r}\right)\right] u_{t+1}$
  - Collapse (probability $1-\pi$): $B_{t+1} = \delta\, u_{t+1}$
  - u_{t+1}: positive noise with mean 1, drawn independently of survival
- The bubble condition still holds: $\mathbb{E}_t[B_{t+1}] = \pi\left[\delta + \frac{1+r}{\pi} B_t - \frac{\delta}{\pi}\right] + (1-\pi)\delta = (1+r)B_t$
- Small-bubble branch: if $B_t \leq \alpha$, then $B_{t+1} = (1+r)B_t u_{t+1}$
  - The two branches above apply only for $B_t > \alpha$
  - Without the threshold, the survival branch turns negative for $B_t < \delta(1-\pi)/(1+r)$
  - With $0 < \delta < (1+r)\alpha$ the bubble stays positive; the simulation uses $\alpha = \delta$

A Simulated Rational Bubble (1/2)



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A Simulated Rational Bubble (2/2)

- Simulation
  - Random-walk dividends D_t
  - Discount rate $r = 1\%$, survival probability $\pi = 0.97$
  - Evans form with $\delta = \alpha = 5$
  - Fundamental value: $\mathbb{E}_t[D_{t+i}] = D_t$, so $F_t = \sum_{i \geq 1} D_t/(1+r)^i = D_t/r$
  - While alive and large, the bubble grows by $(1+r)/\pi - 1 \approx 4.1\%$ per period
- The chart: price $P_t = F_t + B_t$ (blue) and fundamental value F_t (green), over 400 periods
- The bubble grew large and burst 4 times
  - At its largest the price was 1.46 times the fundamental value
- Between episodes the price tracks fundamentals
  - A full-sample test sees mostly "normal" data: the tests of this chapter use **windows**

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Bubbles Beyond Rational Expectations



Robert J. Shiller, Nobel Prize 2013

Photo: Presidential Office Building, Taiwan (2017); CC BY 2.0; Wikimedia Commons

- **Feedback and narratives** Shiller (2015)
  - Rising prices attract new buyers
  - "New era" stories spread by word of mouth
- **Disagreement and short-sale limits** Scheinkman & Xiong (2003)
  - The price includes the option to resell to a more optimistic buyer
- **Riding the bubble** Abreu & Brunnermeier (2003), Brunnermeier & Nagel (2004)
  - Informed traders stay in as long as others do, since no one can burst it alone
- Common signature of all three
  - Prices grow faster than fundamentals for a while: the explosive pattern we will test

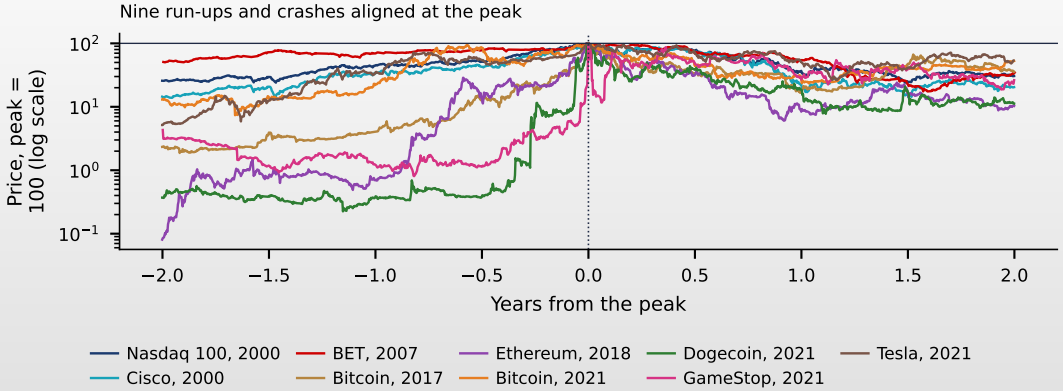
Nine Episodes: Run-Up and Crash

- **Question**: are very fast run-ups followed by crashes, and how often?

Episode	Peak	2-year run-up	Max. fall	Trough	Back at peak
Nasdaq 100, 2000	27 Mar 2000	+288%	−83%	Oct 2002	Nov 2015
Cisco, 2000	27 Mar 2000	+592%	−89%	Oct 2002	Aug 2021
BET, 2007	24 Jul 2007	+97%	−83%	Feb 2009	Mar 2021
Bitcoin, 2017	16 Dec 2017	+4 186%	−83%	Dec 2018	Nov 2020
Ethereum, 2018	13 Jan 2018	+124 004%	−94%	Dec 2018	Feb 2021
Bitcoin, 2021	8 Nov 2021	+667%	−77%	Nov 2022	Mar 2024
Dogecoin, 2021	7 May 2021	+26 897%	−92%	Jun 2022	not yet
GameStop, 2021	27 Jan 2021	+2 218%	−88%	Apr 2024	not yet
Tesla, 2021	4 Nov 2021	+1 837%	−74%	Jan 2023	Dec 2024

- Peak: highest daily close in the episode
- Run-up: simple percentage change of the price over the two years before the peak
  - $P_{\mathbf{peak}} / P_{\mathbf{peak-2y}} - 1$
- Maximum fall: to the lowest close within four years
- BET 2007 (Bucharest Exchange Trading, index): the only episode without a spectacular run-up in the last two years
  - Yet with one of the deepest falls

Run-Ups and Crashes Aligned at the Peak (1/2)



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Run-Ups and Crashes Aligned at the Peak (2/2)

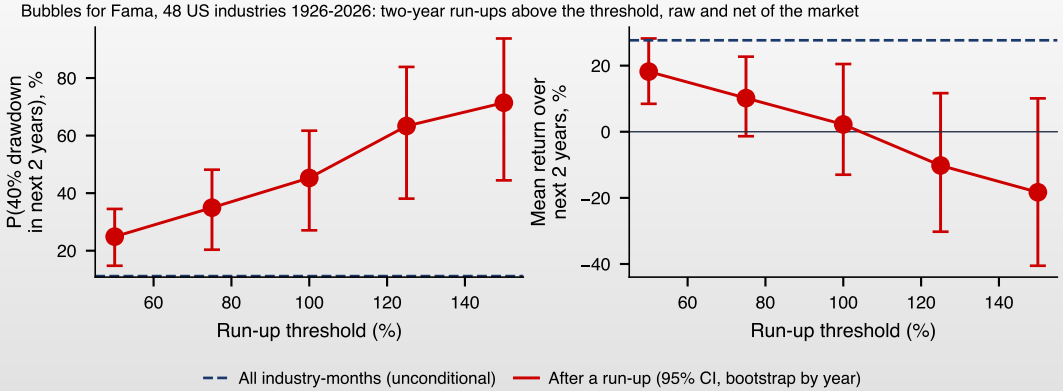
- Daily prices two years before and after each peak, peak = 100, log scale
- Two-year run-ups differ by orders of magnitude, the falls much less (74–94%)
- Common shape: an accelerating rise, a sharp top, a deep fall
  - Convex on the log scale: the growth rate itself rises, the **explosive** behaviour the tests look for
- Selection bias: we picked these episodes because they crashed
  - Many run-ups were not followed by crashes

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Bubbles for Fama: The Design

- Fama's objection: a "bubble" is only a bubble if its end is predictable Fama (2014)
  - Otherwise it may be a rational reaction to news
- Greenwood, Shleifer and You answer with **all** US industries since 1926 Greenwood, Shleifer & You (2019)
  - Not with famous episodes only
  - Do sharp run-ups predict crashes, or low returns?
- **Run-up**: value-weighted industry return over the past 2 years above a threshold (e.g. 100%)
  - Each firm weighted by its market capitalisation
  - Both conditions at once: the raw return **and** the return net of the market exceed the threshold
  - And a raw return of at least 50% over the past 5 years (from 1931), so that recoveries from slumps are not counted
  - First month of the run-up; a new episode in the same industry only two years later
- **Crash**
  - A drawdown of at least 40% from a previous high within the next 2 years
  - Each crash frequency comes with a 95% CI (Confidence Interval)
- The sample: the first 48 of the 49 industries of the Kenneth French Data Library (without "Other")
  - Months with at least ten firms: 41 721 industry-months, 1926–2026

What Do Sharp Run-Ups Predict? (1/2)



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What Do Sharp Run-Ups Predict? (2/2)

- Crash frequency above threshold c: $\hat{p}(c) = n_{\text{crash}}(c)/n(c)$
  - The share of run-ups above c followed by a 40% crash
  - The chart: left, $\hat{p}(c)$ with its 95% CI at the thresholds 50, 75, 100, 125 and 150%; right, the mean 2-year return; dashed line: all industry-months
- After a run-up above 100%, a 40% crash followed in 45% of cases
  - 53 episodes in 27 industries
  - 95% CI [27%, 62%]
  - Against 11% for a random industry-month
  - Above 150%: 71% ($n = 21$)
- CI: bootstrap over calendar years, because run-ups cluster in time
  - E.g. 1928–1929, 1967–1968, 1980
  - Effective sample size: about 31 independent observations (Appendix)
- The mean return over the next 2 years is not significantly different from zero
  - 2% after a 100% run-up, CI [−13%, 20%]
- Run-ups raise the **risk** of a crash, not the predictable loss Greenwood, Shleifer & You (2019)
  - The result does not establish the profitability of a market-timing strategy
- Type of evidence: association (the variables move together; this does not prove causation)
  - a conditional frequency: the run-up precedes the crash, with no mechanism identified

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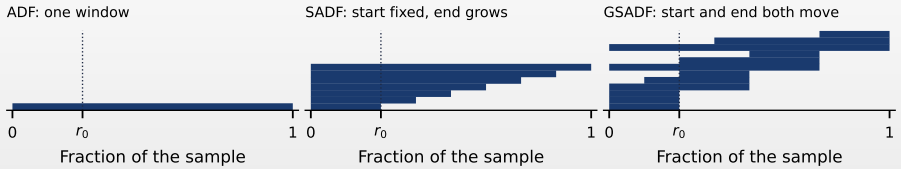
Right-Tailed ADF (1/2): Regression and Hypothesis

- **Question**: how do we show, on a single price series, that there was an explosive phase and when it started?
- Regression on the log price y_t: $\Delta y_t = a + b\, y_{t-\mathbf{1}} + \varepsilon_t$
  - $\Delta y_t = y_t - y_{t-\mathbf{1}} = \ln(P_t/P_{t-\mathbf{1}})$: the log return; ε_t: shock; here no lagged Δy
  - $\mathrm{ADF} = \hat{b}/\mathrm{SE}(\hat{b})$: the Augmented Dickey–Fuller (ADF) t-statistic; $\hat{b}$: the OLS (Ordinary Least Squares) estimate; SE: its Standard Error
  - $b < 0$: mean reversion; $b = 0$: random walk (unit root); $b > 0$: each rise feeds the next, an explosive path
- Bubble test: $H_{\mathbf{0}} : b = 0$ (unit root) vs $H_{\mathbf{1}} : b > 0$ (explosive root), the **right tail** Phillips, Wu & Yu (2011)
  - The left-tail Dickey–Fuller table does not apply

Right-Tailed ADF (2/2): Collapsing Bubbles

- Evans: a bubble that collapses periodically leaves the **full-sample** test with low power Evans (1991)
  - The whole series looks stationary or like a random walk
  - Collapses give the full-sample slope a negative sign (▸ Appendix)
- Solution: run the ADF regression on many **windows** and take the largest statistic
  - SADF (Sup Augmented Dickey–Fuller) Phillips, Wu & Yu (2011) and GSADF (Generalised Sup Augmented Dickey–Fuller) Phillips, Shi & Yu (2015a) differ in the windows they search

Three Ways to Choose Windows (1/2)



- $\mathrm{ADF}_{r_1}^{r_2}$: the ADF statistic on the window from fraction r_1 to fraction r_2 of the sample
  - Each bar in the chart is one ADF regression
  - r_0: the minimum window, as a fraction of the T changes Δy_t; sup: the largest value over the windows allowed
- Minimum window rule: $r_0 = 0.01 + 1.8/\sqrt{T}$ Phillips, Shi & Yu (2015a)
  - E.g. $T = 440$ months: $r_0 = 0.096$, i.e. 42 months
  - Too short a window: an imprecise ADF and more false alarms; too long: short bubbles are missed
  - $w_0 = r_0 T = 0.01T + 1.8\sqrt{T}$: the minimum share r_0 falls towards 1%, but the minimum number of observations grows
  - An episode of fixed length can become too short for the minimum window of a very long sample

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Three Ways to Choose Windows (2/2)

- SADF Phillips, Wu & Yu (2011)
  - $\mathrm{SADF}(r_0) = \sup_{r_2 \in [r_0, 1]} \mathrm{ADF}_0^{r_2}$
  - Windows start at the first observation and end anywhere after r_0
- GSADF Phillips, Shi & Yu (2015a)
  - $\mathrm{GSADF}(r_0) = \sup_{r_2 \in [r_0, 1]} \ \sup_{r_1 \in [0, r_2 - r_0]} \mathrm{ADF}_{r_1}^{r_2}$
  - Windows start and end anywhere, with length at least r_0
- BSADF (Backward Sup Augmented Dickey–Fuller): backward SADF at the end point r_2 Phillips, Shi & Yu (2015a)
  - $\mathrm{BSADF}_{r_2} = \sup_{r_1 \in [0, r_2 - r_0]} \mathrm{ADF}_{r_1}^{r_2}, \ \ \mathrm{GSADF} = \sup_{r_2} \mathrm{BSADF}_{r_2}$
  - BSADF_{r_2} uses only data up to r_2: it can be computed **in real time**
- A larger search needs a higher bar: the critical value is simulated for each T, r_0 and lag order

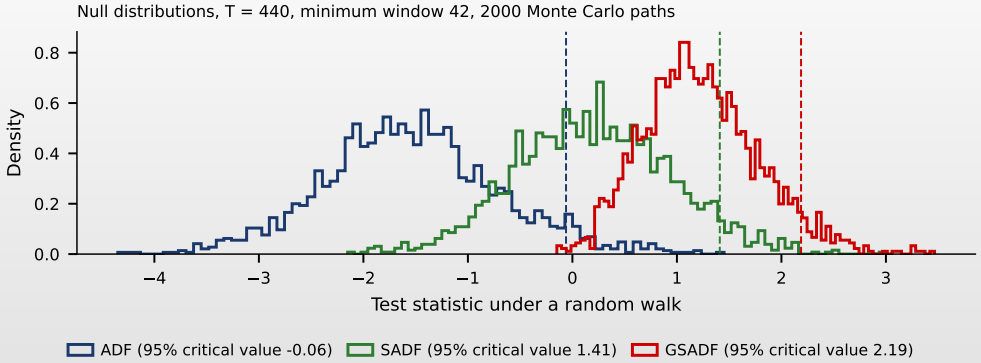
Date-Stamping an Episode (1/2): the PSY Rules

- **Origination**, in the PSY (Phillips, Shi and Yu) procedure Phillips, Shi & Yu (2015a)
  - The first date at which BSADF_{r_2} exceeds its critical value cv_{r_2}
  - cv_{r_2}: the 95% quantile of BSADF_{r_2} under the null, date by date
- **Termination** (PSY, eq. 8) Phillips, Shi & Yu (2015a)
  - The first date after $\hat{r}_e + \delta \ln(T)/T$ at which BSADF falls below cv_{r_2}
  - $\hat{r}_e$: the estimated start, as a fraction of the sample; T: number of changes Δy_t
  - $\delta \ln(T)/T$: a minimum duration, with δ chosen for the data frequency (not the δ of the Evans form)
  - Why $\ln T$: an isolated exceedance often occurs by chance; a duration that grows slowly with T removes short alarms without losing long bubbles
- PWY (Phillips, Wu and Yu) date-stamping uses the recursive $\mathrm{ADF}_0^{r_2}$ instead Phillips, Wu & Yu (2011)
  - It may miss a second bubble after a collapse

Date-Stamping an Episode (2/2): Minimum Duration, Real Time and Chains

- Minimum-duration rule of the R package exuber Vasilopoulos, Pavlidis & Martínez-García (2022)
  - Runs of consecutive exceedances shorter than L periods are dropped
  - $L = \lceil \ln T \rceil$, i.e. $\delta = 1$; $\lceil \cdot \rceil$: rounded up; $T = 440$ months gives $L = 7$
- Start: the first exceedance; end: the last exceedance
  - The PSY rule (eq. 8) would turn a single exceedance into an episode of L periods
- Real time: the episode is **confirmed** only after L consecutive exceedances
  - $L - 1$ periods after its start, which is dated in hindsight
- **Chain**: kept runs separated by a short gap
  - Monthly data: the next run starts at most 93 days after the previous one ends (at most two months without an exceedance)

Null Distributions and Critical Values (1/2)



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Null Distributions and Critical Values (2/2)

- Random walk with a small drift, $y_t = T^{-1} + y_{t-1} + \varepsilon_t$, 2 000 paths of $T = 440$
- 95% critical values Phillips, Shi & Yu (2015a)
  - A single ADF -0.06 (negative)
  - SADF 1.41, GSADF 2.19
- The supremum over about 80 000 windows shifts the GSADF distribution to the right
- Asymptotics (Appendix)
  - The null limit is pivotal: it does not depend on unknown parameters
  - Against mildly explosive roots the tests are consistent: power tends to 1
- Date-stamping keeps a 5% false-alarm chance at every date Phillips, Shi & Yu (2015b)

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Changing Volatility: Wild Bootstrap

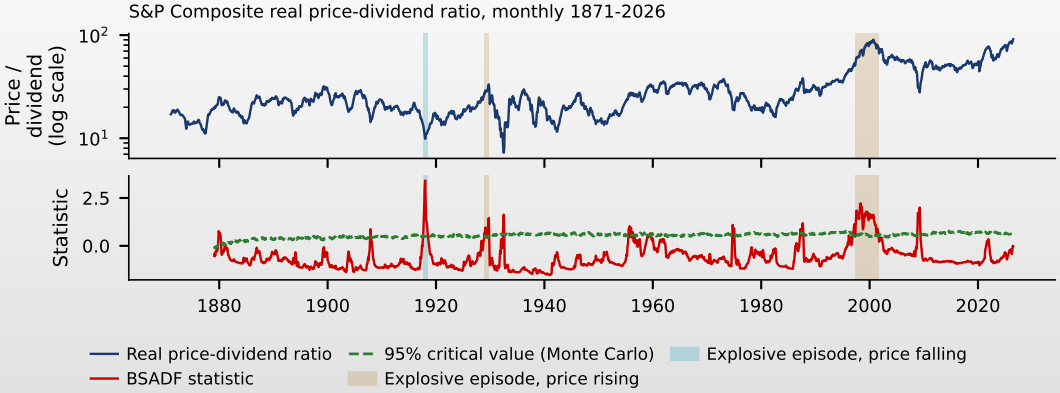
- Monte Carlo critical values simulate $y_t = T^{-1} + y_{t-1} + \varepsilon_t$, $\varepsilon_t \sim N(0,1)$
  - Shocks with constant variance
  - Real prices show volatility clustering and shifts in volatility
  - A burst of volatility can mimic explosiveness and inflate the false rejection rate Harvey et al. (2016)
- **Wild bootstrap**: the demeaned returns with randomly flipped signs Phillips & Shi (2020)
  - $\Delta y_t^* = w_t(\Delta y_t - \overline{\Delta y})$, with $w_t = \pm 1$, each with probability $1/2$
  - $y_t^* = y_0 + \sum_{s \le t} \Delta y_s^*$; then recompute BSADF on y^*
  - The bootstrap data have a unit root by construction (no explosive root) and inherit the time pattern of $\mathrm{Var}(\Delta y_t)$ Harvey et al. (2016)
  - Demeaning removes the drift, as in the null model
- Retrospective and real-time versions
  - Centring with the full-sample mean and resampling the whole path give **retrospective** critical values
  - In real time, recompute them with data up to each date

Serial Dependence: A One-Lag Check

Series	$\hat{\rho}_1$	Ljung–Box p	GSADF, $k = 0$	GSADF, $k = 1$	95% critical value, $k = 1$
Nasdaq 100, monthly	0.05	0.58	2.57	3.08	2.40
Bitcoin, weekly	0.07	0.35	3.85	3.60	2.37
BET, monthly	0.18	0.008	2.90	2.29	2.39

- The chapter's ADF regressions have no lags of Δy; autocorrelated returns can distort their size
  - $\hat{\rho}_1$: first-order autocorrelation of returns
  - Ljung–Box p: the test with 12 lags, not corrected for heteroskedasticity
  - k: number of lags of Δy in the ADF regression
  - The Monte Carlo critical value is recomputed for the one-lag regression (2 000 paths)
- Nasdaq 100 and Bitcoin: the conclusion is the same with one lag
- BET: monthly returns are autocorrelated, and with one lag GSADF falls below its critical value ($p = 0.063$)
- The wild bootstrap keeps the time pattern of the variance, not the autocorrelation: independent signs destroy it
  - Nor does it cover infinite variance

S&P Composite (Shiller) Price-Dividend Ratio, 1871–2026 (1/2)



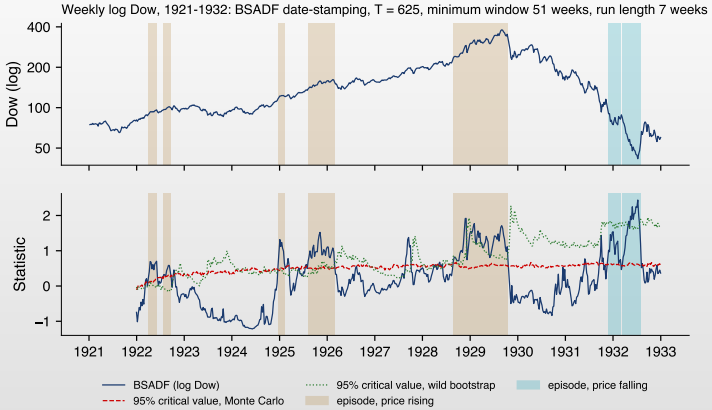
MFM_ch17_sp_pd ▶ Interactive

S&P Composite (Shiller) Price-Dividend Ratio, 1871–2026 (2/2)

- Real price over real dividend, monthly, Jan 1871–Jun 2026
  - $T = 1\,865$, minimum window 96 months
  - In the chart legend, "price rising/falling" refers to the plotted series, here the ratio
- GSADF $= 3.41$ vs 95% critical value 2.39 (99%: 2.88)
  - The full-sample ADF (-1.11) sees nothing
- Episodes: Aug 1917–Jun 1918; Nov 1928–Sep 1929; May 1997–Aug 2001
  - 1917–1918: an accelerating **fall** of the ratio, explosive but not a bubble: always report the direction
- Dot-com: peak 90.2 in Aug 2000, long-run mean 29.8
  - The sample maximum, 91.2 in May 2026, lies in no episode: a high ratio is not necessarily explosive
- A ratio can explode through its denominator: check both price and dividend
- Type of evidence: description (it shows what happened in the data, without testing a cause)
  - the test dates explosive episodes; it does not separate them from explosive fundamentals

MFM_ch17_sp_pd

The Dow Jones, 1921–1932: The Tests in Real Time (1/2)



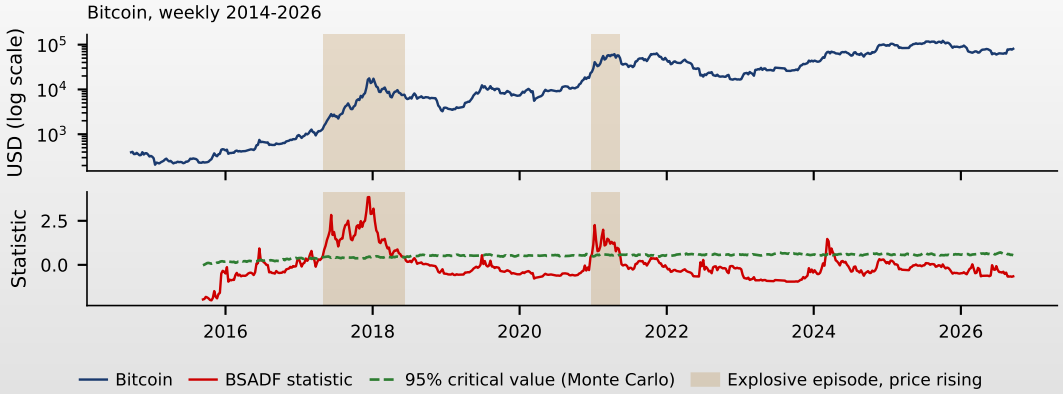
Data source: Williamson (2026)

MFM_ch17_case1929

The Dow Jones, 1921–1932: The Tests in Real Time (2/2)

- Log weekly closes, 1921–1932: $T = 625$, minimum window 51 weeks, minimum run length 7 weeks
- GSADF $= 2.44$: above the 95% Monte Carlo critical value (2.30), below the wild bootstrap one (3.92)
  - The volatility of 1929–1932 changes the conclusion: with heteroskedasticity-robust critical values the overall test does not reject the null
- Main episode: 25 Aug 1928–19 Oct 1929
  - The start is dated in hindsight: the real-time alarm comes 7 weeks later, on 6 Oct 1928, with the index at 240.17
  - The index then rises another 59% to the peak: the alarm comes early and does not date the end
  - The signal ends in the week before Black Thursday: the test measures acceleration, and the rise had stopped in September
- Four other, shorter rising episodes, 1922–1926: falls of at most 19% over the following two years
- 1931–1932: episodes of accelerating decline, hence not bubbles
- The Shiller price-dividend ratio dates a close episode: Nov 1928–Sep 1929

Bitcoin: Explosive Episodes (1/2)



MFM_ch17_bitcoin ▶ Interactive

Bitcoin: Explosive Episodes (2/2)

- Weekly log closes in US dollars (USD) 2014–2026, $T = 626$, minimum window 51 weeks
- GSADF $= 3.85$: rejected with either critical value
  - 95% critical value: Monte Carlo 2.27, wild bootstrap 2.82
- Episodes with Monte Carlo critical values: Apr 2017–Jun 2018; Dec 2020–May 2021
  - With wild bootstrap critical values: May 2017–Jan 2018; Dec 2020–Feb 2021
- The 2017 episode starts 8 months before the peak of 16 Dec 2017 and ends 6 months after it
- The 2024–2025 rise to new highs is **not** flagged: steady growth is not explosive acceleration
- Type of evidence: description

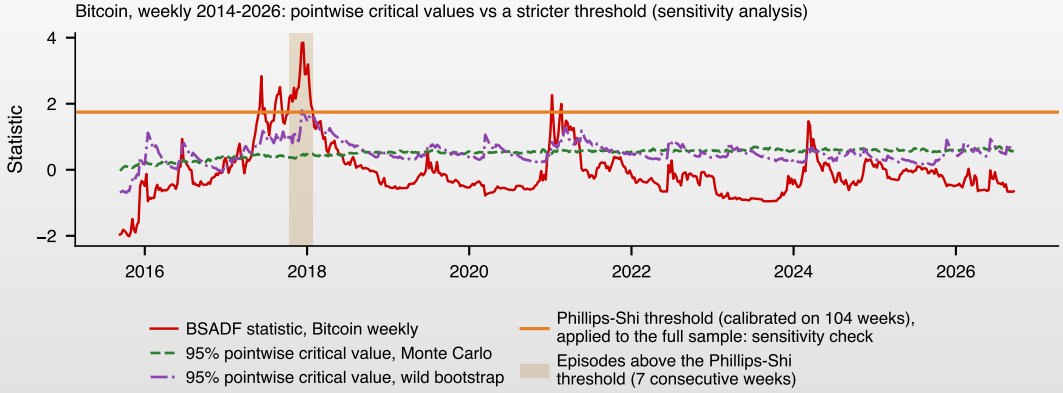
MFM_ch17_bitcoin

Date-Stamping Is a Multiple Test: Bitcoin (1/2)

- BSADF vs its critical value at each of 626 weeks: hundreds of dependent tests
- Family-wise error rate (FWER)
  - The probability of at least one false alarm over the whole monitoring window
  - 2 000 random walks with the Bitcoin T and w_0: a crossing in 95%, an "episode" in 59% of paths
- Phillips and Shi: bootstrap the **maximum** of BSADF Phillips & Shi (2020)
  - Over the last τ_b monitoring dates; here $\tau_b = 104$ weeks
  - Phillips–Shi threshold: 1.75, against Monte Carlo pointwise values of at most 0.71
- Episodes above the Phillips–Shi threshold applied to the whole history: Oct 2017–Jan 2018
  - The 2020–2021 episode found with Monte Carlo pointwise values does not survive
- A sensitivity check, not proved FWER control (design in the Appendix)
  - The threshold is calibrated on 104 weeks; applied to all 626, under the null it is crossed in 19% of paths
  - A confirmed episode (7 consecutive crossings) occurs in 2% of paths

MFM_ch17_inference

Date-Stamping Is a Multiple Test: Bitcoin (2/2)



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Real Time vs Hindsight

Episode	Start / Peak	Confirmed	Lead (months)	Lead from conf.	Rise after conf.	Fall after peak
Nasdaq 100, monthly	Jun 1995 / Mar 2000	Dec 1995	57	51	+716%	−76%
BET, monthly	Nov 2003 / Jul 2007	Apr 2004	44	39	+299%	−83%
Bitcoin 2017, weekly	Apr 2017 / Dec 2017	Jun 2017	8	6	+590%	−83%
Bitcoin 2021, weekly	Dec 2020 / Apr 2021	Feb 2021	4	2	+66%	−75%
GameStop, daily	13 Jan 2021 / 27 Jan 2021	22 Jan 2021	0.5	0.2	+435%	−88%

- Columns (caveats in the ▸ Appendix)
  - Start: first exceedance of the chain of BSADF runs that leads to the peak (hindsight); confirmed: after L consecutive exceedances (real time)
  - Lead: months to the peak, from the start or from confirmation; rise: from confirmation to the peak; fall: to the lowest close within 2 years (GameStop: within 1 year)
- Even at confirmation the tests **warn early**, but they do not time the crash Phillips, Shi & Yu (2015b)
  - Selling at confirmation would have missed most of the rise
- Type of evidence: description
  - five episodes that ended in crashes: the lead is not a hit rate of the alarms

Reading the Real-Time Table

- Largest gap inside a chain
  - 93 days for monthly data
  - 28 days for weekly data
  - 10 days for daily GameStop
- Nasdaq 100: the chain joins 3 runs, from Jun 1995
  - The strict rule keeps only the run containing the peak
  - Strict rule: start Sep 1998, confirmed Mar 1999, leads 18 and 12 months instead of 57 and 51
  - For BET, Bitcoin and GameStop the two rules give the same start
- Two falls for the same dot-com crash
  - This table: -76%, the lowest daily close within 2 years of the peak
  - Nine Episodes table: -83% within 4 years; the trough of Oct 2002 came 30 months after the peak, outside the 2-year window
- Bitcoin 2021: the peak here is the local high of 13 Apr 2021, not the all-time high of 8 Nov 2021
  - The explosive chain ended in May 2021; the later high came after a deep fall and a slower recovery that BSADF did not flag

Crashes as Critical Points



Didier Sornette, ETH (Eidgenössische Technische Hochschule) Zurich

Photo: Didier Sornette (2012); CC BY-SA 3.0 (Germany); Wikimedia Commons

- **Question**: can the shape of the run-up tell when the bubble ends?
- Johansen, Ledoit and Sornette (JLS) Johansen, Ledoit & Sornette (2000)
  - Imitation among traders makes the **hazard rate** of a crash rise as a critical time t_c approaches
  - Hazard rate: the crash probability per unit time, given no crash yet
- Rational investors stay in only if the price grows fast enough
  - The extra growth pays for the rising crash risk
- Result: **faster-than-exponential** growth
  - Decorated by oscillations that speed up near t_c
- t_c is the most probable end of the bubble, not a certain crash date Sornette (2017)

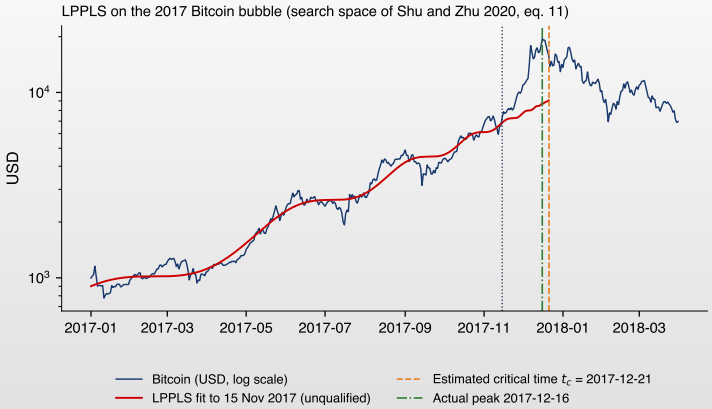
The LPPLS Equation

- $\ln p(t) = A + B\,(t_c - t)^m + C\,(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi), \quad t < t_c$
  - In words: the log price is a power-law trend towards the critical time plus oscillations whose frequency rises as t_c approaches; $p(t)$: price at time t; t_c: critical time
- A: log price at t_c
- $B < 0$: the super-exponential trend for a rising bubble
- $0 < m < 1$: power-law exponent
  - The growth rate of the power-law trend, $-Bm(t_c - t)^{m-1}$, rises without bound as $t \to t_c$
- C, ω, ϕ: amplitude, angular log-frequency and phase of the log-periodic oscillations
- Seven parameters: $A, B, C, \phi, t_c, m, \omega$; after the rewrite with $C_1 = C\cos\phi$, $C_2 = C\sin\phi$ only t_c, m, ω stay nonlinear (Further Topics)
- LPPL: the log-periodic power law pattern
  - LPPLS: the model with the finite-time singularity at t_c

Qualified Fits: Conditions on the Parameters and the Residuals

- A fit on the estimation window $[t_1, t_2]$ is **qualified**
  - If it passes all conditions of Shu & Zhu (2020a, eq. 12)
  - And $B < 0$ for a rising bubble
  - $m \in [0.01, 0.99]$, $\omega \in [2, 25]$, $t_c \in [t_2, t_2 + (t_2 - t_1)/5]$
  - Damping $\frac{m|B|}{\omega|C|} \geq 1$ (already imposed in the search)
  - Number of oscillations (half-periods) $O = \frac{\omega}{\pi} \ln \frac{t_c - t_1}{t_c - t_2} \geq 2.5$, so the oscillations are not noise
  - Largest relative error of the fitted price $\max_t |\hat{p}_t - p_t|/p_t \leq 0.15$
- Lomb periodogram of the detrended residual $(t_c - t)^{-m}(\ln p_t - A - B(t_c - t)^m)$
  - A heuristic filter with nominal level 0.10: the trend is fitted on the same data, so the p-value is not calibrated
  - Lomb periodogram: spectral power for unevenly spaced points, here the values of $\ln(t_c - t)$
- The residual $\ln \hat{p}_t - \ln p_t$ must be stationary
  - Dickey–Fuller and Phillips–Perron reject a unit root at the nominal level 0.10, again a heuristic filter
  - Phillips–Perron: a Dickey–Fuller-type test with a nonparametric correction for serial correlation

LPPLS on Bitcoin, 2017 (1/2)



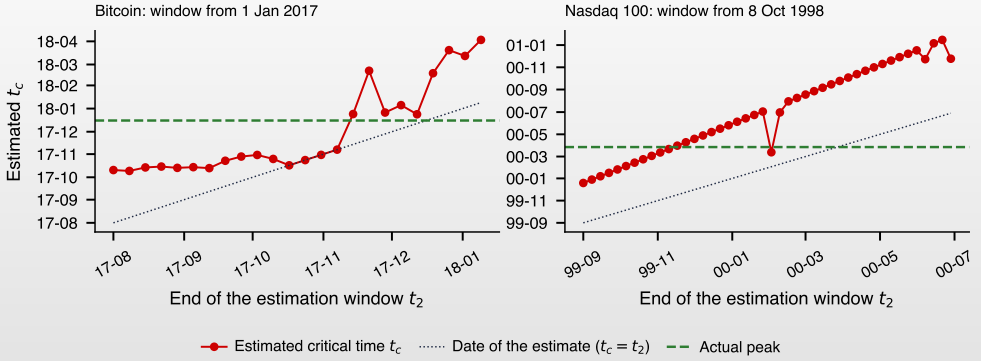
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LPPLS on Bitcoin, 2017 (2/2)

- Fit on daily log prices, 1 January to 15 November 2017
  - The path after 15 November is out of sample
- The fit reproduces the 2017 trend and oscillations, but its largest relative error exceeds 0.15
  - $m = 0.95$, $\omega = 10.54$, $B = -2.62$
  - Damping ratio $\frac{m|B|}{\omega|C|} = 1.00$, at the bound imposed in the search; the exact condition $m|B| \geq |C|\sqrt{m^2 + \omega^2}$ fails (ratio 0.996)
- Estimated t_c: 21 Dec 2017; actual peak: 16 Dec 2017
- The fit does **not** pass the filter, even though it looks convincing

MFM_ch17_lppls

How Stable Is the Critical Time? (1/2)



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How Stable Is the Critical Time? (2/2)

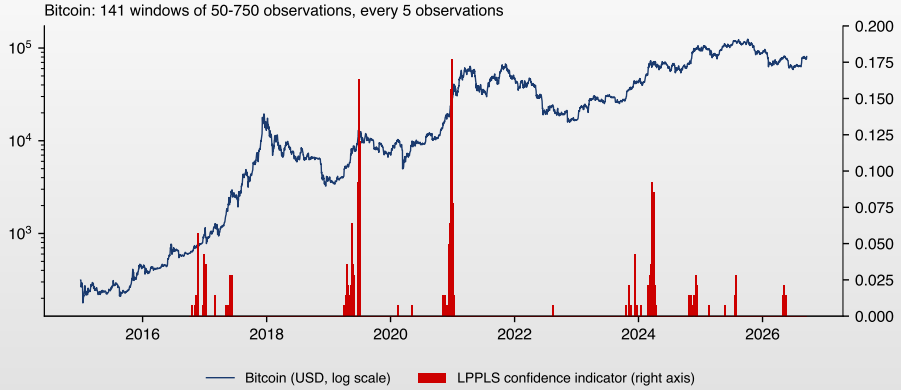
- A good in-sample fit is not a forecast
  - Each point: one fit on a window with a fixed start, ending on the date on the horizontal axis (every 7 days)
- Bitcoin: before the peak (20 fits) t_c ranged from Oct 2017 to Feb 2018
  - Mean absolute error 48 days; within 30 days of the peak: 20% of the fits
- Nasdaq 100: before the peak (30 fits) t_c ranged from Dec 1999 to Sep 2000
  - Mean absolute error 71 days; $t_c \geq t_2$ is a constraint of the search, not a forecast
- Qualified fits before the peak: 0% (Bitcoin), 0% (Nasdaq 100)
  - One window is not enough: the confidence indicator pools many windows

MFM_ch17_lppls

The LPPLS Confidence Indicator: Definition and Evaluation

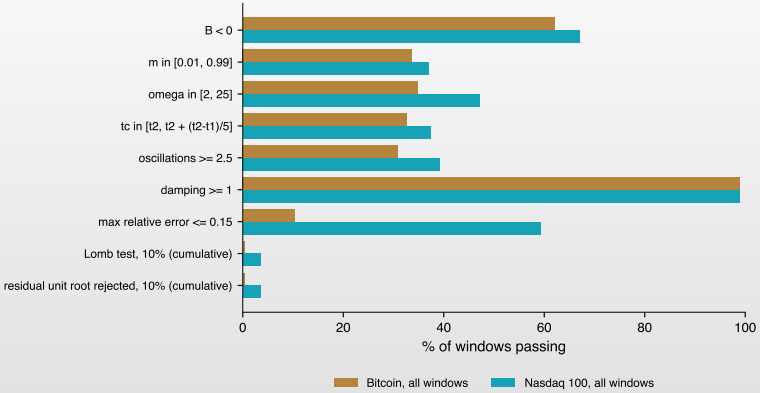
- At each date t_2, fit many windows $[t_2 - \ell, t_2]$ and report the **share of qualified fits** Demirer et al. (2019)
  - $I_{t_2} = \frac{1}{N} \sum_\ell 1\{\text{the fit on } [t_2 - \ell, t_2] \text{ is qualified}\}$, between 0 and 1
  - Grid of Shu and Zhu: $N = 141$ windows, ℓ from 750 down to 50 observations in steps of 5 Shu & Zhu (2020a)
  - t_2 moves every 5 observations (calendar days for Bitcoin); data up to t_2 only
- A high share means the bubble signature is robust to the choice of window
- The birth of a bubble is easier to diagnose than its burst Demos & Sornette (2017)
- Evaluation: is a signal ($I_{t_2} > 0$) followed by a fall of at least 20% within 90 days
  - More often than no signal?
  - Fall measured from the price on the signal date
  - Only dates whose next 90 calendar days are fully observed
  - The 20% threshold and the 90-day horizon are fixed in advance; the AI section checks a grid of 4 horizons $\times$ 3 thresholds

Bitcoin: LPPLS Confidence Indicator



MFM_ch17_lppls_confidence ▶ Interactive

LPPLS: Qualification Filters



MFM_ch17_lppls_confidence ▶ Interactive

The Confidence Indicator and Its Filters: Results

- Price panel: Bitcoin price and the share of the 141 windows with a qualified fit
  - The share is shown as bars on the right axis, every 5 days, 2016–2026
  - Filter panel: share of all windows passing each condition, Bitcoin and, for comparison, the Nasdaq 100
  - Cumulative rows: the Lomb and residual tests count only windows that pass every condition above them
- Positive on 14% of the 2017 dates, 3% in 2021, 12% in other years
  - Maximum 0.18 in Dec 2020
- A 20% fall within 90 days followed 22% of signals, 37% of no-signal dates
  - Bitcoin: 709 dates; Nasdaq 100, 1997–2026: 4% vs 11% (1483 dates)
- The raw difference is not yet evidence (test on the next slide)
  - Signals come in runs and the 90-day outcome windows overlap

MFM_ch17_lppls_confidence

Is the Indicator Better than Chance? A Test (1/2)

- H_0: $P(\text{fall} \geq 20\% \text{ within 90 days} \mid \text{signal}) = P(\cdot \mid \text{no signal})$
  - Statistic $\Delta = \hat{p}_{\text{on}} - \hat{p}_{\text{off}}$: share of signal dates followed by such a fall minus the share of no-signal dates
- Overlap: dates 5 days apart share up to 85 of their 90 outcome days
  - The two-sample z-test assumes independent dates and so understates the standard error
- Dependence-robust inference
  - Circular block bootstrap of the (signal, outcome) pairs: blocks of 90 days, 9 999 replications
  - Linear probability regression of the outcome on the signal, with Newey–West standard errors with 18 lags (90 days at 5-day spacing)

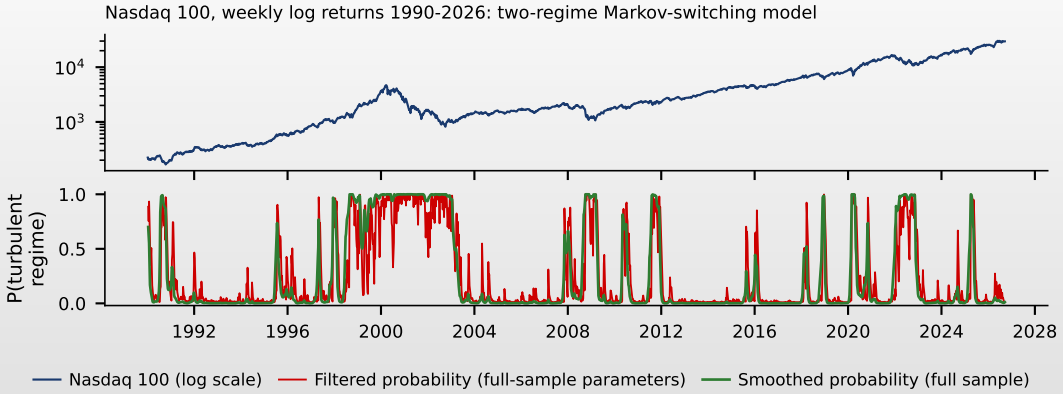
Is the Indicator Better than Chance? A Test (2/2)

- Bitcoin: $\Delta = -15.5$ percentage points, 95% block-bootstrap CI $[-30.0, 1.8]$, which contains zero
  - Block SE 8.1 vs independent-data SE 5.0
  - Blocks of 36 and 72 dates: CI $[-30.8, 2.4]$ and $[-30.2, 1.0]$
- Newey–West regression: two-sided $p = 0.049$ for $H_0 : \Delta = 0$, at the edge of the 5% level
  - The percentile CI and the Newey–West t-test use different standard errors, so they need not agree near the 5% boundary
- Weak evidence, and the point estimate is negative: if anything, a lower crash rate after signals
  - Signals come in steady rises, not just before crashes
  - The negative direction was not stated in advance: the one-sided $p = 0.024$ is only exploratory
- Type of evidence: association
  - between the signal and later falls: weak, and the point estimate is negative

The Markov-Switching Model

- **Question**: can run-ups and crashes be described as alternating risk states, without dating a bubble?
- Weekly log return $r_t = \mu_{s_t} + \sigma_{s_t}\varepsilon_t$, $\varepsilon_t \sim N(0,1)$, regime $s_t \in \{1,2\}$ unobserved Hamilton (1989)
  - $r_t = 100\ln(P_t/P_{t-1})$, in %
  - Each regime has its own mean μ and volatility σ: a calm and a turbulent regime
- Regimes follow a Markov chain: $p_{ij} = P(s_t = j \mid s_{t-1} = i)$
  - Expected duration of regime i: $1/(1-p_{ii})$
- Estimation by maximum likelihood with the Hamilton filter
  - The filter updates $P(s_t = j \mid r_1, \ldots, r_t)$ week by week
- **Filtered** probability: returns up to t for given parameters
- **Smoothed** probability: the whole sample (hindsight)
  - Our parameters are estimated on the full sample, so our filtered probabilities are only **pseudo real time**; genuine real time needs parameters re-estimated with data up to t

Nasdaq 100: Two Regimes (1/2)



MFM_ch17_markov_switching ▶ Interactive

Nasdaq 100: Two Regimes (2/2)

Regime	$\hat{\mu}$ (SE)	$\hat{\sigma}$	$\hat{p}_{ii}$	Dur.
Calm	0.43 (0.06)	2.30	0.987	75.4
Turbulent	−0.32 (0.27)	5.40	0.957	23.5

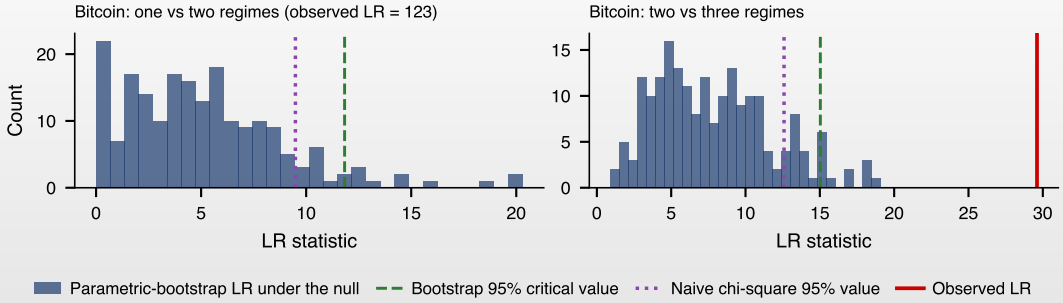
- Weekly log returns in %, 1990–2026, 1 915 weeks
  - Duration in weeks; calm regime in 76% of the weeks
- Turbulent: volatility more than twice as high, negative (insignificant) mean
- The turbulent spell around the 2000 peak began in Jun 1998 (smoothed probability above 0.5)
  - Filtered probability 0.99 at the end of March 2000, in the first falling week after the peak
  - Weekly close: from 4 691.61 (24 March, the peak) to 4 397.84 (31 March), a log return of −6.5%

MFM_ch17_markov_switching

How Many Regimes? A Bootstrap LR Test (1/3)

- H_0: one regime (iid Normal returns: independent and identically distributed) vs H_1: two Markov regimes
  - Statistic $\mathrm{LR} = 2(\hat{\ell}_2 - \hat{\ell}_1)$; $\hat{\ell}_1$, $\hat{\ell}_2$: maximised log-likelihoods with one and two regimes
- Non-standard null distribution (Appendix) Hansen (1992)
  - p_{11}, p_{22} are not identified under H_0, so LR (Likelihood Ratio, test) is not χ^2
- **Parametric bootstrap**
  - Simulate from the fitted H_0 model, refit both models, recompute LR; $B = 199$ replications, so the smallest attainable p is $1/(B+1) = 0.005$
  - $p = (1 + \#\{\mathrm{LR}^* \geq \mathrm{LR}\})/(B+1)$; $\#\{\cdot\}$: number of bootstrap values LR^* at least as large as the observed LR

How Many Regimes? A Bootstrap LR Test (2/3)



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How Many Regimes? A Bootstrap LR Test (3/3)

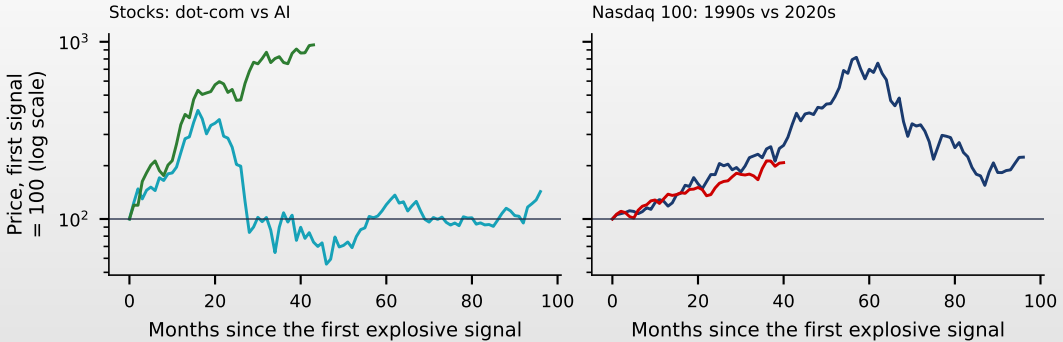
- One vs two regimes: bootstrap $p = 0.005$, the smallest attainable
  - After refitting, 5 (Bitcoin) and 1 (Nasdaq 100) of 199 bootstrap LRs stay negative, of order 10^{-3}: the numerical optimisation of the two-regime model can stop at a local maximum
  - Bitcoin: LR $= 123.2$, 95% critical value 11.8, largest replication 20.3
  - Nasdaq 100: LR $= 475.7$, 95% critical value 11.2, largest replication 18.1
- Two vs three regimes, Bitcoin: LR $= 29.6$, above every replication (maximum 19.1), so again $p = 0.005$
  - Bootstrap 95% value 15.0 vs naive χ^2_6 value 12.6
- One regime is rejected for both series
  - A third Bitcoin regime: an estimated transition probability of order 10^{-9}, so a boundary maximum
  - 154 of 199 replications also end on the boundary: the three-regime p is only indicative
  - Caveat: the one-regime null is Normal, so fat tails alone can favour two regimes
- Type of evidence: description

MFM_ch17_inference

Are Crypto-Assets Bubbles?

- **Question**: without a fundamental value, can we still test the price: is it explosive, and when?
- Crypto-assets pay no dividends: the fundamental value of Bitcoin is hard to define
  - One estimate puts it at zero Cheah & Fry (2015)
- Alternative: value from network adoption (Metcalfe's law: network value grows with the square of the number of users), with LPPLS bubbles on top Wheatley et al. (2019), Pele & Mazurencu-Marinescu-Pele (2019)
- Crypto-assets increasingly behave like an alternative asset class, not a pure bubble Pele et al. (2023)
- Part of the 2017 Bitcoin rise has been tied to the issuance of Tether, a dollar-pegged cryptocurrency Griffin & Shams (2020)

Dot-Com vs AI (1/2)



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Dot-Com vs AI (2/2)

- Monthly closes, rebased to 100 at the first explosive signal of each episode (BSADF, monthly data)
  - First signal: start of the first date-stamped run beginning in 1995 or later (dot-com) or in 2023 or later (AI), not of the run that contains the peak
- Cisco: first signal Nov 1998; NVIDIA (NVIDIA Corporation, a US chip designer): first signal Feb 2023
  - Cisco peaked after 16 months at 4.1 times the signal price
  - NVIDIA, 43 months later: 9.6 times, episode still open
- Nasdaq 100: the AI rise is slower than in the 1990s
  - From Jun 1995: peak after 57 months at 8.2 times; from May 2023: 2.1 times after 40 months

MFM_ch17_crypto_ai

Is There an AI Bubble? What the Tests Can Say

- Monthly BSADF flags NVIDIA and the Nasdaq 100
  - NVIDIA: Mar 2016–Nov 2018; Feb 2020–May 2022; Feb 2023–ongoing; Nasdaq 100: current episode from May 2023
  - Monthly dates ($T = 332$, $w_0 = 36$, $L = 6$ months); the weekly timeline ($T = 1\,443$, $w_0 = 82$, $L = 8$ weeks) shifts the dates by up to a few months, because T, w_0 and L change with the frequency
- The tests cannot separate a bubble from explosive fundamentals Pástor & Veronesi (2009)
  - Fast-growing expected profits of a new technology can also make prices explosive
- What we can say
  - The tests detect explosive price dynamics, not their cause
  - The industry run-up evidence Greenwood, Shleifer & You (2019) motivates watching crash risk, but does not quantify it for these stocks
  - The timing of any crash is unknown
- Type of evidence: description

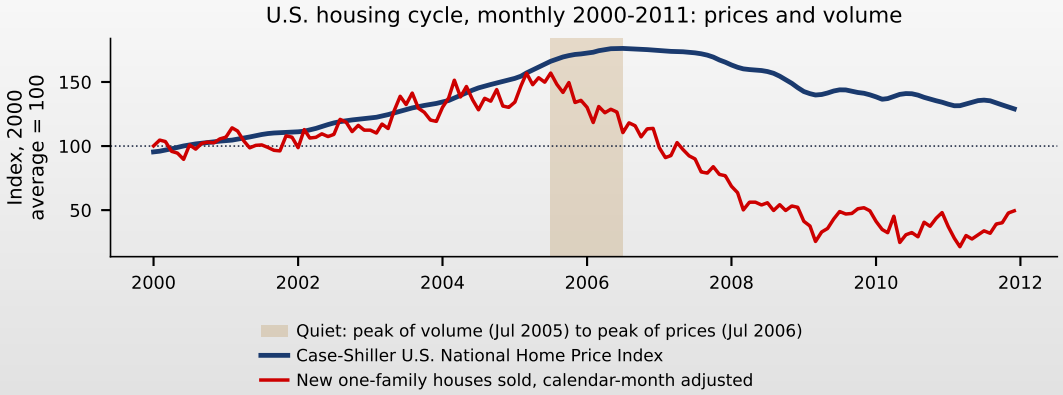
Case Study (1/5): the Research Question

- Paper: DeFusco, Nathanson & Zwick (2022), *Journal of Financial Economics*, 146(1), 205–229
  - Setting: the US housing cycle of 2000–2011, the boom and bust behind the 2008 crisis
- Questions
  - Who buys during a boom?
  - Can speculation explain the joint path of prices, transaction volume and unsold listings?
- Speculation, in the Harrison–Kreps definition the authors adopt
  - Paying more for an asset because one has the right to resell it
- Three phases
  - Boom: prices and volume rise
  - "Quiet": volume falls while prices still rise and unsold listings pile up
  - Bust

Case Study (2/5): Headline Results

- Figures 2 and 4
  - Prices correlate most with volume 24 months earlier: volume leads prices
  - Short-term sales explain 39% of the 2000–2005 rise in sales
- Table 2: cities with more speculation had larger booms and larger busts
  - One standard deviation more speculative volume: boom larger by 24.9 points, bust deeper by 7.4 points
- Model (Sections 5–6, Table 6): extrapolating short-horizon buyers amplify the cycle
  - A tax of 5% of the purchase price, paid at sale, cuts the simulated log-price bust from 8.2 to 1.1 log points
- Type of evidence: association
  - between speculation and the size of the cycle, across cities; the tax effect comes from the model, not the data

Case Study (3/5): Prices and Volume, 2000–2011



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Case Study (4/5): Prices and Volume, 2000–2011

- FRED (Federal Reserve Economic Data)
  - Case-Shiller U.S. National Home Price Index
  - New one-family houses sold
  - 2000 average = 100; volume adjusted by calendar-month means
- Boom: sales peak at 157 in Jul 2005
  - Prices keep rising to 176 in Jul 2006
- Quiet: 12 months from the volume peak to the price peak
  - Sales already down to 110 at the price peak (with seasonally unadjusted sales the lag is 16 months)
- Bust: 2011 averages 35 for sales, 133 for prices
  - New-home sales exclude resales, so they cannot show who trades

MFM_ch17_case_study

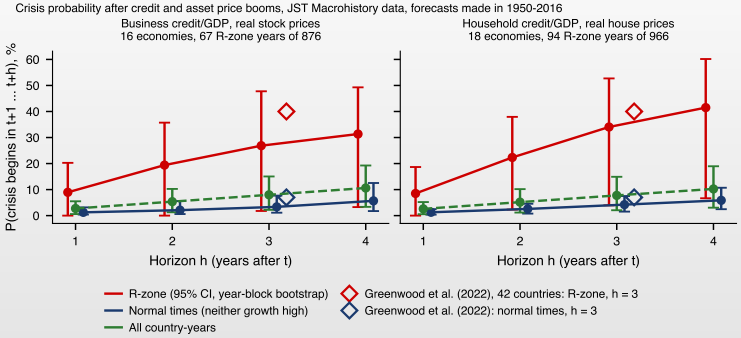
Case Study (5/5): Limits and Open Questions

- Data limit of our illustration
  - The public series are national: volume covers new homes, Case–Shiller the prices of existing homes (repeat sales)
  - The different universes limit the price–volume comparison
  - Short-term-resale and non-occupant volume cannot be built from them
  - Figures 1–2 can be illustrated, Table 2 cannot
- Limits stated by the authors
  - Holding periods are realised, not intended (instrumental-variable check in the paper's Online Appendix)
  - The non-occupant proxy misclassifies owners who give the property address for tax mail
  - Selling horizons are exogenous in the model, so the counterfactuals may overstate short-term buyers
  - Credit and new construction are left out
- Open questions
  - Does trading volume also lead prices in crypto booms?
  - Is there a quiet (falling volume, rising prices) before the end of the Bitcoin episodes dated by BSADF?

Second Case Study (1/4): Are Financial Crises Predictable?

- Paper: Greenwood, Hanson, Shleifer & Sørensen (2022), *Journal of Finance*, 77(2), 863–921
  - Setting: postwar systemic banking crises in 42 countries, 1950–2016
- The question
  - Are financial crises unpredictable "bolts from the sky"?
  - Or do they predictably follow credit booms, as in the Kindleberger–Minsky sequence at the start of the chapter?
- Why it matters for this chapter
  - The explosiveness tests use prices only; the paper adds the credit that finances the boom
  - A signal years before a crisis can inform macroprudential policy that leans against the wind
  - It does not justify it alone: the decision also needs the costs of intervening and of false alarms
- Main claim (the paper's abstract)
  - After 3 years of rapid credit and asset price growth, a crisis begins within the next 3 years with probability of about 40%
  - In normal times, with neither growing rapidly, the probability is about 7%
  - The authors call the combination of the two rapid rises the R-zone (red zone)

Second Case Study (2/4): Crisis Probability After the R-Zone



- **Horizontal axis: horizon h in years; vertical axis: share of country-years t after which a crisis begins in $t+1, \ldots, t+h$**
- **Red: R-zone; blue: normal times; green: all country-years; diamonds: the numbers of the paper's abstract**

MFM_ch17_predictable_crises ▶ Interactive

Second Case Study (3/4): Method and the Course Replication

- The paper's variables, for country i and year t (Eq. 2a–2c)
  - $\Delta_{\mathbf{3}}(D/Y)_{it}$: 3-year change of credit to gross domestic product (GDP), in percentage points
  - $\Delta_{\mathbf{3}} \log P_{it}$: 3-year growth of the real price: stocks for firms, houses for households
  - R-zone: $\Delta_{\mathbf{3}}(D/Y)$ above the 80th and $\Delta_{\mathbf{3}} \log P$ above the 66.7th percentile of the whole panel
  - $\mathrm{Crisis}_{i,t+\mathbf{1}\to t+h} = 1$ if a crisis begins in country i in one of the years $t+1, \ldots, t+h$
- The paper: 42 countries, total credit, the crisis chronology of Baron, Verner & Xiong (2021)
  - Frequencies in 15 cells (price terciles $\times$ credit quintiles); in the R-zone cell, at 3 years: 45.3% for firms, 36.8% for households
  - Regressions with country fixed effects: Driscoll–Kraay errors, Kiefer–Vogelsang corrected p-values
  - Forecast origins stop in 2012, so every horizon has the same observations
- Course replication: the Jordà, Schularick & Taylor (2017) Macrohistory Database (JST), forecast origins 1950–2016, so up to 2016, not 2012
  - Firms: 16 economies; households: 18 economies; bank credit instead of total credit
  - JST chronology: 25 crises
  - R-zone thresholds, firms: credit +4.9 percentage points, real price +24.3 log points (about +27.5%)
  - Households: credit +6.3 percentage points, real price +12.7 log points (about +13.6%)
  - 95% confidence interval (CI): bootstrap over blocks of 5 calendar years, all countries of a year kept together
  - The thresholds are re-estimated in every replication; the CIs are pointwise, not simultaneous across sectors and horizons

Second Case Study (4/4): Result and Limits

- Households: after the R-zone, a crisis begins within 3 years in 34% of cases, CI [4%, 53%]
  - Normal times: 4%; the CI of the difference, [1.7, 46.2] percentage points, excludes zero
- Firms: 27%, CI [2%, 48%], against 3% in normal times
  - The CI of the difference, [0.1, 42.7] percentage points, barely excludes zero
- The significance depends on the block length
  - 3-year blocks: CIs of the difference [4.1, 46.0] (households) and [1.8, 40.8] (firms)
  - 8-year blocks: $[-0.7, 44.3]$ and $[-0.7, 41.8]$, both contain zero
- Crises are slow to arrive: after a household R-zone, 9% within one year and 41% within four
  - An R-zone occurs in the 3 years before 16 of the 25 crises (either sector)
- Type of evidence: association
  - conditional frequencies, with no identified mechanism; the direction is the paper's, at a lower level
- Limits
  - Only 25 crises, many in 2007–2008: wide CIs, and a single global wave weighs heavily
  - The thresholds come from the whole sample, so they use future information; the paper also checks real-time thresholds

Can Bubble Alarms Warn of Crashes?

Real-time bubble alarms

- Questions
  - Can bubble alarms be raised in real time with a controlled false-alarm rate?
  - Do they carry crash information?
- BSADF date-stamping controls the size date by date only, not over the whole sample
  - Live research: monitoring rules for bubbles *and* crashes Whitehouse, Harvey & Leybourne (2023); "econometric thermometers" for housing Shi & Phillips (2023)
- Formal version: alarm $A_t \in \{0,1\}$, crash $C_{t,h} \in \{0,1\}$ (not the LPPLS amplitude C) = a fall of at least 20% within $h = 90$ days
  - H_0: $P(C_{t,h} \mid A_t = 1) = P(C_{t,h} \mid A_t = 0)$; H_1: larger after an alarm
  - The criterion, fixed in advance: reject H_0 if the test robust to overlapping windows rejects it at 5%; otherwise, the data do not support the hypothesis
  - The absence of an effect is supported only by an equivalence test with a margin fixed in advance
- Identification Pástor & Veronesi (2009)
  - Explosive expected cash flows of a new technology leave the same signature as a bubble

The Discovery Loop and Its Checks

- **Literature**: ask for papers on the false-alarm rate of sequential bubble monitoring
  - Resolve every DOI (Digital Object Identifier) on Crossref; find the table the answer quotes
- **Hypotheses**: rational bubble, explosive expected cash flows, falling discount rate
  - Each with its own observable implication
  - E.g. an explosive price-to-earnings ratio vs explosive earnings
- **Code**: replicate a number from the chapter first, then extend
- **Robustness**: pre-register the grid of horizons and crash thresholds, with a Holm correction
  - Holm: the k-th smallest of m p-values is compared with $0.05/(m-k+1)$, stopping at the first non-rejection
- **Critique**: ask the assistant to attack the result as a referee
- **Checks** the researcher must run (▸ Appendix)
  - Full-sample information, the tail of the test
  - Changing volatility, multiplicity, overlapping outcomes

Mini-Case: Does the LPPLS Alarm Predict Crashes?

- A typical assistant proposal
  - "A positive LPPLS confidence indicator announces a crash; compare hit rates with Fisher's exact test"
  - Alarms come in runs and outcomes overlap, so we test by circular rotations of the alarm series (shifted against the outcomes, keeping its clustering)
- One-sided p for $H_1 : \Delta > 0$ (higher hit rate after an alarm)
  - Bitcoin: $\Delta = -15.5$ percentage points, $p = 0.96$ (658 rotations); Nasdaq 100: $\Delta = -7.7$ points, $p = 0.98$
- Pre-registered grid of 4 horizons $\times$ 3 thresholds
  - Smallest raw p 0.76 (Bitcoin) and 0.69 (Nasdaq 100)
  - No cell rejects H_0 at 5%, before or after the Holm correction
- Interpretation (assumptions in the Appendix)
  - The indicator dates accelerating rises, but these tests do not support a higher probability of large falls after a signal
- Type of evidence: association
  - tested between alarm and crash; no test of the grid finds it

Your Turn: a Project Seed

- **Questions**
  - Can a monitoring rule for Bitcoin, the Nasdaq 100 and NVIDIA keep the probability of any false alarm at 5% over the whole period?
  - Can it still warn before large falls?
- **Replicate first**
  - GSADF $= 3.85$ for weekly Bitcoin
  - The crash probability after run-ups above 100% in Greenwood, Shleifer & You (2019) (ours: 45%)
- **Extend**
  - Calibrate the alarm threshold by simulation over the whole monitoring period Whitehouse, Harvey & Leybourne (2023)
  - Evaluate the alarms with rotations or a block bootstrap

Key Takeaways

- A bubble is price minus fundamental value
  - Rational bubbles must be explosive; bursting bubbles hide from full-sample tests
- GSADF tests for explosive episodes
  - BSADF date-stamps them in real time
  - With Monte Carlo critical values a late volatility rise distorts the size (false rejection rate); the wild bootstrap restores it
  - Date-stamping over many dates needs family-wise error rate (FWER) control
- Across our episodes the tests warned months before the peak, but did not time the crash
- LPPLS describes the acceleration
  - Its critical time moves with the window
  - The confidence indicator, judged by a dependence-robust test, did not anticipate crashes in our data
- Sharp run-ups raise crash risk without a statistically established average loss
  - A risk-management signal, not a trading rule
- Credit matters: after the R-zone, a household crisis begins within 3 years in 34% of cases, against 4% in normal periods

Which Tool When?

Question	Tool
Has there been any explosive episode?	GSADF with simulated critical values
When did it start and end?	BSADF date-stamping, minimum duration $\lceil \ln T \rceil$; family-wise critical values for long monitoring
Does volatility change a lot (crypto, crises)?	Wild bootstrap critical values
Is the rise accelerating super-exponentially?	LPPLS confidence indicator over many windows
Has the risk state changed?	Markov-switching filtered probabilities
How likely is a crash after a run-up?	Run-up conditioning across many assets Greenwood, Shleifer & You (2019)

Check Yourself, and Next

Retrieval questions

- Why does a full-sample right-tailed ADF miss a bubble that collapses periodically?
- Why is the 95% critical value of GSADF far above that of a single ADF?
- Why is a single LPPLS critical time not a crash forecast?
- Why must hit rates after alarms be tested with a block bootstrap or rotations?

Chapter 18: systemic risk

- When a bubble bursts, losses spread through banks and markets: contagion and systemic risk
- How much one institution adds to the risk of the system
  - CoVaR: the value-at-risk (VaR) of the system when an institution is in distress
  - SRISK: the capital shortfall of an institution in a crisis
- Further reading: Franke, Härdle & Hafner (2019), Kindleberger & Aliber (2005), Shiller (2015), Sornette (2017)

The Practical Decision

- **What I choose**
  - for the existence of an episode: GSADF with wild-bootstrap critical values (Bitcoin: 3.85 against 2.82)
  - for dating: BSADF with the minimum duration $\lceil \ln T \rceil$ and a family-wise critical value for long monitoring
  - for crash risk: the crash frequency after sharp run-ups (45% after +100%, CI [27%, 62%]), as a risk-management signal
  - the LPPLS alarm: the rule evaluated gives no robust evidence that it raises the probability of the crash defined here ($\Delta = -15.5$ percentage points for Bitcoin)
- **Under which conditions**
  - I read the test as dating explosive dynamics, not as proof of a bubble: explosive fundamentals leave the same footprint
  - an episode is confirmed only after L consecutive exceedances, so the real-time lead is shorter
- **What result would make me drop it**
  - an episode that fails the family-wise critical value, as Bitcoin 2020–2021 does
  - on a new sample, a CI of the crash frequency after run-ups above 100% that contains the 11% rate of a random industry-month

The Series Tested: Stocks, Industries, Crypto

Series	Frequency	Period	Use
S&P Composite, real price and dividend	monthly	1871–2026	price-dividend ratio
48 US industry portfolios	monthly	1926–2026	run-ups and crashes
Nasdaq 100, S&P 500	daily, monthly	1990–2026	dot-com, AI
BET, BET-FI	daily, monthly	1997–2026	Bucharest Stock Exchange indices
Bitcoin, Ethereum, Dogecoin	daily, weekly	2014–2026	crypto booms
Dow Jones Industrial Average	daily, weekly	1921–1955	the 1929 crash
NVIDIA, Cisco, Tesla, GameStop, Strategy (formerly MicroStrategy)	daily, monthly	1990–2026	single stocks

- Closing values up to 18 September 2026
  - Stocks adjusted for dividends and splits
  - Weekly and monthly series use the last price of the period
- Sources: shillerdata.com for the S&P Composite; Kenneth French Data Library for industries

The Dot-Com Bubble



The Pets.com sock puppet, mascot of an online pet-supply retailer that listed and closed in 2000

Photo: Atomic Taco (2013); CC BY-SA 2.0; Wikimedia Commons

- Hedge funds rode the bubble and cut positions before the fall Brunnermeier & Nagel (2004)
- Yet the internet did change the economy: a technology revolution can create both value and a bubble Pástor & Veronesi (2009)

Levels Explode, Log Prices Accelerate

- Theory: the bubble is explosive in **levels**, $\mathbb{E}_t[B_{t+\mathbf{1}}] = (1+r)B_t$
  - The test regresses the **log** price
  - A pure bubble path $B_t = B_{\mathbf{0}}(1+r)^t$ has a constant log growth $\ln(1+r)$: Δy_t does not depend on $y_{t-\mathbf{1}}$, so $b = 0$
  - What $b > 0$ detects: a **rising** growth rate of $\ln P_t$, which occurs while the bubble share of $P_t = F_t + B_t$ grows
    ▸ Appendix
- Toy example ($1 + r = 2$): $F = 100$ constant, $B = 1, 2, 4, 8$, so $P = 101, 102, 104, 108$
  - Bubble share of the price: 1.0%, 2.0%, 3.8%, 7.4%
  - Log increments: $\ln(102/101) = 0.0099$, $\ln(104/102) = 0.0194$, $\ln(108/104) = 0.0377$
  - They rise with $y_{t-\mathbf{1}} = \ln P_{t-\mathbf{1}}$, so the fitted $b > 0$; the bubble alone, $B_t = 2^t$, has every increment equal to $\ln 2 = 0.693$, so $b = 0$
- Check: add $B = 16$, so $P = 116$
  - Is the next increment larger?
  - Yes, $\ln(116/108) = 0.0715$; once $B \gg F$ the increments approach $\ln 2$ and b falls back towards 0: windows that catch the **transition** have the power

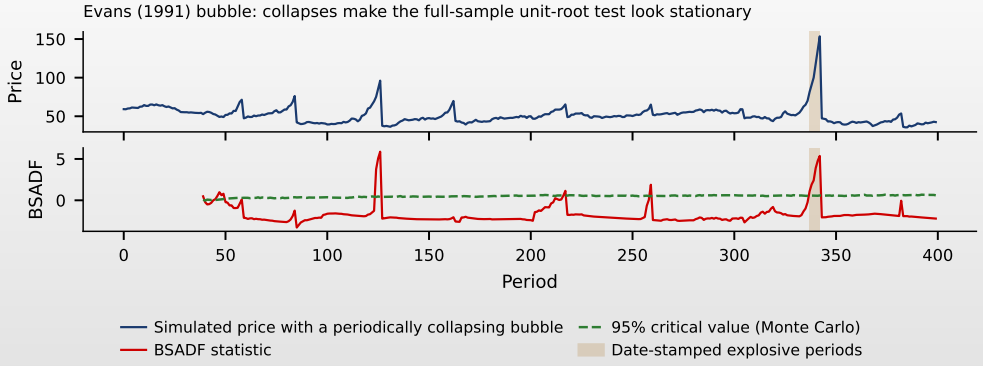
A Hand-Worked Date-Stamping Example

- Invented numbers; $L = 3$; critical value $cv_t = 1.0$ at every date
  - Flag $= 1$ if $\mathrm{BSADF}_t > cv_t$
  - s: start, c: confirmed, e: end, d: dropped

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
BSADF_t	0.4	1.2	1.5	1.8	1.3	0.8	1.1	1.4	1.9	1.6	0.7	0.5	1.3	1.2	0.6
Flag	0	1	1	1	1	0	1	1	1	1	0	0	1	1	0
Event		s		c	e		s		c	e			d	d	

- Runs $t = 2$–5 and $t = 7$–10: four exceedances each, at least L, so both are kept
  - Confirmed at $t = 2 + L - 1 = 4$ and $t = 7 + L - 1 = 9$; each ends at its last exceedance, $t = 5$ and $t = 10$
  - Run $t = 13$–14: only two exceedances, fewer than L: dropped, never confirmed
- A toy chain links 2–5 and 7–10 (one quiet date, $t = 6$)
  - The chain allows at most one quiet date between runs
  - Episode containing $t = 10$: strict rule start $t = 7$; chain start $t = 2$, so the measured lead grows by five periods
- Check: flags 1 1 0 1 1 1 1 with $L = 3$
  - When is the first episode confirmed?
  - Answer: at $t = 6$; the run at $t = 1$–2 is too short

Evans Bubble: Full Sample vs Windows (1/2)



MFM_ch17_evans

Evans Bubble: Full Sample vs Windows (2/2)

- 400 simulated periods
  - BSADF against its 95% critical value from 2 000 random-walk paths
- Full-sample right-tailed ADF: -5.49 vs 95% critical value -0.09
  - No rejection, the bubble is invisible
- SADF 0.98 (critical value 1.46): no rejection
- GSADF 5.88 (critical value 2.21): the null is rejected
- BSADF crosses its critical value in several bursts
  - Only one lasts long enough to be date-stamped

MFM_ch17_evans

Size of GSADF When Volatility Changes: Design

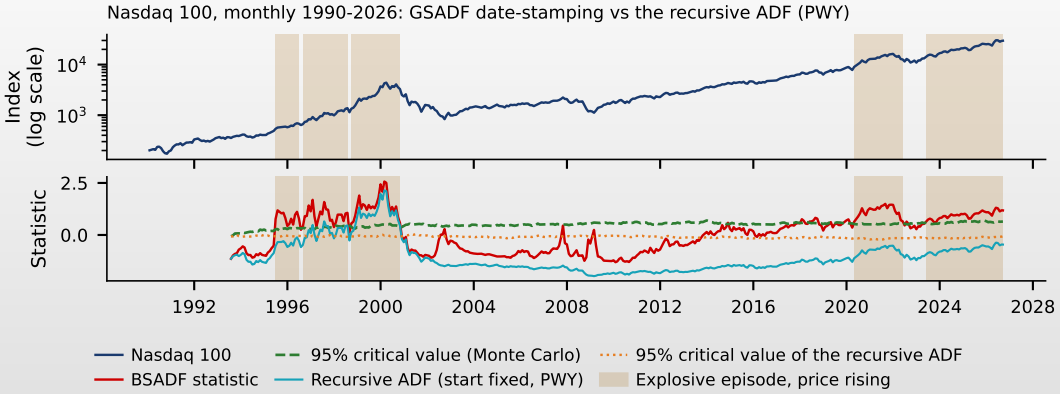
- Null model without a bubble: $y_t = T^{-1} + y_{t-1} + e_t$, a random walk with a small drift
- Four variance patterns of the mean-zero shocks e_t
  - Constant; standard deviation multiplied by 3 at $T/2$, so the variance by 9; standard deviation divided by 3 at $T/2$
  - GARCH (Generalised AutoRegressive Conditional Heteroskedasticity) of order (1,1): $\sigma_t^2 = \omega + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2$, with $\alpha = 0.10$, $\beta = 0.88$
- Sample: $T = 400$, minimum window $w_0 = \lfloor r_0 T \rfloor = 40$ observations
  - 1 000 paths per scenario, each with its own wild bootstrap (199 replications)
  - Monte Carlo error: ± 1.4 percentage points near 5%
- Two outcomes per path
  - Size: GSADF above its 95% critical value (Monte Carlo, MC in the table, or wild bootstrap)
  - False episode: BSADF above its pointwise 95% value for 6 consecutive periods

Size of GSADF When Volatility Changes: Results

Null model, no bubble	GSADF, MC crit. value	GSADF, wild crit. value	False episode, MC	False episode, wild
Constant volatility	4.1%	3.9%	49.1%	51.9%
Volatility $\times 3$ at $T/2$	32.0%	4.1%	63.8%	49.6%
Volatility $\times 1/3$ at $T/2$	4.3%	6.1%	48.0%	54.8%
GARCH(1,1), $\alpha = 0.10$, $\beta = 0.88$	16.6%	6.8%	58.1%	52.8%

- A late rise in volatility is mistaken for explosiveness Harvey et al. (2016)
  - Under GARCH the wild bootstrap removes most, not all, of the distortion; a fall in volatility is harmless
- Even with constant volatility, 49.1% of the paths contain a false date-stamped episode
  - The size is controlled date by date, not over the whole sample

Nasdaq 100: GSADF vs PWY Date-Stamping (1/2)



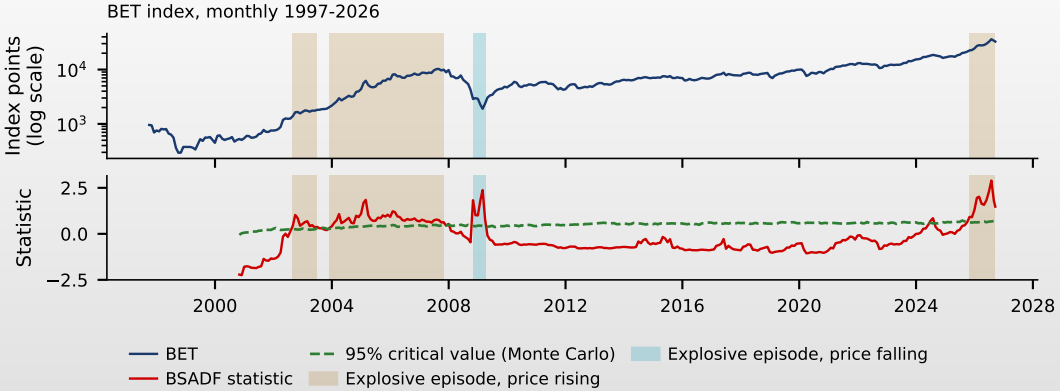
MFM_ch17_nasdaq

Nasdaq 100: GSADF vs PWY Date-Stamping (2/2)

- Monthly log closes 1990–2026, $T = 440$, minimum window 42 months
- GSADF $= 2.57$ (95% critical value 2.19); SADF $= 2.14$ (1.41)
- BSADF episodes
  - Jun 1995–Jun 1996; Aug 1996–Jul 1998; Sep 1998–Oct 2000
  - Apr 2020–May 2022; May 2023–ongoing
  - PWY (fixed start) episodes: Apr 1997–Nov 1997; Jan 1998–Jul 1998; Sep 1998–Oct 2000
  - The recursive ADF misses the post-2020 episodes: it starts in 1990 and is dragged down by the long flat period after 2000
- Dot-com episode dated from Jun 1995: the start of a chain of 3 BSADF runs
  - 57 months before the peak of Mar 2000; confirmed in real time in Dec 1995

MFM_ch17_nasdaq

BET: The 2007 Boom and 2025 (1/2)



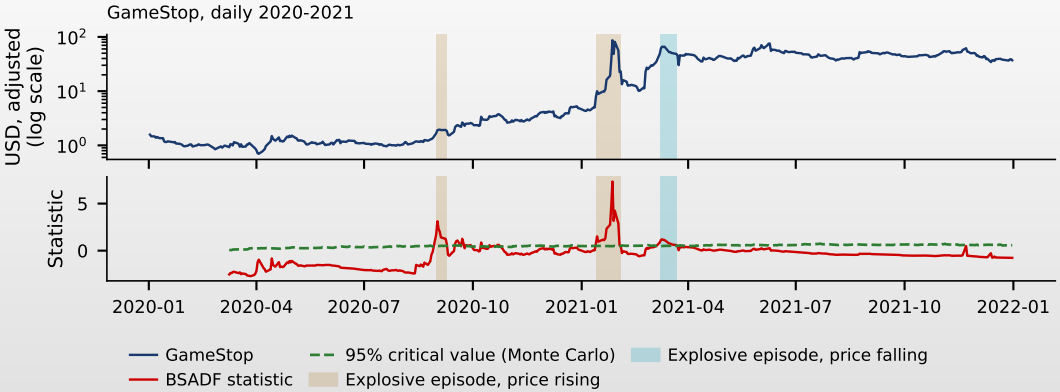
MFM_ch17_bet_gamestop

BET: The 2007 Boom and 2025 (2/2)

- Monthly log closes of the official BET, 1997–2026
  - $T = 348$, minimum window 37 months
- GSADF $= 2.90$ vs 95% critical value 2.21
  - Episodes: Aug 2002–Jun 2003; Nov 2003–Oct 2007; Oct 2008–Mar 2009; Oct 2025–ongoing
  - With one lag of the (autocorrelated) returns: GSADF $= 2.29$, below the critical value 2.39
- 2003–2007: the run-up around Romania's accession to the European Union (January 2007)
- Oct 2008–Mar 2009: an accelerating **fall**
- The current episode (from Oct 2025) is still open
  - Explosive does not mean it will crash: the rise is accelerating

MFM_ch17_bet_gamestop

GameStop, January 2021 (1/2)



MFM_ch17_bet_gamestop

GameStop, January 2021 (2/2)



A GameStop store, 3 February 2021, a week after the peak

Photo: Will Buckner (2021); CC BY 2.0; Wikimedia Commons

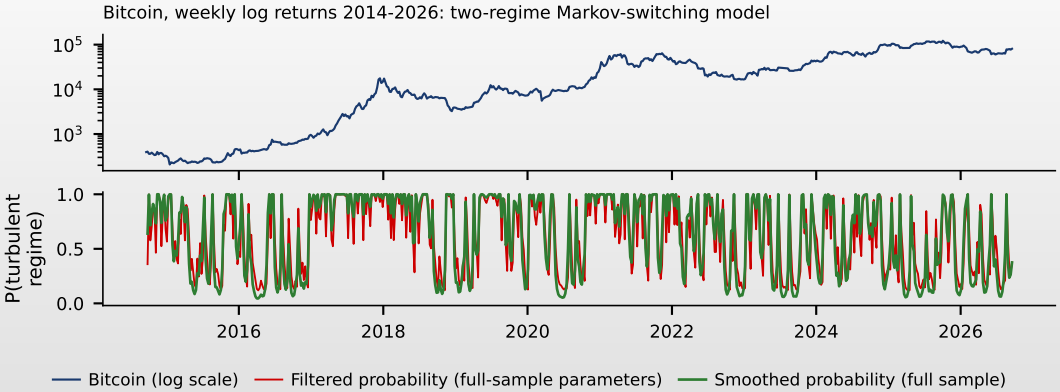
- Daily log closes 2020–2021, $T = 504$, window 45 days
- GSADF $= 7.34$, critical value 2.21
- January episode dated from 13 Jan 2021
  - 14 days before the closing high of 27 Jan 2021
- The episode is confirmed on 22 Jan 2021
- The chart dates two more short episodes: 31 Aug–9 Sep 2020 and 8–22 Mar 2021
- A surge of a few weeks: detecting it needs daily data
  - The test does not identify the mechanism (short covering: buying back shares sold short)

MFM_ch17_bet_gamestop

Calibration: Filimonov and Sornette

- Rewrite the cosine: $C\cos(\omega\ln(t_c - t) - \phi) = C_1\cos(\omega\ln(t_c - t)) + C_2\sin(\omega\ln(t_c - t))$ Filimonov & Sornette (2013)
  - $C_1 = C\cos\phi$, $C_2 = C\sin\phi$; A, B, C_1, C_2 are now **linear**: for given (t_c, m, ω) they come from OLS on the window $[t_1, t_2]$
- Search only over (t_c, m, ω)
  - A grid, then a Nelder–Mead refinement (a derivative-free simplex search) of the sum of squared residuals
- Search space Shu & Zhu (2020a, eq. 11)
  - $m \in [0, 1]$, $\omega \in [1, 50]$, $t_c \in [t_2, t_2 + (t_2 - t_1)/3]$
  - Damping $\frac{m|B|}{\omega|C|} \geq 1$: an approximate condition for a non-negative crash hazard rate
  - Exact condition: $m|B| \geq |C|\sqrt{m^2 + \omega^2}$
- Many local minima remain: different windows and starting points give different t_c Geraskin & Fantazzini (2013)

Bitcoin: Two Regimes (1/2)



MFM_ch17_markov_switching

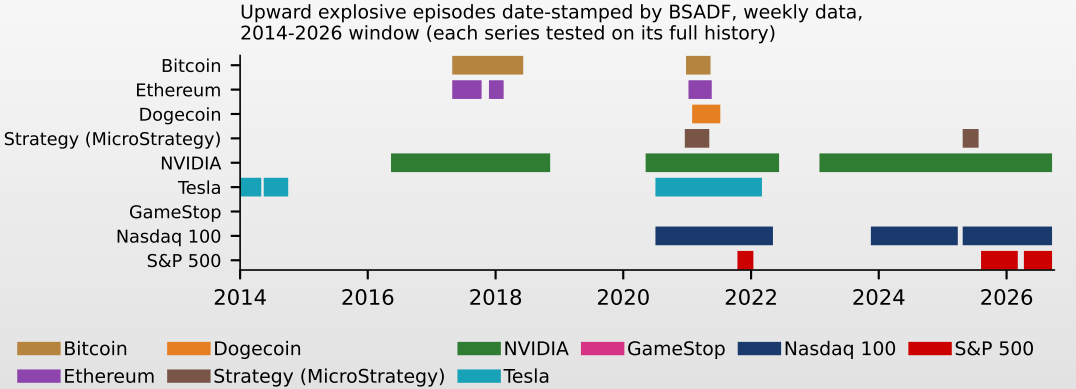
Bitcoin: Two Regimes (2/2)

Regime	$\hat{\mu}$ (SE)	$\hat{\sigma}$	$\hat{p}_{ii}$	Duration
Calm	0.26 (0.28)	2.93	0.736	3.8
Turbulent	1.20 (0.58)	11.22	0.840	6.3

- Weekly log returns in %, 2014–2026, 626 weeks
  - Duration in weeks; the turbulent regime covers 62% of the weeks
- Unlike equities, the turbulent regime has the **higher** mean
  - Regimes are short: volatility states, not bubble phases
- Regimes ignore the price level: a slow, calm bubble stays calm
  - BSADF says "the price is accelerating"; the regime says "risk has changed"

MFM_ch17_markov_switching

Explosive Episodes in Crypto and Tech Stocks (1/2)



MFM_ch17_crypto_ai

Explosive Episodes in Crypto and Tech Stocks (2/2)

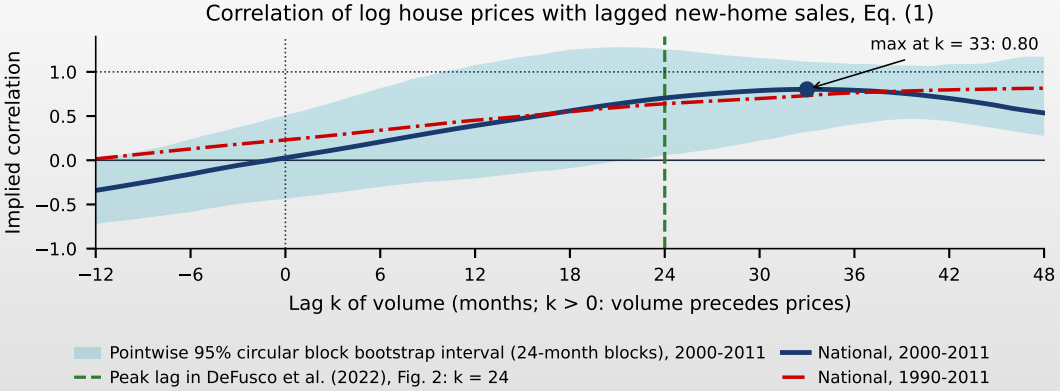
- Weekly log closes; rising episodes only
  - Each series tested on its own history, with its own Monte Carlo critical values
- Crypto booms cluster in 2017 and late 2020–2021
  - Bitcoin Apr 2017–Jun 2018; Dec 2020–May 2021; Ethereum Apr 2017–Oct 2017; Nov 2017–Feb 2018; Jan 2021–May 2021; Dogecoin Jan 2021–Jul 2021
- Stocks
  - Strategy (formerly MicroStrategy; holds Bitcoin): Dec 2020–May 2021; Apr 2025–Jul 2025; Tesla: Nov 2013–May 2014; May 2014–Oct 2014; Jul 2020–Mar 2022; NVIDIA: May 2016–Nov 2018; May 2020–Jun 2022; Jan 2023–ongoing
  - GameStop, weekly: the 3-week surge of January 2021 is shorter than the minimum duration
  - Indices: Nasdaq 100 Jul 2020–May 2022; Nov 2023–Mar 2025; Apr 2025–ongoing; S&P 500 Oct 2021–Jan 2022; Aug 2025–Mar 2026; Apr 2026–ongoing

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Case Study (1/3): Why It Matters, Data and Method

- The explosiveness tests of this chapter use prices only, yet bubbles come with high turnover Scheinkman & Xiong (2003)
  - Investors trade the resale option of Bubbles Beyond Rational Expectations
  - Quantities show who drives a boom, as blockchain flows did for the 2017 Bitcoin boom Griffin & Shams (2020)
- Data: 51 580 408 home sales from deeds records, 1995–2014, 115 metropolitan areas
  - **Speculators**: short-term buyers who resell within three years, and non-occupant buyers
- Lead–lag (Eq. 1): $p_{i,t} = \beta_k v_{i,t-k} + \eta_{i,t}$, $k = -12, \ldots, 48$ months
  - i: metropolitan area; t: month; p: demeaned log price; v: demeaned sales rate; $\eta_{i,t}$: error
  - $k > 0$: volume k months earlier; implied correlation $\beta_k \, \mathrm{sd}(v_{i,t-k})/\mathrm{sd}(p_{i,t})$, sd: standard deviation

Case Study (2/3): Volume Leads Prices



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Case Study (3/3): Volume Leads Prices

- The lead–lag regression (Eq. 1) on the national series, without a constant
  - Pointwise 95% band: circular block bootstrap, 24-month blocks, 2000 replications
- The implied correlation is a standardised slope, not a Pearson correlation: its bounds can exceed 1
- 2000–2011: maximum at $k = 33$ (0.80, CI $[0.32, 1.11]$)
  - At the paper's $k = 24$: 0.70, CI $[0.06, 1.26]$
- The band excludes zero for $k = 22, \ldots, 48$
  - But one national cycle holds few independent blocks: wide intervals
- Direction as in the paper: volume first, by two to three years
  - On 1990–2011 the maximum falls at the edge of the grid ($k = 48$, 0.82): the lag is not identified

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Your Turn: a Project Seed

- **Deliverables**
  - A pre-registered grid, fixed in a dated document before the evaluation
  - A notebook that regenerates every number; false-alarm and hit rates with valid p-values
- **AI log** (templates)
  - `AI_USE.md`: the prompts that shaped the design
  - `AI_ERRORS.md`: each wrong claim (left-tail critical values, a full-sample flag) and the test that caught it

Transversality: Where the Bubble Term Comes From

- Substitute the pricing equation into itself h times (law of iterated expectations)
  - $P_t = \sum_{i=1}^{h} \frac{\mathbb{E}_t[D_{t+i}]}{(1+r)^i} + \frac{\mathbb{E}_t[P_{t+h}]}{(1+r)^h}$
- Let $h \to \infty$: $P_t = F_t + B_t$ with $B_t = \lim_{h\to\infty} \mathbb{E}_t[P_{t+h}]/(1+r)^h$
- **Transversality condition**: $\lim_{h\to\infty} \mathbb{E}_t[P_{t+h}]/(1+r)^h = 0$
  - It holds $\Leftrightarrow B_t = 0 \Leftrightarrow P_t = F_t$
- Shift the limit one period: $\mathbb{E}_t[B_{t+1}] = \lim_h \mathbb{E}_t[P_{t+1+h}]/(1+r)^h = (1+r)\,B_t$
  - The condition that the discounted bubble is a martingale
- The condition $B_t \geq 0$ rules out a bubble that starts from zero Diba & Grossman (1988)
  - A bubble that is zero today stays zero

Back

Bubbles for Fama: Effective Sample Size

- The run-ups fall in 32 calendar years and overlap
  - Consecutive 2-year windows of the same boom share months
- Year-block bootstrap SE: 8.9 percentage points
  - Binomial SE $\sqrt{\hat{p}(1-\hat{p})/n} = 6.8$, with $\hat{p} = 0.45$ and $n = 53$
- Design effect $\mathrm{deff} = (\mathrm{SE}_{\mathsf{boot}}/\mathrm{SE}_{\mathsf{bin}})^2 = 1.71$
  - Effective sample size $n_{\mathsf{eff}} = n/\mathrm{deff} \approx 31$ independent observations
- Blocks of 2, 3 and 5 consecutive years: SE = 9.0, 8.6, 7.8
  - 2-year windows of adjacent years overlap
- Unconditional mean 2-year return over all industry-months: 28%
  - Above the CI of the mean return after a 100% run-up (−13% to 20%): returns after run-ups look lower than usual
  - A descriptive comparison, not a formal test of the difference of two dependent estimates

Back

Why the Full-Sample ADF Fails: the Leverage of Collapses

- OLS slope: $\hat{b} = \sum_t \tilde{y}_{t-\mathbf{1}}\,\Delta y_t \big/ \sum_t \tilde{y}^{\mathbf{2}}_{t-\mathbf{1}}$
  - $\tilde{y}_{t-\mathbf{1}}$: the lagged log price minus its sample mean
  - Expansion: $\Delta y_t > 0$ while $\tilde{y}_{t-\mathbf{1}}$ rises above its mean: positive terms
  - Collapse: the most negative Δy_t at the highest $\tilde{y}_{t-\mathbf{1}}$: a few very large negative terms
- Evans form in logs, $y = \ln B$, collapse to $\delta = e^c$ with probability $1 - \pi$ Evans (1991)
  - For $B \gg \delta$ the survival log growth is $\approx \ln \frac{\mathbf{1}+r}{\pi}$
  - $\mathbb{E}[\Delta y_t \mid y_{t-\mathbf{1}}] \approx \pi \ln \frac{\mathbf{1}+r}{\pi} + (1-\pi)(c - y_{t-\mathbf{1}})$
  - Slope $-(1-\pi) < 0$: apparent mean reversion, although $\mathbb{E}_t B_{t+\mathbf{1}} = (1+r)B_t$
- Windows with the expansion but not the collapse have $\hat{b} > 0$
  - When the log price accelerates (next slide)
  - A supremum over windows restores the power Phillips, Shi & Yu (2015b)

Back

Explosive in Levels, Accelerating in Logs

- A rational bubble is explosive in the **level**: $\mathbb{E}_t[B_{t+\mathbf{1}}] = (1+r)B_t$
  - An autoregressive root $1 + r > 1$
- A deterministic example: F constant or growing more slowly than r, $B_t = (1+r)^t B_{\mathbf{0}}$
  - The log price $\ln(F_t + B_t)$ **accelerates**: its growth rises from that of F towards $\ln(1+r)$, a convex path
  - A stochastic bubble does not guarantee this shape: for Blanchard–Watson, while it survives, the rate is $\ln((1+r)/\pi)$, and noise and collapses change the path
- So the log-price test targets a different alternative
  - **accelerating** log prices, not the level condition itself
  - The specification of the null and of the deterministic terms matters Phillips, Shi & Yu (2014)
- In this chapter we test log prices and log ratios (like PWY for the Nasdaq) Phillips, Wu & Yu (2011)
  - PSY test the price-dividend ratio Phillips, Shi & Yu (2015a)

Back

Limit Theory (1): the Null Distribution

- Null model: $y_t = d\,T^{-\eta} + y_{t-1} + \varepsilon_t$, $\eta > 1/2$, ε_t iid $(0, \sigma^2)$ Phillips, Shi & Yu (2015a)
  - A random walk with an asymptotically negligible drift $d\,T^{-\eta}$ (d: a constant)
- Limit (PSY, Theorem 1, eq. 5), by the functional central limit theorem and continuous mapping
  - $$\mathrm{ADF}_{r_1}^{r_2} \Rightarrow \frac{\frac{1}{2} r_w \left[W(r_2)^2 - W(r_1)^2 - r_w\right] - \int_{r_1}^{r_2} W\,dr\,\left[W(r_2) - W(r_1)\right]}{r_w^{1/2} \left\{ r_w \int_{r_1}^{r_2} W^2 dr - \left(\int_{r_1}^{r_2} W\,dr \right)^2 \right\}^{1/2}}$$
  - $r_w = r_2 - r_1$; W: standard Brownian motion; $\Rightarrow$: convergence in distribution as $T \to \infty$
- σ and d disappear: the limit is pivotal
  - SADF and GSADF are suprema of the same functional: their critical values depend only on r_0 and the deterministic terms
- $r_1 = 0$, $r_2 = 1$: the Dickey–Fuller limit with a constant
  - Its 95% quantile is negative (simulated: -0.06), far from 1.645
  - The drift and intercept specification changes the limit and the size Phillips, Shi & Yu (2014)

Back

Limit Theory (2): Mildly Explosive Alternatives

- Alternative: $y_t = \rho_T\, y_{t-\mathbf{1}} + \varepsilon_t$ with $\rho_T = 1 + c/k_T$, $c > 0$, $k_T \to \infty$, $k_T = o(T)$
  - PSY use $k_T = T^{\alpha}$, $\alpha \in (0, 1)$
  - c: how explosive; k_T: the speed at which the root approaches 1; $k_T = o(T)$: $k_T/T \to 0$
  - Closer to 1 than any fixed explosive root, farther than a local-to-unity root $1 + c/T$: the "mild" speed of real bubbles
- Phillips and Magdalinos: $\dfrac{\rho_T^T}{\rho_T^{\mathbf{2}} - 1}\,(\hat{\rho}_T - \rho_T) \Rightarrow \mathcal{C}$ Phillips & Magdalinos (2007)
  - A standard Cauchy law, without assuming Gaussian errors
  - $\hat{\rho}_T$: the OLS estimate of ρ_T; $\mathcal{C}$: the standard Cauchy distribution
  - Rate $k_T \rho_T^T$: exponentially fast; the ADF statistic diverges to $+\infty$
- Hence SADF and GSADF diverge too: the tests are consistent, power $\to 1$ Phillips, Shi & Yu (2015b)
  - For GSADF any window of at least $r_{\mathbf{0}} T$ observations inside the bubble suffices
- Finite samples: power depends on c, α and the duration
  - Short bursts may stay below the bar, as in the Evans simulation

Back

Limit Theory (3): Consistent Date-Stamping

- Crossing times (PSY, eqs. 7–8)
  - $\hat{r}_e = \inf\{r_2 \geq r_0 : \mathrm{BSADF}_{r_2} > scv_{r_2}^{\beta_T}\}$, $\hat{r}_f = \inf\{r_2 \geq \hat{r}_e + \delta \ln(T)/T : \mathrm{BSADF}_{r_2} < scv_{r_2}^{\beta_T}\}$
  - $\hat{r}_e$, $\hat{r}_f$: estimated start and end, as fractions of the sample; inf: the first r_2 that meets the condition
  - $scv_{r_2}^{\beta_T}$: the 100$(1-\beta_T)$% quantile of BSADF on $\lfloor Tr_2 \rfloor$ observations; β_T: the level; $\lfloor \cdot \rfloor$: rounded down
- Consistency (PSY, eq. 11): $(\hat{r}_e, \hat{r}_f) \to_p (r_e, r_f)$ if $\dfrac{1}{scv^{\beta_T}} + \dfrac{scv^{\beta_T}}{T^{1-\alpha/2}} \to 0$
  - $\to_p$: convergence in probability to the true dates r_e, r_f
  - The critical value must diverge, so that false alarms vanish ($\beta_T \to 0$)
  - But slower than $T^{1-\alpha/2}$, so that true bubbles are still found; e.g. like $\ln T$ Phillips, Shi & Yu (2015a)
- In practice β_T is fixed at 5% at every date
  - Over hundreds of dates a false alarm becomes likely (the Monte Carlo size study in the wild bootstrap section)

Back

The Price-Dividend Ratio: Comparison with PSY

- Our critical values: 1 000 Monte Carlo paths of the sample length
- PSY also date 1879–1880, 1955–1956 and 1986–1987 Phillips, Shi & Yu (2015a)
  - On data to 2010 and without our filter of 8 consecutive months
- With a shorter minimum window 1879–1880, 1954–1956 and 2008–2009 appear in our data too

Back

Family-Wise Critical Values: Bootstrap Design

- Phillips and Shi: the 95% quantile of the maximum of BSADF over τ_b consecutive dates Phillips & Shi (2020)
  - Sets the FWER at 5% for the bootstrap design: τ_b monitoring dates after a start-up of w_0 observations
- Resampling
  - Residuals e_t of $\Delta y_t = a + e_t$, drawn with replacement and multiplied by $N(0,1)$ weights
  - Paths of $w_0 + \tau_b - 1$ changes; 999 replications
- Why the control is only approximate here
  - BSADF over the full history: later dates search more start points than the bootstrap design
  - Drawing with replacement loses the time order: unlike the pointwise wild bootstrap, which keeps each Δy_t at its date, these paths ignore the volatility clustering of Bitcoin
- The pointwise rule gives a crossing within the last 104 weeks in 42% of null paths

Back

Real Time vs Hindsight: Caveats

- Column definitions as on the main slide, with two caveats
  - Confirmed is only pseudo real time: w_0 and L come from the full-sample T (a truly real-time design fixes them ex ante)
  - GameStop: leads in calendar days converted to months

Back

Real-Time Monitoring: Newer Procedures

- Repeating a one-shot test every period inflates the false-positive rate Astill et al. (2018)
  - Astill et al. compare monitoring statistics with the maximum over a training period
  - The false-positive rate is then set for a chosen horizon, robust to time-varying volatility
- Bubbles and crashes Whitehouse, Harvey & Leybourne (2023)
  - After a bubble is detected, monitor for the crash with the sign of the mean of Δy_t
  - It never signals a crash before the crash starts
- Earlier detection Whitehouse, Harvey & Leybourne (2025)
  - Alternative variance standardisations keep the false-detection probability but find bubbles sooner (Bitcoin, housing)
- Collapse phase: PSY applied to the reversed sample dates the crisis and the recovery Phillips & Shi (2018)
- Alternatives to sup-ADF
  - A Chow-type break test, random walk to explosive, has the highest power in Homm & Breitung (2012)
  - Recursive tests with a start-point search Harvey, Leybourne & Sollis (2015)

Back

LPPL in Cryptocurrency and Emerging Markets

- Bitcoin 2012–2018: bubble episodes on several time scales, dated with LPPLS Gerlach, Demos & Sornette (2019)
- Bitcoin: real-time prediction of crashes with the LPPLS confidence indicator Shu & Zhu (2020b)
- Emerging markets in Latin America and Asia: log-periodic bubbles and anti-bubbles (accelerating declines) Johansen & Sornette (2001)
- Bucharest Stock Exchange
  - Herding and log-periodic patterns before the 2007–2008 crash Pele, Mazurencu & Nijkamp (2013)
  - Crash modelling for the same market Pele & Mazurencu-Marinescu (2012)
- Cryptocurrencies: a fundamental value from Metcalfe's law (network size)
  - Combined with LPPL bubbles Pele & Mazurencu-Marinescu-Pele (2019), Conda et al. (2026)
- Dogecoin 2021: LPPL together with sentiment from social media Pele (2025)
- Common lesson: LPPL describes the acceleration well
  - The crash date remains the weak point

Back

Estimation: Hamilton Filter, Kim Smoother, EM

- Log-likelihood of θ (all μ_j, σ_j, p_{ij}), by prediction and update Hamilton (1989)
  - $\ell(\theta) = \sum_t \ln \sum_j P(s_t = j \mid \mathcal{F}_{t-1})\, \varphi(r_t; \mu_j, \sigma_j^2)$
  - $\mathcal{F}_t$: returns up to t; $\varphi(x; \mu, \sigma^2)$: the Normal density with mean μ and variance σ^2
- Kim smoother, backwards Kim (1994)
  - $P(s_t = j \mid \mathcal{F}_T) = P(s_t = j \mid \mathcal{F}_t) \sum_k p_{jk} \dfrac{P(s_{t+1} = k \mid \mathcal{F}_T)}{P(s_{t+1} = k \mid \mathcal{F}_t)}$
- The EM (Expectation-Maximisation) algorithm: the M-step is a weighted average Hamilton (1990)
  - Smoothed $\xi_{tj} = P(s_t = j \mid \mathcal{F}_T)$, $\xi_{t,ij} = P(s_{t-1} = i, s_t = j \mid \mathcal{F}_T)$
  - $\hat{\mu}_j = \dfrac{\sum_t \xi_{tj} r_t}{\sum_t \xi_{tj}}$, $\hat{\sigma}_j^2 = \dfrac{\sum_t \xi_{tj}(r_t - \hat{\mu}_j)^2}{\sum_t \xi_{tj}}$, $\hat{p}_{ij} = \dfrac{\sum_{t \geq 2} \xi_{t,ij}}{\sum_{t \geq 2} \xi_{t-1,i}}$
- Unbounded likelihood ($\sigma_j \to 0$ on one return) and several local maxima
  - 20 random sets of starting values, then the full optimisation; if LR < 0, a refit with more starts
  - No penalty: a $\hat{p}_{ij} < 10^{-4}$ or a $\hat{\sigma}_j$ below 5% of the sample volatility flags a boundary maximum
  - Standard errors from the inverse numerical Hessian assume a known number of regimes

Back

Why the LR Test for Regimes Is Non-Standard

- Under H_0 the regime means and variances coincide Hansen (1992)
  - p_{11}, p_{22} are not identified: nuisance parameters present only under H_1
  - The information matrix is singular and some scores are identically zero
- Hansen: bounds on the standardised LR via empirical-process theory Hansen (1992)
- Carrasco, Hu and Ploberger: an optimal test Carrasco, Hu & Ploberger (2014)
  - It needs only the estimate under H_0, with bootstrap critical values
- Two vs three regimes: the same logic
  - The null model is then the fitted two-regime chain

Back

Case Study, Data and Method (1/2)

- Data (Section 1.1): arm's-length sales only
  - 115 metropolitan areas (48% of the US housing stock); listings linked to sales in 57 of them
  - The cycle analysis covers 2000–2011: realised holding periods need consecutive sales
- Speculators (Section 1.2)
  - Short-term buyers: resell within three years (realised holding period)
  - Non-occupant buyers: a mailing address different from the property address (102 areas)
- Lead–lag (Eq. 1, Figure 2): construction of the variables
  - p: log price, demeaned by city; v: sales over the 2000 housing stock, demeaned by city and calendar month
  - Standard errors clustered by month

Back

Case Study, Data and Method (2/2)

- Cross-section (Table 2): OLS across cities on the speculative volume boom (2000–2005)
  - Outcomes: price boom (2000–2006), price bust (2006–2011), listings, volume, foreclosures
- Model (Sections 5–6): extrapolating buyers with different horizons
  - Not every listing sells
  - Counterfactuals: rational expectations, no speculators, transaction taxes

Back

Case Study, Complete Results (1/2)

- Figure 4: sales rise from 2.73 to 3.82 million (2000 to 2005)
  - Short-term sales: from 510 to 940 thousand, 39% of the rise; non-occupant purchases: 53%
  - Interpretation: speculators account for much of the volume boom
- Table 2, Panel A: price-boom coefficient 1.930 (standard error 0.297, $R^2 = 0.272$)
  - One standard deviation more (12.9%, Table 1): price rise larger by 24.9 percentage points
  - Price bust -0.571 (0.083): 7.4 points deeper
  - Interpretation: more speculation, larger booms and larger busts

Back

Case Study, Complete Results (2/2)

- Table 2, Panels B–C (Section 3.2)
  - Quantities in % of 2003 total volume
  - Unsold listings in the quiet: 5.961 (1.353), 76.9 points more per standard deviation
  - Foreclosures: 0.895 (0.398), 11.5 points, about 370 thousand more across the 115 areas
  - Interpretation: speculators who bought in the boom list in the quiet and fail to sell
  - That this turns the cycle is a model result (Sections 5–6), not identified by the regressions
- Figure 7, Panel C: a 10% price gain over the past year
  - Goes with an 8.2 points larger short-term buyer share
  - Interpretation: extrapolation draws short-horizon buyers in
- Table 6 (model): a tax of 5% of the purchase price, paid at sale
  - On all buyers: the simulated log-price bust falls from 8.2 to 1.1 log points (100$\times$ the change in log price)
  - On non-occupants only: to 5.8 points
  - Interpretation: short horizons, not non-occupancy, amplify the cycle

Back

Case Study, Lead–Lag Inference

- p and v demeaned over the full sample
- At $k = 0$: 0.03; at $k = -12$: -0.34
  - Level correlations, not a directional test of whether prices lead volume
- Eq. (1) has no constant and the lag-aligned series need not have zero mean
  - Hence bootstrap limits above 1 are possible
- The block bootstrap assumes stable, weakly dependent series
  - Doubtful for one boom–bust cycle in levels: the band is pointwise and exploratory

Back

The Discovery Loop, Reusable Prompts (1/2)

- **Literature**: map what is known, then verify every reference
  - "`Which papers control the false-alarm rate of sequential bubble monitoring, and what size and power do they report? Give DOI and simulation design.`"
  - Resolve each DOI on Crossref and find the simulation table the answer quotes; search with Semantic Scholar, Elicit or Google Scholar
- **Hypotheses**: competing explanations, each with its own observable
  - "`For the current NVIDIA episode, list competing explanations (rational bubble, explosive expected cash flows, falling discount rate) and the observable each one predicts.`"
  - E.g. a bubble predicts an explosive price-to-earnings ratio; explosive cash flows predict explosive earnings with a stable ratio
- **Data and code**: on the course data, replicating a known number first
  - "`Implement GSADF with r0 = 0.01 + 1.8/sqrt(T) on weekly Bitcoin closes from the course data and simulate its 95% critical value.`"
  - Replicate first: GSADF = 3.85 and critical value 2.27 on 626 weeks

Back

The Discovery Loop, Reusable Prompts (2/2)

- **Robustness**, fixed before the alarms are evaluated
  - "`Pre-register the evaluation of the alarm: horizons 30/60/90/180 days, crash thresholds 10/20/30%, inference robust to overlapping windows, Holm correction over the 12 cells.`"
  - Twelve cells tried and one reported is data snooping; twelve cells pre-registered and corrected is a test
- **Critique**: ask the assistant to attack the result
  - "`Act as a referee: why could an alarm-crash association be an artefact of overlapping windows, look-ahead or the choice of assets?`"
  - Expect: dependent observations inflate significance; full-sample critical values; famous episodes only, the selection that Greenwood, Shleifer & You (2019) avoid
- **Report**: hit rates with valid *p*-values, all 12 cells, the alarms that failed
  - The prompts that shaped the design go into `AI_USE.md`

Back

What the Researcher Must Check

- **Full-sample information in date-stamping**
  - Critical values, lag order or detrending fitted on the whole sample
  - Test: cut the sample at t and recompute; the flag at t must not change
- **Wrong tail**
  - Left-tail ADF critical values (about -2.86) used against an explosive alternative
  - Test: on simulated random walks the right-tailed test must reject in about 5% of the paths
- **Changing volatility**
  - Test: wild bootstrap critical values; for Bitcoin 2.82 instead of 2.27
- **Multiplicity**: many dates, assets and windows
  - Test: run the rule on simulated random walks and count paths with at least one alarm
- **Overlapping outcomes**: Fisher's exact or χ^2 tests overstate significance
  - Test: block bootstrap or circular rotations of the alarm series; drop windows cut by the end of the data

Back

Mini-Case: What the Result Does and Does Not Say

- A large one-sided p is not evidence of **no** information
  - It only fails to support a **higher** crash probability after alarms
  - Both point estimates are **negative**: one-sided rotation p for $\Delta < 0$, a lower crash rate after alarms, 0.07 (Bitcoin), 0.02 (Nasdaq 100)
  - Bitcoin: this 0.07 and the Newey–West one-sided 0.024 answer the same question under different null assumptions (stationary rotations vs a regression with HAC (Heteroskedasticity and Autocorrelation Consistent) errors)
  - Post-hoc, two-sided reading: alarms come in steady rises that did not end in a crash within 90 days
- Assumption behind the rotation p-values
  - Under H_0 every circular shift of the alarm series (at least 26 steps, ends joined) is as likely as the observed alignment
  - Stronger than equal hit rates: it needs stationary series and alarms independent of the crash process
  - Hence exploratory p-values; check their size by simulation under plausible null processes

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Further Study on Quantinar

Advanced courses on quantinar.com

- Cryptocurrency: Speculative Asset and Medium of Exchange
- Are Cryptos becoming alternative Assets?
- Statistics of Financial Markets

Code and data

- Quantlets: github.com/danpele/MFM/tree/main/Quantlets/Ch_17

Back

Acronyms used in this material (1/2)

- **ADF**: Augmented Dickey–Fuller (test)
- **AI**: Artificial Intelligence
- **AR**: AutoRegressive model
- **BET**: Bucharest Exchange Trading (index)
- **BET-FI**: BET Financial Investment companies index
- **BSADF**: Backward Sup Augmented Dickey–Fuller (statistic)
- **CI**: Confidence Interval
- **CoVaR**: Conditional Value-at-Risk (of the system, given an institution)
- **DOI**: Digital Object Identifier
- **EM**: Expectation-Maximisation (algorithm)
- **FRED**: Federal Reserve Economic Data
- **FWER**: Family-Wise Error Rate
- **GARCH**: Generalised AutoRegressive Conditional Heteroskedasticity
- **GDP**: Gross Domestic Product
- **GSADF**: Generalised Sup Augmented Dickey–Fuller (test)
- **HAC**: Heteroskedasticity and Autocorrelation Consistent (standard errors)

Acronyms used in this material (2/2)

- **JLS**: Johansen–Ledoit–Sornette (model)
- **JST**: Jordà–Schularick–Taylor (Macrohistory Database)
- **LPPL**: Log-Periodic Power Law
- **LPPLS**: Log-Periodic Power Law Singularity (model)
- **LR**: Likelihood Ratio (test)
- **MC**: Monte Carlo (simulation)
- **NVIDIA**: NVIDIA Corporation (company name, US chip designer)
- **OLS**: Ordinary Least Squares
- **PSY**: Phillips–Shi–Yu (bubble test and date-stamping procedure)
- **PWY**: Phillips–Wu–Yu (bubble test and date-stamping procedure)
- **SADF**: Sup Augmented Dickey–Fuller (test)
- **SE**: Standard Error
- **SRISK**: Systemic RISK (capital shortfall measure)
- **US**: United States
- **USD**: United States dollar
- **VaR**: Value-at-Risk

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