








Modelling Financial Markets

Seminar 18: Systemic Risk and Networks — Hands-on



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Which Banks Are Systemic?

- Questions of the seminar
  - **which banks are systemic?**
  - **how sure can we be?**
- Deliverables
  - the answers to the A exercises
  - one table and one chart per B exercise
  - the corrected artificial intelligence (AI) answer in C1 and its error log
  - one interpretation sentence per exercise
- Textbook: Franke, Härdle & Hafner (2019)
- Notebook for today: open the seminar notebook in Colab
  - each task names its notebook section
- Seminar: Prof. Daniel Traian Pele, danpele@ase.ro; Antoaneta Amza, doctoral (PhD) candidate, antoaneta.amza@csie.ase.ro

Exposure or Contribution to the System?

- A bank may suffer a large loss in a market crisis but add little to the distress of other banks: exposure and contribution need not produce the same ranking.

Data Used

Series	Assets	Price used	Used in
US banks	JPMorgan, Bank of America, Citigroup, Goldman Sachs, Morgan Stanley, Wells Fargo	adjusted close; daily high and low	B1–B6, B8, B9, C2, C3
European banks	Deutsche Bank, BNP Paribas, Santander, ING, HSBC	adjusted close; daily high and low	B2–B5, B7–B9, C2, C3
Romanian banks	Banca Transilvania, BRD-Groupe SG (days without trades removed)	adjusted close; daily high and low	B2, B3, B7, B8, B9, B10, C2, C3
Market indices	S&P 500, Euro Stoxx 50, BET, VIX (Cboe implied volatility)	close	A5, B6, B7, B9, B10, C3

- Daily prices in data/market, one series per asset (field used: adjusted close for the banks, close for the indices; daily high and low for volatility); sample 5 January 2010 – 18 September 2026
- Returns R: prices aligned first on the 3 962 dates common to all 17 series, then the 3 961 log returns between consecutive common dates, in %; volatilities V: log Parkinson volatility (from the daily high–low range) of the 13 banks

Setup: Run and Verify

```
# notebook section "Data" builds
# R (returns, %) and V (log volatility)
print(R.shape, V.shape)
print(R.index[0].date(),
      R.index[-1].date())
print(R[['JPM', 'DBK', 'TLV',
         'SPX']].head(3).round(2))
```

- Expected output: (3961, 17) (3966, 13); first and last day 2010-01-05, 2026-09-18; V needs only the 13 banks, not the indices, so it has more common days (3966)

date	JPM	DBK	TLV	SPX
2010-01-05	1.92	1.60	2.73	0.31
2010-01-06	0.55	-1.05	3.09	0.05
2010-01-07	1.96	0.45	0.00	0.40

- If your counts or first values differ, check the symbols, that you use the dividend-adjusted price, and the alignment on common days

Systemic Risk, Quantiles, VaR and ES

- Details: Chapter 18
- Two questions about a bank
  - **Exposure**: how much does the bank lose when its market index is in trouble? (Marginal Expected Shortfall, MES, and its crisis versions, LRMES and SRISK)
  - **Contribution**: how much worse is the system when the bank is in trouble? (ΔCoVaR)
- The α-quantile $q_\alpha(X)$ of a return X: the value it falls below with probability α

$$P(X \le q_\alpha(X)) = \alpha, \qquad \mathrm{VaR}_\alpha = -q_\alpha(X), \qquad \mathrm{ES}_\alpha = -\mathbb{E}\big[X \mid X \le q_\alpha(X)\big]$$

  - α: tail probability (1% or 5%); Value at Risk (VaR): the loss exceeded on a fraction α of days; Expected Shortfall (ES): the average loss on those days, so $\mathrm{ES}_\alpha \ge \mathrm{VaR}_\alpha$
  - $\mathbb{E}[\cdot \mid A]$: mean conditional on the event A; VaR, ES, CoVaR, MES are reported as positive losses (%)

CoVaR and ΔCoVaR: Definitions

- CoVaR_α: the VaR of the system X^{sys} on the bank's distress day Adrian & Brunnermeier (2016)

$$P\left(X^{sys} \leq -\mathrm{CoVaR}_\alpha \,\middle|\, X_i = q_\alpha(X_i)\right) = \alpha$$

  - X_i: return of bank i; X^{sys}: return of the system (here the other banks of the region); $X_i = q_\alpha(X_i)$: the bank is at its own VaR

- The same quantity in the bank's median state, $X_i = q_{\mathbf{50\%}}(X_i)$: $\mathrm{CoVaR}_\alpha^{\mathbf{50\%}}$
  - $q_{\mathbf{50\%}}(X_i)$: the median return, a normal day for the bank

- The contribution of the bank: the difference

$$\Delta\mathrm{CoVaR}_\alpha = \mathrm{CoVaR}_\alpha - \mathrm{CoVaR}_\alpha^{\mathbf{50\%}}$$

  - the extra tail loss of the system when the bank moves from a normal day to distress; > 0 when bank and system move together
  - own risk (VaR of the bank) and contribution (ΔCoVaR) are different quantities (A1, B4)

Estimating CoVaR by Quantile Regression

- **Quantile regression**: a line for the α-quantile of X^{sys}, not for its mean

$$q_\alpha(X^{sys} \mid X_i) = a_\alpha + b_\alpha X_i, \qquad (\hat{a}_\alpha, \hat{b}_\alpha) = \arg\min_{a,\, b} \sum_{t=1}^{n} \rho_\alpha\left(X_t^{sys} - a - bX_{i,t}\right)$$

  - b_α: by how much the α-quantile of the system moves per point of X_i; n: number of days; hat: estimated value

- The **check function** replaces the squared residual of Ordinary Least Squares (OLS)

$$\rho_\alpha(u) = u\left(\alpha - 1\{u < 0\}\right)$$

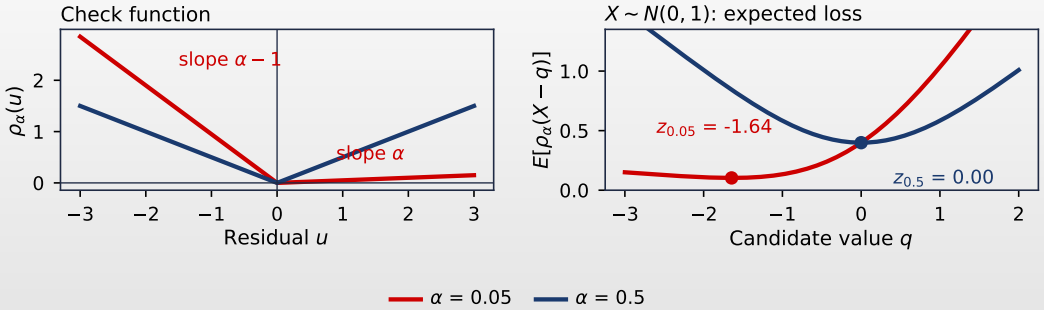
  - $1\{\cdot\}$: 1 if the condition holds, else 0; negative residuals weigh $1-\alpha$, positive ones α, so a share α of the points lies below the line

- Plug in the empirical quantiles $\hat{q}_\alpha(X_i)$, $\hat{q}_{\mathbf{50\%}}(X_i)$ of the bank

$$\widehat{\mathrm{CoVaR}}_\alpha = -\left(\hat{a}_\alpha + \hat{b}_\alpha\, \hat{q}_\alpha(X_i)\right), \qquad \widehat{\Delta\mathrm{CoVaR}}_\alpha = \hat{b}_\alpha\left(\hat{q}_{\mathbf{50\%}}(X_i) - \hat{q}_\alpha(X_i)\right)$$

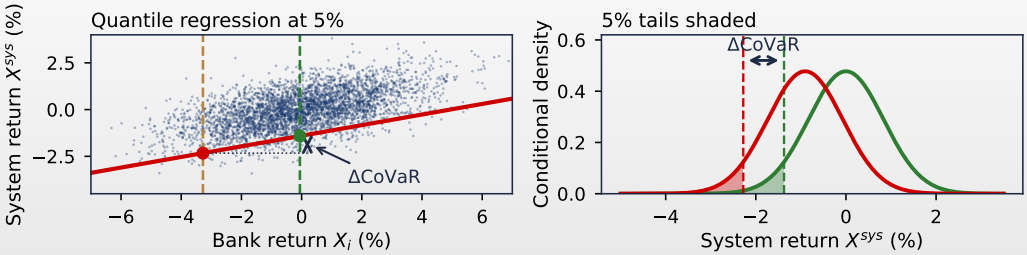
  - the slope times the distance from the median day to the distress day; $\hat{a}_\alpha$ cancels

The Check Function



- Left: $\rho_\alpha(u)$ is a tilted absolute value; at $\alpha = 0.5$ it is $|u|/2$ (median regression)
- Right: for $X \sim N(0,1)$, $\mathbb{E}[\rho_\alpha(X-q)]$ is smallest at the α-quantile: -1.64 at 5%, 0 at 50%
  - minimising ρ_α estimates a quantile, as minimising squares estimates a mean

ΔCoVaR on Simulated Data



- Simulated bivariate Normal returns: $\sigma_i = 2\%$, $\sigma_{sys} = 1\%$, $\rho = 0.55$, $n = 4\,000$ days
  - left: slope $\hat{b}_{5\%} = 0.28$; from the median day to the distress day ($\hat{q}_{5\%} = -3.28\%$) the line falls by $\Delta\text{CoVaR} = 0.91\%$
  - right: the 5% quantiles (dashed) of the two conditional densities of X^{sys} are ΔCoVaR apart
- Normal closed form Adrian & Brunnermeier (2016): $\Delta\text{CoVaR}_\alpha = \rho\,\sigma_{sys}|z_\alpha|$, $z_\alpha = \Phi^{-1}(\alpha)$ (Φ: standard Normal distribution function); here $0.55 \cdot 1\% \cdot 1.645 = 0.90\%$

A Worked ΔCoVaR Example

- Estimates: $\hat{b}_{5\%} = 0.5$, $\hat{q}_{50\%} = 0.05\%$, $\hat{q}_{5\%} = -2.95\%$
  - $\Delta\mathrm{CoVaR}_{5\%} = 0.5 \cdot (0.05 + 2.95) = 1.5\%$ of extra system loss
- Question for the class: a bank doubles its volatility, with the same correlation with the system. What happens to its ΔCoVaR?
  - Answer: nothing; $\Delta\mathrm{CoVaR}_\alpha = \rho\,\sigma_{sys}|z_\alpha|$ does not depend on σ_i (A1)
- In B1, B4: the system of bank i excludes bank i
  - $X_t^{sys} = \frac{1}{N-1}\sum_{j\neq i} X_{j,t}$, the equal-weighted return of the other $N-1$ banks of the region

Exposure: MES and ES

- **Marginal expected shortfall** Acharya et al. (2017): the bank's average loss on the market's worst α days

$$\mathrm{MES}_{i,\alpha} = -\mathbb{E}\big[X_i \mid X_m \le q_\alpha(X_m)\big]$$

  - X_m: return of the market (or of the portfolio); the condition is on the **market's** tail, not the bank's
- Additivity (A2): with portfolio weights w_i, the weighted MES add up to the ES of the portfolio

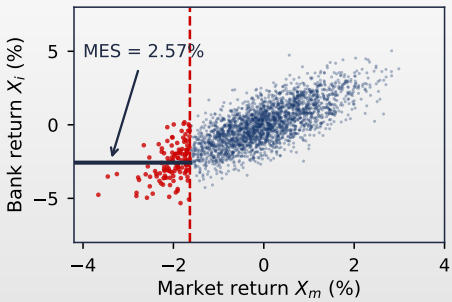
$$\sum_i w_i \,\mathrm{MES}_{i,\alpha} = \mathrm{ES}_\alpha(X_p), \qquad X_p = \sum_i w_i X_i$$

- Normal case: MES is beta times the ES of the market

$$\mathrm{MES}_{i,\alpha} = \beta_i \,\mathrm{ES}_\alpha(X_m), \qquad \beta_i = \frac{\rho\,\sigma_i}{\sigma_m}, \qquad \mathrm{ES}_\alpha(X_m) = \frac{\sigma_m\,\varphi(z_\alpha)}{\alpha}$$

  - σ_i, σ_m: standard deviations of bank and market; ρ: their correlation; φ: standard Normal density
  - e.g. $\beta = 1.2$, $\sigma_m = 1\%$, $\alpha = 5\%$: $\mathrm{ES} = 2.06\%$, $\mathrm{MES} = 2.47\%$

MES on a Scatter Plot



- Simulated days: $X_i = 1.2\,X_m + e_i$, $\sigma_m = 1\%$
  - e_i: the bank's own shock, independent of the market, standard deviation 1%
- Red: the 125 days with $X_m \leq q_{5\%}(X_m)$
  - MES = minus the mean of X_i on these days (horizontal line)
  - estimate 2.57%, against the Normal value $\beta\,\mathrm{ES} = 2.47\%$: the difference is sampling error from 125 days

Exposure in a Crisis: LRMES

- **Long-run MES** (LRMES) Brownlees & Engle (2017): the expected fall of the bank's equity if the market crashes

$$\mathrm{LRMES}_i = -\mathbb{E}\big[R_{i,t+\mathbf{1}:t+h} \,\big|\, R_{m,t+\mathbf{1}:t+h} < C\big]$$

  - $R_{t+\mathbf{1}:t+h}$: arithmetic return over the next h days; C: crisis threshold of the market
  - scenarios: $h = 22$ days, $C = -10\%$ (one month); $h = 126$ days, $C = -40\%$ (six months)

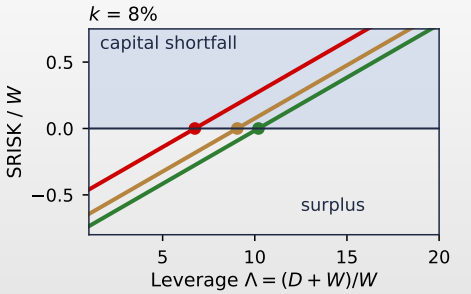
- By simulation (B6): Generalised Autoregressive Conditional Heteroskedasticity (GARCH) volatilities and Dynamic Conditional Correlation (DCC), many paths, average over the crisis paths

- By a shortcut from daily data Acharya, Engle & Richardson (2012)

$$\mathrm{LRMES}_i \approx 1 - \exp\big(-18\,\mathrm{MES}_i^{\mathbf{2\%}}\big), \qquad \mathrm{MES}_i^{\mathbf{2\%}} = -\mathbb{E}\big[X_i \mid X_m < -2\%\big]$$

  - X as fractions; $\mathrm{MES}^{\mathbf{2\%}}$: average daily bank loss on days when the market falls by more than 2%; the 18 is derived in A5

Exposure in a Crisis: SRISK



- **SRISK** Brownlees & Engle (2017): capital the bank would lack in the crisis

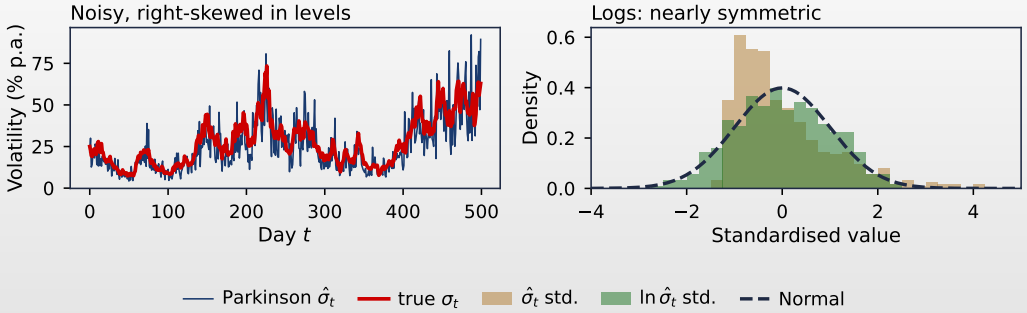
$$\mathrm{SRISK} = k\big[D + (1 - \mathrm{LRMES})W\big] - (1 - \mathrm{LRMES})W$$

  - k: prudential capital ratio; D: book debt; W: market equity; required capital minus remaining equity
  - equivalently $kD - (1 - k)(1 - \mathrm{LRMES})W$
- Zero at the break-even leverage Λ^*

$$\Lambda^* = 1 + \frac{(1 - k)(1 - \mathrm{LRMES})}{k}, \qquad \Lambda = \frac{D + W}{W}$$

  - $\Lambda > \Lambda^*$: shortfall (chart: above zero); a larger LRMES lowers Λ^*
- Example: $W = 50$, $D = 500$, $k = 8\%$, LRMES $= 30\%$
  - $\mathrm{SRISK} = 40 - 0.92 \cdot 0.70 \cdot 50 = 7.8$; $\Lambda = 11 > \Lambda^* = 9.05$

Parkinson Volatility from the Daily Range



- Parkinson estimate $\hat{\sigma}_t = \sqrt{252\,(\ln H_t - \ln L_t)^2/(4 \ln 2)}$; H_t, L_t: daily high and low
  - $4 \ln 2$ makes $\hat{\sigma}_t^2$ unbiased for a Brownian price without drift; 252: annualisation, so $\hat{\sigma}_t = 0.20$ means 20% per year
  - left: one-day estimates (blue) are noisy around the simulated true volatility (red)
  - right: skewness 1.3 in levels, 0.0 in logs; the Vector AutoRegression (VAR, not VaR) is therefore fitted to $\ln \hat{\sigma}_t$

The Vector Autoregression (VAR)

- VAR(p) for the vector y_t of the N log volatilities: each series on p lags of all series

$$y_t = c + A_{\mathbf{1}} y_{t-\mathbf{1}} + \cdots + A_p y_{t-p} + \varepsilon_t, \qquad \mathbb{E}[\varepsilon_t] = 0, \quad \mathrm{Cov}(\varepsilon_t) = \Sigma = (\sigma_{ij})$$

  - c: N intercepts; A_l: $N \times N$ matrix, $(A_l)_{ij}$ = effect of $y_{j,t-l}$ on $y_{i,t}$; ε_t: shocks, serially uncorrelated but correlated across banks
  - estimated by OLS equation by equation; $\hat{\Sigma}$: covariance of the residuals
- Lag order by Bayesian Information Criterion (BIC) (B3): the smallest value of

$$\mathrm{BIC}(p) = \ln \det \hat{\Sigma}_p + \frac{\ln T}{T} p N^{\mathbf{2}}$$

  - $\det \hat{\Sigma}_p$: generalised residual variance with p lags (fit); $pN^{\mathbf{2}}$ slope coefficients; T: number of days (penalty)
- Moving-average form of a stable VAR (eigenvalues of the companion matrix inside the unit circle)

$$y_t = \mu + \sum_{h \geq \mathbf{0}} \Phi_h \varepsilon_{t-h}, \qquad \Phi_{\mathbf{0}} = I, \quad \Phi_h = \sum_{l=\mathbf{1}}^{\min(h,p)} A_l \Phi_{h-l}$$

  - μ: mean of y_t; Φ_h: effect after h days of a shock today; VAR(1): $\Phi_h = A^h$

Impulse Responses: Generalized and Cholesky

- Shocks are correlated, so "a shock to bank j alone" must be defined
  - e_j: the j-th unit vector; σ_{jj}: variance of $\varepsilon_{j,t}$
- **Generalized** response Pesaran & Shin (1998): shock j by one standard deviation, the other shocks at their conditional mean

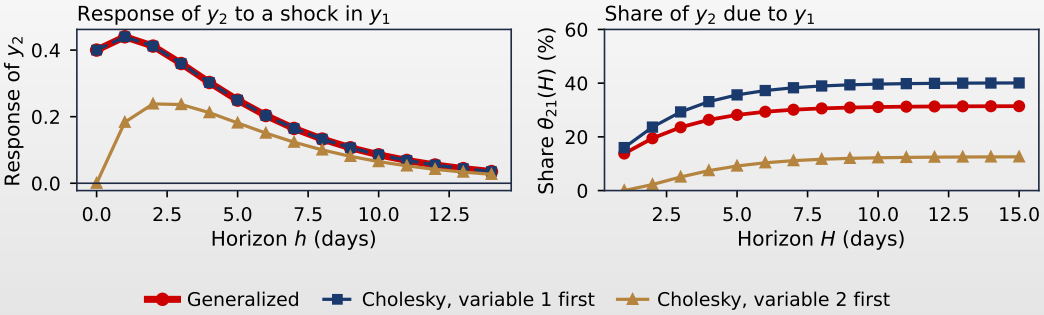
$$\psi_j^g(h) = \Phi_h \Sigma e_j / \sqrt{\sigma_{jj}}$$

  - $\Sigma e_j / \sqrt{\sigma_{jj}} = \mathbb{E}[\varepsilon_t \mid \varepsilon_{j,t} = \sqrt{\sigma_{jj}}]$ for Normal shocks: the correlated banks move too; independent of the ordering
- **Cholesky** (orthogonalised) response: $\Sigma = PP^\top$, P lower triangular

$$\psi_j^c(h) = \Phi_h P e_j$$

  - the first variable takes the whole common shock, the last none: the ordering decides
  - for the variable ordered first the two responses coincide

Impulse Responses of a Simulated VAR(1)



- Two variables: $A = \begin{pmatrix} 0.7 & 0.1 \\ 0.2 & 0.6 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & 0.4 \\ 0.4 & 1 \end{pmatrix}$
  - left: with y_2 first, y_2 does not react on impact ($h = 0$); generalized and Cholesky with y_1 first coincide
  - right: share of y_2 due to y_1 at $H = 10$: 40% or 12% by Cholesky ordering, 31% generalized

Forecast Error Variance Decomposition (FEVD)

- Forecast error of y_i at H steps and its variance

$$y_{t+H} - \mathbb{E}_t[y_{t+H}] = \sum_{h=0}^{H-1} \Phi_h \varepsilon_{t+H-h}, \qquad \mathrm{Var}_i(H) = \sum_{h=0}^{H-1} e_i^\top \Phi_h \Sigma \Phi_h^\top e_i$$

- **Generalized FEVD** (Forecast Error Variance Decomposition) Pesaran & Shin (1998): share of that variance due to shocks in j

$$\theta_{ij}(H) = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{H-1} \left(e_i^\top \Phi_h \Sigma e_j\right)^2}{\mathrm{Var}_i(H)}, \qquad \tilde{\theta}_{ij}(H) = \frac{\theta_{ij}(H)}{\sum_{k=1}^{N} \theta_{ik}(H)}$$

  - correlated shocks are counted for every bank they hit, so $\sum_j \theta_{ij} > 1$; the rows are normalised to sum to 1 Diebold & Yilmaz (2014)

- **Cholesky FEVD**: the same with Pe_j in place of $\Sigma e_j / \sqrt{\sigma_{jj}}$

$$\theta_{ij}^c(H) = \sum_{h=0}^{H-1} \left(e_i^\top \Phi_h P e_j\right)^2 \Big/ \mathrm{Var}_i(H)$$

  - rows sum to 1 exactly, but the shares depend on the ordering (B2); with diagonal Σ both coincide

Connectedness Measures

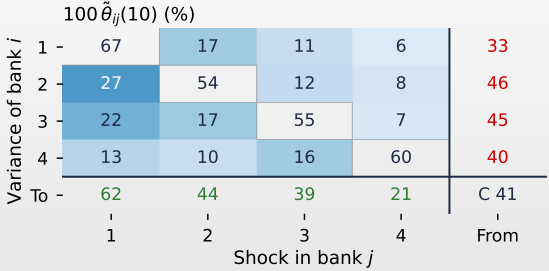
- From the normalised table $\tilde{\theta}_{ij}$ (rows: receivers i, columns: sources j) Diebold & Yilmaz (2014)

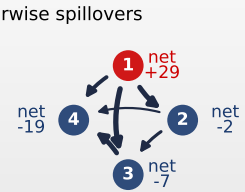
$$F_i = \sum_{j \neq i} \tilde{\theta}_{ij}, \qquad T_j = \sum_{i \neq j} \tilde{\theta}_{ij}, \qquad \mathrm{Net}_i = T_i - F_i, \qquad C = \frac{100}{N} \sum_{i \neq j} \tilde{\theta}_{ij}$$

  - F_i: received from others (row sum off the diagonal); T_j: transmitted to others (column sum)
  - $\mathrm{Net}_i > 0$: net transmitter; C: total connectedness, the share (%) of all forecast error variance that crosses banks

- Example: two banks, rows $(0.7, 0.3)$ and $(0.2, 0.8)$
  - $F_1 = 0.3$, $F_2 = 0.2$; $T_1 = 0.2$, $T_2 = 0.3$
  - Net: bank 1 -0.1 (receiver), bank 2 $+0.1$ (transmitter); $C = \frac{100}{2}(0.3 + 0.2) = 25\%$

A Connectedness Table and Its Network





Net transmitter ($T_i - F_i > 0$) Net receiver Arrow $j \to i$: $\tilde{\theta}_{ij} > \tilde{\theta}_{ji}$

- A simulated VAR(1) of four banks in which bank 1 drives the others with a lag; $H = 10$
  - Left: the table in %; last column F_i, last row T_j, corner $C = 41\%$
  - Right: bank 1 is the only net transmitter (+29); arrows show the net pairwise flows $\tilde{\theta}_{ij} - \tilde{\theta}_{ji}$

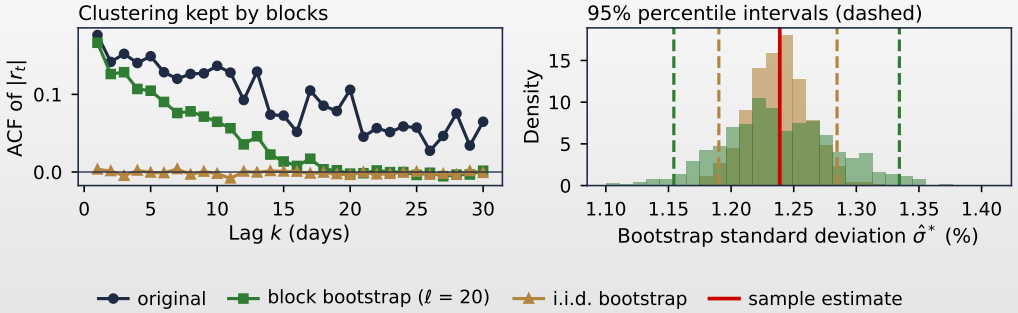
The Moving-Block Bootstrap

- Sampling uncertainty of an estimate $\hat{\theta}$ (e.g. ΔCoVaR, C) without a formula
  - $\hat{\theta}_b^*$: the estimate recomputed on bootstrap sample $b = 1, \ldots, B$ ($B = 999$)
- **Moving blocks** Künsch (1989): glue $\lceil n/\ell \rceil$ blocks $(X_s, \ldots, X_{s+\ell-\mathbf{1}})$ with random starts s
  - $\ell = 20$ days keeps the volatility clustering inside each block; the independent and identically distributed (i.i.d.) bootstrap ($\ell = 1$) destroys it
  - whole days (all banks together) are resampled, so cross-bank dependence is kept
  - VAR (B3): blocks of residuals $\hat{\varepsilon}_t$, then $y_t^* = \hat{c} + \sum_l \hat{A}_l y_{t-l}^* + \varepsilon_t^*$
- What we report: a Confidence Interval (CI)

$$\mathrm{CI}_{\mathbf{95\%}} = \left[\hat{\theta}^*_{(\mathbf{2.5\%})},\ \hat{\theta}^*_{(\mathbf{97.5\%})}\right], \qquad \mathrm{se} = \mathrm{sd}(\hat{\theta}^*_{\mathbf{1}}, \ldots, \hat{\theta}^*_B), \qquad z = \hat{\theta}/\mathrm{se}$$

  - percentile interval; bootstrap standard error; for $H_{\mathbf{0}}$: $\theta = 0$, $p = 2\,(1 - \Phi(|z|))$, reject at 5% if $|z| > 1.96$

Blocks against i.i.d. Resampling



- A simulated GARCH series ($T = 2\,000$ days) with volatility clustering
  - Left: block resamples keep the autocorrelation of $|r_t|$ up to about $\ell = 20$ lags; i.i.d. resamples have none
  - Right: the block interval for the standard deviation is about twice as wide ($[1.15, 1.33]$ against $[1.19, 1.29]$): the i.i.d. bootstrap overstates the precision

Many Tests at Once: Holm, Wald and Spearman

- **Step-down correction** Holm (1979): m tests, p-values sorted $p_{(1)} \leq \cdots \leq p_{(m)}$

$$\text{reject } H_{(k)} \text{ while } p_{(k)} \leq \frac{0.05}{m-k+1}, \quad k = 1, 2, \ldots; \text{ stop at the first failure}$$

  - family-wise error rate (FWER) $\leq 5\%$: probability of at least one false rejection, valid under any dependence

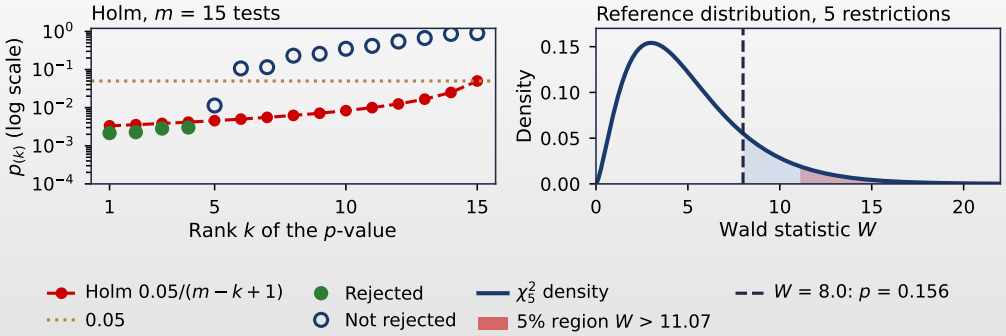
- **Wald test** that m contributions are equal (B1, B4)

$$W = \hat{d}^{\top} \hat{V}^{-1} \hat{d} \;\sim\; \chi^2_{m-1} \text{ under } H_0$$

  - $\hat{d}$: the $m-1$ differences $\Delta\widehat{\mathrm{CoVaR}}_i - \Delta\widehat{\mathrm{CoVaR}}_m$; $\hat{V}$: their covariance across common bootstrap draws
  - uses the correlation of the estimates, which separate intervals ignore; reject if W exceeds the 95% quantile of χ^2_{m-1}

- **Spearman** rank correlation of two rankings of n banks (B4)
  - $r_S = 1 - 6\sum_i d_i^2 / \big(n(n^2-1)\big)$, d_i: difference of the two ranks of bank i; 1: same order, -1: reversed

Holm Thresholds and the χ^2 Reference Distribution



- Left: 15 simulated p-values (4 from false nulls); 5 are below 0.05, Holm rejects 4
  - the threshold rises from $0.05/15 = 0.0033$ to 0.05; testing stops at the first point above its threshold
- Right: χ^2_5, the Wald reference for six banks; 5% critical value 11.07
  - p-value = area to the right of the observed W: $W = 8.0$ gives $p = 0.156$, no rejection

Extension Packs: CoVaR Variants and Tests

- CoVaR conditional on losses **at least** VaR Girardi & Ergün (2013) (A4, B5)

$$P\left(X^{sys} \leq -\mathrm{CoVaR}_{\alpha}^{\leq} \mid X_i \leq q_{\alpha}(X_i)\right) = \alpha$$

  - conditions on the whole tail of the bank, not on one point; larger than CoVaR when $\rho > 0$

- Kupiec Likelihood Ratio (LR) test of a hit rate Kupiec (1995), applied to systemic measures by Banulescu-Radu, Hurlin, Leymarie & Scaillet (2021): x hits in n days, nominal rate π_0, $\hat{\pi} = x/n$

$$LR = -2\left[x \ln \pi_0 + (n-x)\ln(1-\pi_0) - x \ln \hat{\pi} - (n-x)\ln(1-\hat{\pi})\right] \;\sim\; \chi^2_1$$

  - reject at 5% if $LR > 3.84$; for a joint CoVaR event $\pi_0 = \alpha^2$; power: probability of rejecting a wrong rate

- Newey–West standard errors Newey & West (1987) (B8): OLS with autocorrelated errors

$$\hat{\Omega} = \hat{\Gamma}_0 + \sum_{l=1}^{L} \left(1 - \frac{l}{L+1}\right)\left(\hat{\Gamma}_l + \hat{\Gamma}_l^{\top}\right), \qquad \hat{\Gamma}_l = \frac{1}{T}\sum_t \hat{u}_t \hat{u}_{t-l}\, x_t x_{t-l}^{\top}$$

  - $\hat{u}_t$: residual, x_t: regressors, L: lags; $\widehat{\mathrm{Var}}(\hat{\beta}) = (X^{\top}X)^{-1}\, T\hat{\Omega}\, (X^{\top}X)^{-1}$; needed for overlapping rolling windows

Extension Packs: GARCH–DCC and Stress Tests

- Glosten–Jagannathan–Runkle (GJR) GARCH(1,1) (B6, B7): bad news raise the variance more

$$\sigma_t^{\mathbf{2}} = \omega + \left(\alpha_G + \gamma\, 1\{r_{t-\mathbf{1}} < 0\}\right) r_{t-\mathbf{1}}^{\mathbf{2}} + \beta_G\, \sigma_{t-\mathbf{1}}^{\mathbf{2}}$$

  - $\omega > 0$; α_G: reaction to news; $\gamma > 0$: extra reaction to losses; β_G: memory; persistence $\alpha_G + \gamma/2 + \beta_G < 1$; the subscript G separates α_G from the tail probability α
- DCC(1,1): a time-varying correlation of the standardised residuals ε_t
  - $Q_t = (1 - a - b)\bar{Q} + a\, \varepsilon_{t-\mathbf{1}} \varepsilon_{t-\mathbf{1}}^{\top} + b\, Q_{t-\mathbf{1}}$, $\rho_t = Q_{\mathbf{12},t} / \sqrt{Q_{\mathbf{11},t} Q_{\mathbf{22},t}}$; $\bar{Q}$: sample correlation of ε_t
- Reverse stress test (A7, A8): the most likely factor move f with loss ℓ^*, $f \sim N(0, \Sigma)$
  - maximise the density, i.e. minimise the squared Mahalanobis distance $d^{\mathbf{2}} = f^{\top} \Sigma^{-\mathbf{1}} f$, subject to $-b^{\top} f = \ell^*$ (b: exposures)
  - d: distance in standard deviations that accounts for correlation; Student-t factors (A8): same direction, heavier tail probability

Exercise Map

Exercise	Question	Type	Model
A1	ΔCoVaR under the Normal distribution	Solved	–
A2	why MES adds up to the portfolio ES	Solved	–
A3	the generalized variance decomposition	Solved	–
B1	do US banks differ in ΔCoVaR 5%?	Solved	–
B2, B3	robustness and intervals of connectedness	Solved	–
C1	what is wrong in an AI answer?	Proposed	A1, A2
B4	European banks: which is most systemic?	Proposed, homework	B1
Extension packs	A4–A8, B5–B10, C2, C3 (optional, after the lecture)	Solved and Proposed (marked on each task)	named on each task

- **[Solved]**: full solution in the slides and in the notebook, a model to copy; **[Proposed]**: you solve it, the solution is discussed in class
- Part A: derivations on paper; Part B: estimation and inference on real data, each ending with an interpretation question; Part C: critique of an AI answer and open questions

A1 [Solved]: Deriving CoVaR under the Normal Distribution — Task

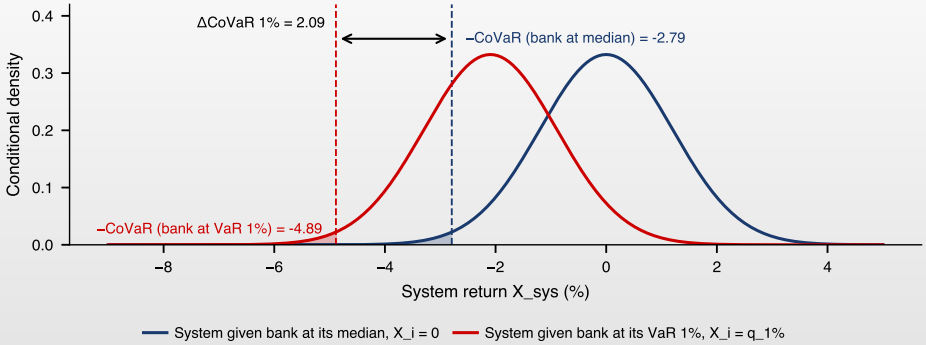
- **Question**: How much does a bank in distress add to the tail loss of the system when returns are Normal?
- **Data and assumptions**
  - (X_i, X^{sys}): daily returns (%) of bank i and of the system, bivariate Normal with zero means, standard deviations σ_i, σ_{sys} and correlation $\rho > 0$
  - Distress of the bank: $X_i = q_\alpha(X_i) = \sigma_i z_\alpha$, with $z_\alpha = \Phi^{-1}(\alpha) < 0$; median state: $X_i = 0$; CoVaR is minus the α-quantile of X^{sys}, a positive loss Adrian & Brunnermeier (2016)
  - Numbers for (c): $\sigma_i = 2\%$, $\sigma_{sys} = 1.5\%$, $\rho = 0.6$, $\alpha = 1\%$
- **Tasks**
  - (a) Project X^{sys} on X_i and derive the conditional distribution of X^{sys} given $X_i = x$ (mean and variance).
  - (b) Derive CoVaR_α (given $X_i = \sigma_i z_\alpha$), the median-state CoVaR (given $X_i = 0$) and $\Delta\mathrm{CoVaR}_\alpha$ as their difference; show that $\Delta\mathrm{CoVaR}_\alpha$ does not depend on σ_i.
  - (c) Compute VaR 1% of the bank, CoVaR 1%, the median-state CoVaR and ΔCoVaR 1% for the numbers above.
- **Report**
  - The conditional mean and standard deviation, the three closed forms of (b) and the four numbers of (c), with every intermediate product
- **Interpretation**
  - A bank twice as volatile, with the same ρ, has the same ΔCoVaR: what does this say about ranking banks by their own VaR?
- Notebook: section A1

Back Exercise list

A1 [Solved]: Solution (1/3)

- (a) Projection: $X^{sys} = \beta X_i + U$ with $\beta = \mathrm{Cov}(X^{sys}, X_i)/\sigma_i^2 = \rho\sigma_{sys}/\sigma_i$ and U Normal, independent of X_i.
- Residual variance: $\mathrm{Var}(U) = \sigma_{sys}^2 - \beta^2\sigma_i^2 = \sigma_{sys}^2(1-\rho^2)$.
- Result: $X^{sys} \mid X_i = x \sim N\big(\rho\frac{\sigma_{sys}}{\sigma_i}x,\ \sigma_{sys}^2(1-\rho^2)\big)$.
- (b) The α-quantile given x is $\rho\frac{\sigma_{sys}}{\sigma_i}x + \sigma_{sys}\sqrt{1-\rho^2}\,z_\alpha$.
- At $x = \sigma_i z_\alpha$: $\mathrm{CoVaR}_\alpha = -\sigma_{sys}z_\alpha(\rho + \sqrt{1-\rho^2})$; at $x = 0$: $\mathrm{CoVaR}_\alpha^{50\%} = -\sigma_{sys}\sqrt{1-\rho^2}\,z_\alpha$.
- Difference: $\Delta\mathrm{CoVaR}_\alpha = -\rho\,\sigma_{sys}z_\alpha$; σ_i cancels because the distress state x grows with σ_i while the slope falls with $1/\sigma_i$.

A1 [Solved]: Solution (2/3)



- Blue: system given the bank at its median; red: given the bank at its VaR 1%; shaded: the 1% tails; the gap between the dashed lines is ΔCoVaR 1%

A1 [Solved]: Solution (3/3)

- (c) $z_{\mathbf{1\%}} = -2.326$; VaR 1% of the bank: $-\sigma_i z_{\mathbf{1\%}} = 2 \cdot 2.326 = 4.65\%$.
- Slope $\rho\sigma_{sys}/\sigma_i = 0.6 \cdot 1.5/2 = 0.45$; conditional standard deviation $1.5\sqrt{1 - 0.36} = 1.20\%$.
- CoVaR 1%: $0.45 \cdot 4.65 + 1.20 \cdot 2.326 = 2.094 + 2.792 = 4.885\%$.
- Median state: $1.20 \cdot 2.326 = 2.792\%$; so $\Delta\mathrm{CoVaR}_{\mathbf{1\%}} = 0.6 \cdot 1.5 \cdot 2.326 = 2.094\%$, the first term above.
- Interpretation: when the bank moves from its median to its VaR, the system loses about 2.09 extra points; doubling σ_i doubles the VaR but leaves ΔCoVaR unchanged.
- Own risk and contribution to systemic risk are different quantities: a ranking by VaR is not a systemic ranking.

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A2 [Solved]: MES as an Euler Contribution — Task

- **Question**: Why do the MES of the assets add up exactly to the ES of the portfolio?
- **Data and assumptions**
  - Portfolio $X_p = \sum_i w_i X_i$ of daily returns with a continuous distribution; $q_\alpha(w)$ is its α-quantile
  - $\mathrm{ES}_\alpha(X_p) = -\frac{1}{\alpha}\mathbb{E}[X_p 1\{X_p \le q_\alpha\}]$ and $\mathrm{MES}_{i,\alpha} = -\mathbb{E}[X_i \mid X_p \le q_\alpha]$ Acharya et al. (2017)
  - Numbers for (c): jointly Normal, zero means, $w = (0.5, 0.5)$, $\sigma = (2\%, 3\%)$, $\rho = 0.5$, $\alpha = 5\%$
- **Tasks**
  - (a) Differentiate ES_α with respect to w_i, including the effect of w_i on the quantile, to show $\partial \mathrm{ES}_\alpha / \partial w_i = \mathrm{MES}_{i,\alpha}$; then use homogeneity to show $\mathrm{ES}_\alpha = \sum_i w_i \mathrm{MES}_{i,\alpha}$.
  - (b) For jointly Normal returns with zero means, show that $\mathrm{MES}_{i,\alpha} = \beta_{i,p}\mathrm{ES}_\alpha(X_p)$ with $\beta_{i,p} = \mathrm{Cov}(X_i, X_p)/\sigma_p^2$ and $\mathrm{ES}_\alpha(X_p) = \sigma_p \varphi(z_\alpha)/\alpha$.
  - (c) Compute σ_p, $\mathrm{ES}_{5\%}$, both betas and both MES for the numbers above, and check the Euler sum.
- **Report**
  - σ_p, $\varphi(z_{5\%})/0.05$, $\mathrm{ES}_{5\%}$, β, MES and the check $\sum_i w_i \mathrm{MES}_i = \mathrm{ES}_{5\%}$
- **Interpretation**
  - Asset 2 has half the total portfolio weight: what share of the ES does it carry?
  - Why is MES, not volatility, the right key for allocating tail risk?
- Notebook: section A2

Back Exercise list

A2 [Solved]: Solution (1/2)

- (a) Write $\alpha \mathrm{ES}_\alpha = -\mathbb{E}[X_p 1\{X_p \le q(w)\}]$; differentiating under the expectation gives the direct term $-\mathbb{E}[X_i 1\{X_p \le q\}]$.
- The boundary term is $-q \frac{\partial}{\partial w_i} P(X_p \le q(w))$: the mass that crosses the boundary, where $X_p = q$.
- $P(X_p \le q(w)) = \alpha$ for every w, so this derivative is zero and $\partial \mathrm{ES}_\alpha / \partial w_i = -\frac{1}{\alpha}\mathbb{E}[X_i 1\{X_p \le q\}] = \mathrm{MES}_{i,\alpha}$.
- Homogeneity: $\mathrm{ES}_\alpha(tX_p) = t\,\mathrm{ES}_\alpha(X_p)$, so Euler's theorem gives $\mathrm{ES}_\alpha = \sum_i w_i\, \partial \mathrm{ES}_\alpha / \partial w_i = \sum_i w_i \mathrm{MES}_{i,\alpha}$ Gouriéroux, Laurent & Scaillet (2000).
- (b) Normal regression: $\mathbb{E}[X_i \mid X_p] = \beta_{i,p} X_p$, so $\mathrm{MES}_{i,\alpha} = -\beta_{i,p}\mathbb{E}[X_p \mid X_p \le q] = \beta_{i,p}\mathrm{ES}_\alpha(X_p)$.
- For $X_p \sim N(0, \sigma_p^2)$, $\mathrm{ES}_\alpha = \sigma_p \varphi(z_\alpha)/\alpha$; and $\sum_i w_i \beta_{i,p} = w^\top \Sigma w / \sigma_p^2 = 1$ confirms the Euler sum.

A2 [Solved]: Solution (2/2)

- (c) Covariance $\rho\sigma_1\sigma_2 = 0.5 \cdot 2 \cdot 3 = 3$; $\sigma_p^2 = 0.25 \cdot 4 + 2 \cdot 0.25 \cdot 3 + 0.25 \cdot 9 = 4.75$, so $\sigma_p = 2.179\%$.
- $\varphi(z_{5\%})/0.05 = 2.063$, so $\mathrm{ES}_{5\%} = 2.179 \cdot 2.063 = 4.50\%$.
- $\mathrm{Cov}(X_i, X_p) = (\Sigma w)_i = (3.5, 6.0)$; $\beta = (3.5, 6.0)/4.75 = (0.737, 1.263)$.
- $\mathrm{MES} = \beta \cdot 4.50 = (3.31, 5.68)\%$; check: $0.5(3.31 + 5.68) = 4.50\% = \mathrm{ES}_{5\%}$.
- Interpretation: with half the weight, asset 2 carries 63% of the ES, because it moves more with the portfolio in its tail.
- MES splits the tail loss additively and uses the co-movement with the portfolio, which volatility alone ignores.

A3 [Solved]: Deriving the Generalized FEVD — Task

- **Question**: How is the forecast error variance of one bank split between shocks to all banks?
- **Data and assumptions**
  - Stable VAR(1) $y_t = Ay_{t-\mathbf{1}} + \varepsilon_t$ (eigenvalues of A inside the unit circle), ε_t serially uncorrelated, zero-mean and jointly Normal with positive definite $\mathrm{Cov}(\varepsilon_t) = \Sigma = (\sigma_{ij})$; moving-average matrices $\Phi_h = A^h$; e_j is the j-th unit vector
  - Generalized impulse Pesaran & Shin (1998): a one-standard-deviation shock $\delta_j = \sqrt{\sigma_{jj}}$ to variable j, the other shocks at their conditional expectation
  - Numbers for (b): $A = \begin{pmatrix} 0.5 & 0.2 \\ 0.1 & 0.4 \end{pmatrix}$, $\Sigma = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$
- **Tasks**
  - (a) Show that $\mathbb{E}[\varepsilon_t \mid \varepsilon_{jt} = \delta_j] = \Sigma e_j / \sqrt{\sigma_{jj}}$ and derive the share $\theta_{ij}(H)$ of the H-step forecast error variance of i due to shocks in j.
  - (b) Compute θ, its row sums, the row-normalised table $\tilde{\theta}$ and total connectedness C at $H = 1$ and $H = 2$.
  - (c) Compute the Cholesky decomposition at $H = 1$ with variable 1 first, and show that with Σ diagonal the generalized and Cholesky decompositions coincide.
- **Report**
  - θ and $\tilde{\theta}$ with row sums and C at $H = 1, 2$, with $A\Sigma$ and $\mathrm{diag}(A\Sigma A^\top)$; the Cholesky table
- **Interpretation**
  - Why do the rows of the generalized table not sum to one?
  - What does the Cholesky order decide?
- Notebook: section A3

Back Exercise list

A3 [Solved]: Solution (1/3)

- (a) For Normal shocks $\mathbb{E}[\varepsilon_t \mid \varepsilon_{jt}] = \Sigma e_j \, \varepsilon_{jt} / \sigma_{jj}$; at $\varepsilon_{jt} = \sqrt{\sigma_{jj}}$ this is $\Sigma e_j / \sqrt{\sigma_{jj}}$.
- Response of variable i after h steps: $e_i^\top \Phi_h \Sigma e_j / \sqrt{\sigma_{jj}}$; the H-step forecast error variance of i is $\sum_{h<H} e_i^\top \Phi_h \Sigma \Phi_h^\top e_i$.
- Share: $\theta_{ij}(H) = \dfrac{\sigma_{jj}^{-1} \sum_{h<H} (e_i^\top \Phi_h \Sigma e_j)^2}{\sum_{h<H} e_i^\top \Phi_h \Sigma \Phi_h^\top e_i}$.
- Normalisation $\tilde{\theta}_{ij} = \theta_{ij} / \sum_j \theta_{ij}$ and total connectedness $C = \frac{100}{N} \sum_{i \neq j} \tilde{\theta}_{ij}$ Diebold & Yilmaz (2014).

A3 [Solved]: Solution (2/3)

- (b) $H = 1$ ($\Phi_0 = I$): $\theta_{ij} = \sigma_{ij}^2/(\sigma_{ii}\sigma_{jj})$, so $\theta = \begin{pmatrix} 1 & 0.25 \\ 0.25 & 1 \end{pmatrix}$, row sums 1.25.
- Normalised: $\tilde{\theta} = \begin{pmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{pmatrix}$, so $C = 100(0.2 + 0.2)/2 = 20.0\%$.
- $H = 2$ adds $\Phi_1 = A$: $A\Sigma = \begin{pmatrix} 0.60 & 0.45 \\ 0.30 & 0.45 \end{pmatrix}$, $\mathrm{diag}(A\Sigma A^\top) = (0.39, 0.21)$; denominators 1.39 and 1.21.
- Numerators $\sigma_{ij}^2 + (A\Sigma)_{ij}^2$: row 1 $(1.3600, 0.4525)$, row 2 $(0.3400, 1.2025)$; so $\theta = \begin{pmatrix} 0.978 & 0.326 \\ 0.281 & 0.994 \end{pmatrix}$, row sums 1.304 and 1.275.
- Normalised off-diagonal shares 0.250 and 0.220, so $C = 100(0.250 + 0.220)/2 = 23.5\%$.

A3 [Solved]: Solution (3/3)

- (c) Cholesky with variable 1 first: $P = \begin{pmatrix} 1 & 0 \\ 0.5 & \sqrt{0.75} \end{pmatrix}$, $PP^\top = \Sigma$; at $H = 1$ the shares are the squared entries of the rows of P: $(1, 0)$ and $(0.25, 0.75)$.
- The whole common shock is given to variable 1, the first in the order.
- Diagonal Σ: $\Sigma e_j / \sqrt{\sigma_{jj}} = \sqrt{\sigma_{jj}}\, e_j = P e_j$ with $P = \Sigma^{\mathbf{1/2}}$ the diagonal Cholesky factor: the impulses coincide and the rows already sum to one.
- Interpretation: rows of θ exceed one because a correlated shock is counted for both variables; normalisation fixes the sum without imposing an order.
- With Cholesky the order, not the data, decides who receives the common shock; the lagged links of A raise C from 20.0% to 23.5% between $H = 1$ and $H = 2$.

B1 [Solved]: ΔCoVaR 5% of US Banks, with Tests — Task (1/2)

- **Question**: Do the six United States (US) banks differ in their contribution to systemic risk?
- **Data and assumptions**
  - Adjusted closes of six US banks (course dataset): JPMorgan, Bank of America, Citigroup, Goldman Sachs, Morgan Stanley, Wells Fargo
  - Sample: 5 January 2010 – 18 September 2026, 3 961 log returns (%) between trading dates common to all 13 banks and the four indices (prices aligned on common dates first)
  - System of bank i: the equal-weighted portfolio of the other five banks Adrian & Brunnermeier (2016); level $\alpha = 5\%$, about 198 tail days
- **Tasks**
  (a) For each bank, estimate the quantile regression $q_{\mathbf{5\%}}(X_t^{sys} \mid X_{i,t}) = a_{\mathbf{5\%}} + b_{\mathbf{5\%}} X_{i,t}$ and compute VaR 5%, CoVaR 5% and ΔCoVaR 5% with the plug-in formulas of Estimating CoVaR by Quantile Regression.
  (b) Build 95% percentile intervals for ΔCoVaR with a moving-block bootstrap (blocks of 20 days) Künsch (1989) and with an i.i.d. bootstrap, 999 resamples each.

Back Exercise list

B1 [Solved]: ΔCoVaR 5% of US Banks, with Tests — Task (2/2)

- **Tasks (continued)**
  - (c) Test $H_0 : b_{5\%} = b_{50\%}$ for each bank with $z = (\hat{b}_{5\%} - \hat{b}_{50\%})/\mathrm{se}$ (block-bootstrap se) at the 5% level, with and without the Holm correction Holm (1979).
  - (d) Test equal ΔCoVaR for the 15 pairs on joint draws (the same resampled days for all banks) with Holm, and for all six banks jointly with a Wald test (χ^2_5).
- **Report**
  - A table with $\hat{b}_{5\%}$, ΔCoVaR, both intervals, $\hat{b}_{5\%} - \hat{b}_{50\%}$ and its p-value; a chart of the two intervals per bank
  - The number of rejections in (c) and (d), raw and after Holm; the Wald statistic, its p-value and the decision
- **Interpretation**
  - Why is the block interval wider than the i.i.d. one?
  - Can overlapping marginal intervals hide a significant difference?
  - Why is bank i left out of its own system?
- Notebook: section B1

B1 [Solved]: Solution (1/4)

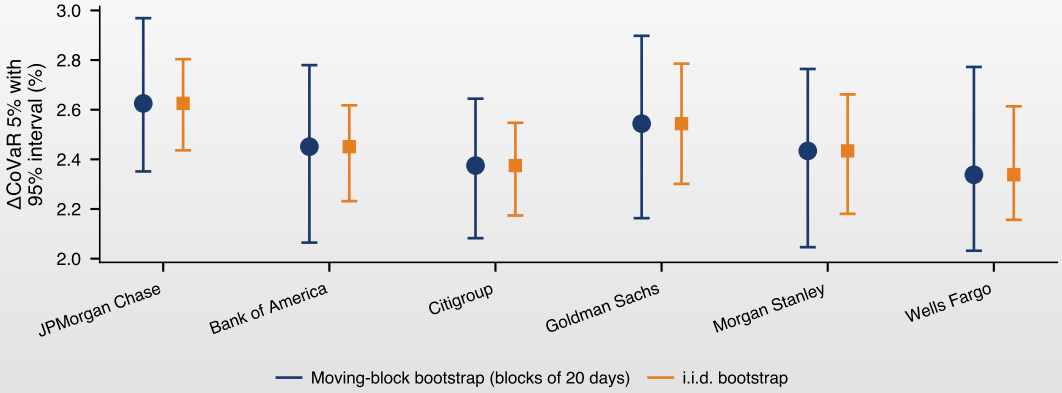
- Step 1: for each bank the system return is $X_t^{sys} = \frac{1}{5}\sum_{j\neq i} X_{j,t}$; estimate the quantile regressions at 5% and at 50%.
- Step 2: plug in the empirical quantiles: $\mathrm{CoVaR}_{5\%} = -(\hat{a} + \hat{b}\hat{q}_{5\%})$ and $\Delta\mathrm{CoVaR}_{5\%} = \hat{b}(\hat{q}_{50\%} - \hat{q}_{5\%})$.
- Example, JPMorgan: VaR 5% $= 2.66\%$, $\hat{q}_{50\%} - \hat{q}_{5\%} = 2.7365$, $\hat{b}_{5\%} = 0.9594$, so $\Delta\mathrm{CoVaR}_{5\%} = 0.9594 \cdot 2.7365 = 2.63\%$; CoVaR 5% $= 3.81\%$.
- Step 3: draw 999 samples of 20-day blocks (block bootstrap) and 999 samples of single days (i.i.d.), re-estimate on each and take the 2.5% and 97.5% percentiles.
- Step 4: in each block draw compute all six $\hat{b}_{5\%} - \hat{b}_{50\%}$ and all six ΔCoVaR on the same days; the standard deviations of the draws are the standard errors, their covariance feeds the Wald test.

B1 [Solved]: Solution (2/4)

	$\hat{b}_{5\%}$	ΔCoVaR	block 95% interval	i.i.d. 95% interval	$\hat{b}_{5\%} - \hat{b}_{50\%}$	p
JPMorgan	0.96	2.63	[2.35, 2.97]	[2.44, 2.80]	−0.002	0.872
Bank of America	0.76	2.45	[2.06, 2.78]	[2.23, 2.62]	−0.017	0.426
Citigroup	0.72	2.38	[2.08, 2.64]	[2.17, 2.55]	−0.040	0.015
Goldman Sachs	0.89	2.54	[2.16, 2.90]	[2.30, 2.79]	0.044	0.045
Morgan Stanley	0.72	2.43	[2.05, 2.76]	[2.18, 2.66]	−0.009	0.751
Wells Fargo	0.81	2.34	[2.03, 2.77]	[2.16, 2.61]	0.032	0.206

- (a) All six contributions lie between 2.34% and 2.63% of extra system loss; slopes lie between 0.72 and 0.96.
- (b) Block intervals are wider (JPMorgan: width 0.62 against 0.37 points): tail days cluster in crises, and the i.i.d. bootstrap breaks the clusters apart.

B1 [Solved]: Solution (3/4)



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B1 [Solved]: Solution (4/4)

- (c) Citigroup: $z = -0.0398/0.0164 \approx -2.42$, $p = 0.015$, rejected at 5%; JPMorgan: $z = -0.16$, $p = 0.872$, not rejected; overall, the null is rejected for 2 of 6 banks, and for 0 after Holm.
- (d) 2 of 15 pairs differ at 5%, 0 after Holm; Wald $\chi^2_5 = 13.8$, $p = 0.017 < 0.05$: we reject that the six contributions are all equal.
- Interpretation: the joint test sees differences that overlapping marginal intervals hide, because it uses the positive correlation of the estimates across banks.
- Bank i is left out of its system: otherwise X_i would appear on both sides of the regression and add a mechanical self-contribution to $\hat{b}$ (with w the weight of the bank in the system and b the slope on the rest, the slope becomes $w + (1 - w)b$).

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B2 [Solved]: How Robust Is the Connectedness Index? — Task

- **Question**: How much do modelling choices move the connectedness index?
- **Data and assumptions**
  - Daily high and low prices of the 13 banks (course dataset; six US, five European, Banca Transilvania and BRD), 2010–2026, common days
  - Parkinson volatility $\hat{\sigma}_t$ of Parkinson Volatility from the Daily Range, used in logs; order of the variables: US banks first, Romanian banks last
  - Generalized connectedness Diebold & Yilmaz (2014) with the decomposition of Pesaran & Shin (1998) (A3)
- **Tasks**
  (a) Estimate VAR(p) by OLS for $p = 1, \ldots, 4$ and compute total connectedness C at $H = 5, 10, 20$ (12 values).
  (b) With $p = 1$, $H = 10$, compute C and the summed net position of the six US banks under the generalized decomposition and under Cholesky in the listed order (US first), then with the entire ordering reversed (Romanian first).
- **Report**
  - The 4×3 table and chart of (a); the three totals and three net positions of (b)
- **Interpretation**
  - Which choice moves the index most?
  - Why does the Cholesky order change the net position of US banks?
- Notebook: section B2

Back Exercise list

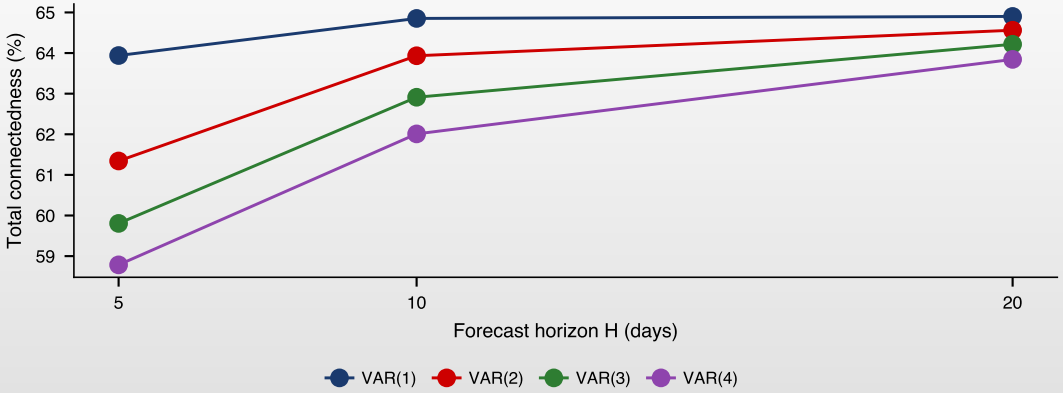
B2 [Solved]: Solution (1/3)

- Steps: (1) log Parkinson volatilities; (2) VAR(p) with a constant by OLS and the residual covariance $\hat{\Sigma}$; (3) Φ_h for $h < H$; (4) generalized shares, rows normalised; (5) $C = \frac{100}{13} \sum_{i \neq j} \tilde{\theta}_{ij}$.

C (%)	$H = 5$	$H = 10$	$H = 20$
VAR(1)	63.9	64.9	64.9
VAR(2)	61.3	63.9	64.6
VAR(3)	59.8	62.9	64.2
VAR(4)	58.8	62.0	63.8

- (a) The index moves between 58.8% ($p = 4$, $H = 5$) and 64.9% ($p = 1$, $H = 20$): a range of 6.1 points over all 12 choices.

B2 [Solved]: Solution (2/3)



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B2 [Solved]: Solution (3/3)

- (b) Totals: Cholesky 44.1% (US first) and 43.5% (Romanian first) against generalized 64.9%.
- Net position of the US banks: $+65.9$ (US first), -96.0 (Romanian first), $+53.2$ (generalized).
- Interpretation: the decomposition moves the index most; Cholesky gives the common shock to the variables ordered first, so the order, not the data, decides who transmits: use the generalized decomposition.

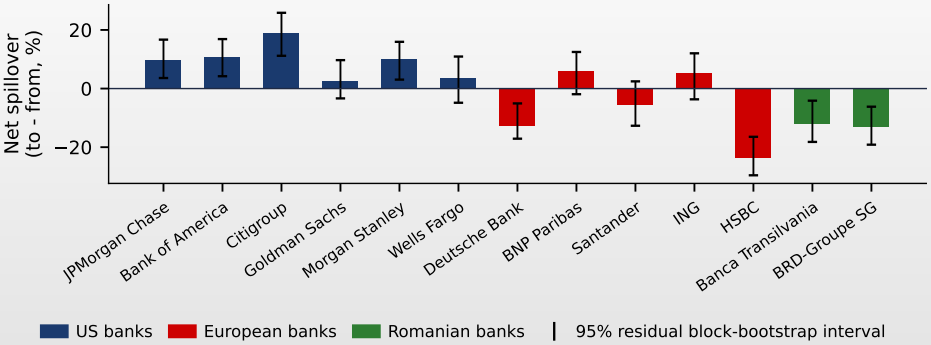
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B3 [Solved]: Which Net Spillovers Are Identified? — Task

- **Question**: Which transmitter and receiver labels of the network survive estimation error?
- **Data and assumptions**
  - Log Parkinson volatilities of the 13 banks (B2), 2010–2026; VAR order $p = 2$ chosen by BIC (up to 5 lags), $H = 10$, generalized decomposition Diebold & Yilmaz (2014)
  - Peak window: the 250 days ending on 2020-08-27, the day on which the rolling total connectedness over 250-day windows (VAR(2), $H = 10$) reaches its maximum
- **Tasks**
  (a) Build 999 bootstrap samples: resample the complete 13-bank residual vectors in common blocks of 20 days, rebuild the series with the estimated VAR, re-estimate the VAR and recompute the connectedness table.
  (b) Report 95% percentile intervals for C and for the 13 net positions; call a bank a transmitter (receiver) only if its interval lies above (below) zero.
  (c) Repeat (a)–(b) for C in the peak window and compare the bootstrap mean with the estimate.
- **Report**
  - C with its interval; a chart of the 13 net positions with intervals; the identified banks; the peak-window estimate, interval and bootstrap mean
- **Interpretation**
  - Which "transmitter" and "receiver" labels survive?
  - Why is the bootstrap mean of C in the short window different from the estimate?
- Notebook: section B3

Back Exercise list

B3 [Solved]: Solution (1/2)



- Net positions with 95% intervals; a label is identified only if its interval excludes zero.

B3 [Solved]: Solution (2/2)

- Steps: (1) VAR(2) by OLS, residuals $\hat{\varepsilon}_t$; (2) resample 20-day blocks of $\hat{\varepsilon}_t$; (3) rebuild $y_t^* = \hat{c} + \sum_{l=1}^{2} \hat{A}_l y_{t-l}^* + \varepsilon_t^*$ ($\hat{c}$, $\hat{A}_l$: estimated VAR constants and coefficients, ε_t^*: resampled residuals) from the first 2 observed values; (4) re-estimate, recompute the table; (5) percentiles over 999 draws.
- (a)–(b) Full sample: $C = 63.9\%$, interval $[59.9, 65.8]$.
- 8 of 13 net positions exclude zero: JPMorgan, Bank of America, Citigroup, Morgan Stanley, Deutsche Bank, HSBC (HSBC Holdings plc), Banca Transilvania, BRD-Groupe SG (SG: Société Générale); the intervals are marginal, with no multiple-testing correction; for the other five the sign of the net position is not statistically distinguishable from zero.
- (c) 2020 peak (250 days to 2020-08-27): $C = 79.3\%$, interval $[65.9, 83.0]$, bootstrap mean 75.8%, i.e. -3.5 points from the estimate.
- Interpretation: the gap between the bootstrap mean and the estimate is a bootstrap estimate of finite-sample bias: in short windows the index is biased as well as noisy; sampling noise in the OLS coefficients is one plausible source Demirer et al. (2018).
- Report the interval next to every net number, and draw only identified labels on a network chart.

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C1 [Proposed]: Critique an AI Answer — the Answer

- **Student prompt**: "Bank i and the system are jointly Normal with zero means: $\sigma_i = 2.5\%$, $\sigma_{sys} = 1.2\%$, $\rho = 0.5$. Compute CoVaR 1%, ΔCoVaR 1%, MES 5% (the system is the market) and SRISK with $W = 100$, $D = 1\,100$, $k = 8\%$, LRMES $= 40\%$."
  - W: market value of equity; D: book value of debt; k: prudential capital ratio
- **Answer of the AI assistant**
  - $\mathrm{VaR}_i = 2.5 \cdot 2.326 = 5.82\%$; $X^{sys} \mid X_i = x \sim N(0.24\,x,\ 1.04^2)$
  - CoVaR conditions on the bank at its VaR, $X_i = \mathrm{VaR}_i = 5.82$: the 1% quantile is $0.24 \cdot 5.82 - 1.04 \cdot 2.326 = -1.02$, so $\mathrm{CoVaR} = 1.02\%$
  - Median state: $1.04 \cdot 2.326 = 2.42\%$; $\Delta\mathrm{CoVaR} = 1.02 - 2.42 = -1.40\%$: the bank **reduces** systemic risk
  - MES is the expected loss of the bank in its own 5% tail: $\mathrm{MES} = 2.5 \cdot \varphi(1.645)/0.05 = 5.16\%$
  - $\mathrm{SRISK} = kD - (1 - \mathrm{LRMES})\,W = 88 - 60 = 28$

C1 [Proposed]: Critique an AI Answer — Task

- The answer on the previous slide contains **three errors**; each affects one or more numbers or interpretations
  - Model: A1, A2 and the SRISK formula of Exposure in a Crisis: SRISK; notebook section C1; conventions of this seminar: α is a tail probability, VaR, CoVaR and MES are positive losses
- **Tasks**
  (a) Identify each of the three errors: the wrong step and why it is wrong.
  (b) For each error, state the check that detects it without the correct answer: a sign, a limiting case or a closed form.
  (c) Recompute CoVaR 1%, ΔCoVaR 1%, MES 5% and SRISK.
  (d) Log each error in `AI_ERRORS.md`.
- **Report**
  - A table with the AI claim, the mistake, the check and the correct value for each error, plus the four correct numbers

B4 [Proposed]: ΔCoVaR of European Banks and the VaR Ranking — Task (1/2)

- **Questions**
  - Can we name one European bank "most systemic"?
  - Does its VaR tell us which?
- **Data and assumptions**
  - Adjusted closes of five European banks (course dataset): Deutsche Bank, BNP Paribas SA, Santander, ING (Dutch banking group ING Groep), HSBC; same sample and returns as B1
  - System of each bank: the equal-weighted portfolio of the other four; levels $\alpha = 1\%$ (about 40 tail days) and $\alpha = 5\%$ (about 198)
  - For (c): the eleven international banks (the six US banks of B1 and these five), each with its own regional system
- **Tasks**

(a) Compute VaR 1%, $\hat{b}_{1\%}$, ΔCoVaR 1% with a block-bootstrap interval (blocks of 20 days, 999 resamples) and ΔCoVaR 5%; rank the banks at each level and compute the Spearman correlation of the two rankings.

Back Exercise list

B4 [Proposed]: ΔCoVaR of European Banks and the VaR Ranking — Task (2/2)

- **Tasks (continued)**
  - (b) At each level, test equal ΔCoVaR for the 10 pairs on joint block-bootstrap draws with the Holm correction, and for all five banks jointly with a Wald test (χ^2_4), at 5%.
  - (c) Compute the Spearman correlation of VaR 1% and ΔCoVaR 1% across the eleven banks, with a 95% block-bootstrap interval (blocks of 20 days, 999 resamples).
- **Report**
  - A table of (a); both rankings and their Spearman correlation; the pair counts (raw, Holm), the Wald statistics, *p*-values and decisions; the Spearman correlation of (c) with its interval
- **Interpretation**
  - How many tail days does each level use?
  - What does this do to the power of the tests?
  - Would you name one European bank "most systemic" from these data?
  - Is the VaR–ΔCoVaR correlation different from zero?
  - Is it different from one?
- **Model**: B1 [Solved]; notebook section B4

Team Project

- Choose a financial system not covered in class (e.g. Central and Eastern European banks, insurers, crypto exchanges)
- Estimate MES, ΔCoVaR and a connectedness table, each with block-bootstrap uncertainty; one historical and one reverse stress test
- Justify the institutions and common sample: report missingness, usable observations and tail-event counts
- **Deliverables**: a Quantlet folder (code, Metainfo.txt, charts) and three slides: numbers, interpretation, limitations
- **Assessment criteria**: reproducibility, justified choices, reported uncertainty and limitations; an impressive point estimate alone is not enough
- Log AI use in `AI_USE.md` and caught errors in `AI_ERRORS.md`; extension packs are optional

Extension Packs: What Each One Needs

Pack	Exercises	Prerequisites
1. CoVaR variants and backtests	A4, B5	A1, B1
2. SRISK	B6 [Solved], A5, B7	A2; the lecture section on SRISK
3. Connectedness	A6, B8, C2	A3, B2, B3
4. Tail networks	B9 [Solved]	Lecture 18, Financial Risk Meter
5. Stress testing	A7 [Solved], A8, B10	Lecture 18, reverse stress test
6. Romanian banks	C3	B1, B3

- Each pack has its own notebook sections; do the exercises in the order of the pack
- Useful for the team project: C2 and C3 are possible project starts

A4 [Proposed]: The Girardi–Ergün CoVaR under the Normal Distribution — Task

- **Question**: How much larger is CoVaR when we condition on losses at least equal to VaR?
- **Data and assumptions**
  - Same bivariate Normal model and numbers as A1(c): $\sigma_{sys} = 1.5\%$, $\rho = 0.6$, $\alpha = 1\%$
  - Definition of Girardi & Ergün (2013): distress is $X_i \leq q_\alpha(X_i)$ (at or beyond the VaR), and $\mathrm{CoVaR}_\alpha^{\leq} = c$ solves $P(X^{sys} \leq -c \mid X_i \leq q_\alpha(X_i)) = \alpha$
  - $\Phi_2(u, v; \rho)$: the standard bivariate Normal cumulative distribution function (cdf) with correlation ρ
- **Tasks**
  - (a) Write the conditional probability as a joint probability divided by α, standardise both coordinates and show that c solves $\Phi_2(z_\alpha, -c/\sigma_{sys}; \rho) = \alpha^2$.
  - (b) Prove that $\mathrm{CoVaR}_\alpha^{\leq} \geq \mathrm{CoVaR}_\alpha$ for $\rho > 0$, using that the law of X^{sys} given $X_i \leq q_\alpha$ is a mixture, over $x \leq q_\alpha$, of the laws of A1(a).
  - (c) Solve the equation of (a) numerically by bisection on c and compare the result with $\mathrm{CoVaR}_{1\%}$ of A1.
- **Report**
  - c, the standardised threshold $-c/\sigma_{sys}$ and the gap to A1 in points and in percent
- **Interpretation**
  - Which departures from the Normal model could widen the gap, and how could you distinguish their effects empirically?
- **Model**: A1 [Solved]; notebook section A4

Back Exercise list

B5 [Proposed]: Backtesting the Girardi–Ergün CoVaR — Task (1/2)

- **Question**: Would a backtest detect a badly calibrated CoVaR?
- **Data and assumptions**
  - The eleven international banks of B1, B4 with their regional systems; daily returns and sample as in B1
  - Each day $t \geq 1\,000$ (zero-based: the 1001st common day onward, 2961 forecasts), using all earlier days: $\hat{q}_t$ is the empirical 5% quantile of X_i; $\mathrm{CoVaR}_t^{\leq}$ is minus the empirical 5% quantile of X^{sys} on the past days with $X_i \leq \hat{q}_t$ (historical version of A4) Girardi & Ergün (2013)
  - Joint hit $I_t = 1\{X_t^{sys} \leq -\mathrm{CoVaR}_t^{\leq},\ X_{i,t} \leq \hat{q}_t\}$; under H_0, $P(I_t = 1) = \alpha^2 = 0.25\%$
- **Tasks**
  (a) Count the joint hits of each bank and test $H_0 : P(I_t = 1) = 0.25\%$ at 5% with the Kupiec test Kupiec (1995), as in Banulescu-Radu, Hurlin, Leymarie & Scaillet (2021), likelihood-ratio statistic $LR = -2[\ln L(0.25\%) - \ln L(\hat{\pi})] \sim \chi_1^2$.

Back Exercise list

B5 [Proposed]: Backtesting the Girardi–Ergün CoVaR — Task (2/2)

- **Tasks (continued)**
  (b) Find the rejection region of the test in terms of the number of hits, and compute its power, assuming independent Bernoulli hits: the probability of rejecting when the true joint rate is 0.5%.
- **Report**
  - The number of forecast days; expected and observed hits; *LR* and *p* for JPMorgan; the range of hits and the smallest *p* over the eleven banks; the rejection region and the power
- **Interpretation**
  - Why is α^2 the right null rate?
  - What does a non-rejection tell you here?
- **Model**: B1 (tests), A4 ($\mathrm{CoVaR}^{=}$); notebook section B5

B6 [Solved]: SRISK: LRMES by Simulation — Task (1/2)

- **Question**: How much would JPMorgan lose in a one-month market crisis, today and in March 2020?
- **Data and assumptions**
  - JPMorgan (adjusted close) and the S&P 500 index (Standard & Poor's 500; close), daily log returns in %, 5 January 2010 – 18 September 2026 Brownlees & Engle (2017)
  - Crisis as in Brownlees & Engle (2017): arithmetic market return below $C = -10\%$ over $h = 22$ days; LRMES is the expected bank loss over the same h days in a crisis
  - For (c): the six-month scenario ($h = 126$ days, $C = -40\%$), the shortcut $1 - \exp(-18\,\mathrm{MES}^{2\%})$ of Lecture 18 and the prudential ratio $k = 8\%$
- **Tasks**

(a) Estimate a zero-mean GJR–GARCH(1,1) for each return series and a DCC(1,1) for their correlation, by two-step Quasi-Maximum Likelihood (QML).

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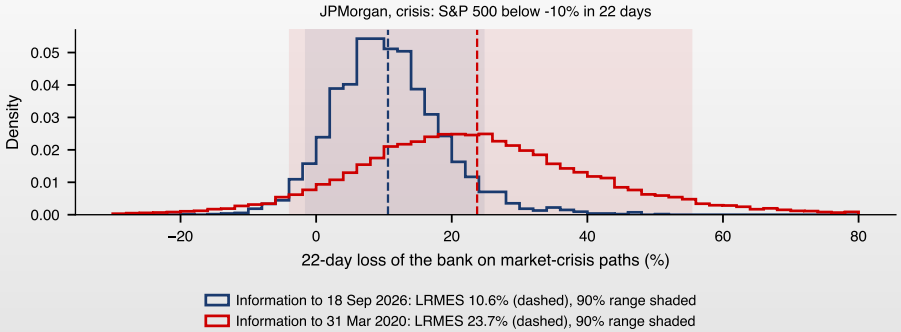
B6 [Solved]: SRISK: LRMES by Simulation — Task (2/2)

- **Tasks (continued)**
  - (b) Simulate 200 000 paths of 22 days from resampled pairs of standardised innovations, keep the crisis paths and compute LRMES with its 90% range (5% and 95% loss quantiles, a scenario range rather than a confidence interval for the mean); repeat with data up to 31 March 2020.
  - (c) Repeat (b) with $h = 126$, $C = -40\%$ (400 000 paths), compare with the shortcut, and compute the break-even leverage Λ^* of Exposure in a Crisis: SRISK for both.
- **Report**
  - JPMorgan's GJR estimates and the DCC estimates; the share of crisis paths on both dates; LRMES with its range, today and on 31 March 2020; the six-month LRMES against the shortcut; both Λ^*
- **Interpretation**
  - Which assumption of the shortcut does the simulation drop?
  - Why does LRMES change between March 2020 and today?
  - What is missing to turn LRMES into SRISK in money?
- Notebook: section B6

B6 [Solved]: Solution (1/3)

- (a) GJR recursion of Extension Packs: GARCH–DCC and Stress Tests; JPMorgan: $\hat{\alpha}_G = 0.016$, $\hat{\gamma} = 0.120$, $\hat{\beta}_G = 0.891$, persistence $\hat{\alpha}_G + \hat{\gamma}/2 + \hat{\beta}_G = 0.967$.
- DCC recursion on the standardised residuals (same slide): $\hat{a} = 0.052$, $\hat{b} = 0.928$; last correlation $\rho_T = 0.53$.
- (b) Steps: draw 22 days of pairs $(\xi_{i\tau}, \epsilon_{m\tau})$, with $\xi_{i\tau} = (\epsilon_{i\tau} - \rho_\tau \epsilon_{m\tau})/\sqrt{1-\rho_\tau^2}$; rebuild $\epsilon_i = \rho\,\epsilon_{m\tau} + \sqrt{1-\rho^2}\,\xi_{i\tau}$ with the simulated ρ; run the GJR and DCC recursions from today's state; keep the paths with market return below -10%; average the bank's arithmetic loss over them.
- Result: 1.3% of the paths are crises (about 2 699 of 200 000); LRMES = 10.6%, 90% range $[-1, 25]$%.
- With data up to 31 March 2020 the same procedure gives 11.9% crisis paths and LRMES = 23.7%.

B6 [Solved]: Solution (2/3)



- Simulated 22-day loss of JPMorgan on the paths where the S&P 500 falls below −10%; dashed: LRMES; shaded: 90% range

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B6 [Solved]: Solution (3/3)

- (c) Shortcut: JPMorgan's $\mathrm{MES}^{2\%} = 3.61\%$, $18 \cdot 0.0361 = 0.650$, so LRMES $= 1 - e^{-0.650} = 48\%$.
- Simulation for $h = 126$, $C = -40\%$: LRMES $= 32.8\%$.
- $\Lambda^* = 1 + 0.92(1 - 0.328)/0.08 = 8.7$ from the simulation against $1 + 0.92(1 - 0.478)/0.08 = 7.0$ from the shortcut.
- Interpretation: the simulation drops the constant beta and the log-return scaling behind the 18 of the shortcut (see A5); it starts from today's volatility and correlation and keeps the joint tail of the innovations.
- On 31 March 2020 volatility and correlation were at crisis levels, so a further -10% month would have caused a larger bank loss; today the starting state is calm.
- SRISK in money also needs the book value of debt D and the market value of equity W.

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A5 [Proposed]: Where the 18 in the LRMES Shortcut Comes From — Task

- **Question**: Where does the constant 18 in the LRMES shortcut come from?
- **Data and assumptions**
  - Crisis scenario Acharya, Engle & Richardson (2012): the market falls by exactly $d = 40\%$ over six months, a deterministic boundary scenario ($R_m = -d$) that motivates the shortcut; the simulated LRMES of B6 instead averages over all paths with $R_m < C$
  - The bank's six-month log return is β times the market's, with constant β; $\mathrm{MES}^{2\%} = \beta\,\mathrm{ES}_m^{2\%}$, where $\mathrm{ES}_m^{2\%}$ is the average market loss on days with a return below -2%
  - Data for (c): S&P 500 daily log returns in percent (close), restricted to the common 17-series sample, 5 January 2010 – 18 September 2026; and a Normal market with $\sigma_m = 1\%$
- **Tasks**
  - (a) Show that in this scenario the bank loses $\mathrm{LRMES} = 1 - (1-d)^{\beta}$.
  - (b) Substitute $\beta = \mathrm{MES}^{2\%}/\mathrm{ES}_m^{2\%}$ to show $\mathrm{LRMES} = 1 - \exp(-\kappa \mathrm{MES}^{2\%})$ with $\kappa = -\ln(1-d)/\mathrm{ES}_m^{2\%}$.
  - (c) Use ES as a fraction (0.025, not 2.5) when computing κ. Compute $\mathrm{ES}_m^{2\%}$ and κ for the S&P 500 and for the Normal market ($\mathrm{ES}_m^{2\%} = \sigma_m \varphi(2)/\Phi(-2)$), and the $\mathrm{ES}_m^{2\%}$ that gives $\kappa = 18$.
- **Report**
  - The number of S&P 500 days below -2%, both values of $\mathrm{ES}_m^{2\%}$ and of κ, and the $\mathrm{ES}_m^{2\%}$ implied by 18
- **Interpretation**
  - What goes wrong in the shortcut if β rises in crises?
- **Model**: A2 [Solved]; notebook section A5

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B7 [Proposed]: SRISK of a European and a Romanian Bank — Task

- **Question**: How exposed are Deutsche Bank and Banca Transilvania to a crisis of their own market?
- **Data and assumptions**
  - Deutsche Bank with the Euro Stoxx 50 as its market; Banca Transilvania with BET (Bucharest Exchange Trading, index); sample of B6
  - Models and crisis as in B6: GJR–GARCH(1,1), DCC(1,1), 200 000 paths of 22 days, $C = -10\%$
  - Prudential ratio $k = 5.5\%$ Engle, Jondeau & Rockinger (2015), so $\Lambda^* = 1 + 0.945(1 - \mathrm{LRMES})/0.055$
- **Tasks**
  (a) Compute LRMES with its 90% range for both banks with data up to 18 September 2026 and up to 31 March 2020, and the share of crisis paths of each market.
  (b) Compute Λ^* from the LRMES and from both ends of its 90% scenario range; order the transformed endpoints because higher losses imply lower admissible leverage.
- **Report**
  - LRMES with range (both dates), crisis shares, Λ^* with its range, for both banks
- **Interpretation**
  - Which bank is more exposed to a crisis of its own market?
  - How wide is the range of Λ^*?
  - What does this range mean for a supervisor?
- **Model**: B6 [Solved]; notebook section B7

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A6 [Proposed]: Connectedness of a Bivariate VAR — Task

- **Question**: Who transmits and who receives in a two-bank VAR?
- **Data and assumptions**
  - VAR(1) for two banks with $A = \begin{pmatrix} 0.5 & 0 \\ 0.4 & 0.5 \end{pmatrix}$ and $\Sigma = \begin{pmatrix} 1 & 0.3 \\ 0.3 & 1 \end{pmatrix}$
  - $A_{21} = 0.4$, $A_{12} = 0$: bank 1 drives bank 2 with a one-day lag, not the reverse
  - Generalized decomposition of A3, rows normalised to one; $C = \frac{100}{2}(\tilde{\theta}_{12} + \tilde{\theta}_{21})$; net position $T_i - F_i$ (to others minus from others)
- **Tasks**
  (a) Compute the generalized table (raw and normalised) and C at $H = 1$.
  (b) Compute $A\Sigma$, $\mathrm{diag}(A\Sigma A^\top)$, the raw and normalised tables and C at $H = 2$.
  (c) Compute the net position of each bank at $H = 2$ and identify the net transmitter.
- **Report**
  - Both tables, C at both horizons and the two net positions
- **Interpretation**
  - Why is the table symmetric at $H = 1$ although bank 1 drives bank 2?
  - Why does C rise with H?
- **Model**: A3 [Solved]; notebook section A6

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B8 [Proposed]: Did Connectedness Change after 2020? — Task (1/2)

- **Question**: Did bank connectedness rise after 2020?
- **Data and assumptions**
  - Rolling total connectedness of Lecture 18: log Parkinson volatilities of the 13 banks (B2), windows of 250 days moved by 5 days, dated by their last observation, VAR(2), $H = 10$; 744 windows
  - Break date: 1 January 2020; for (b) the daily series of B2 in 2010–2019 (2 373 days) and in 2020–2026 (1 593 days)
- **Tasks**

(a) Regress the rolling index on a constant and $D_t = 1\{t \geq 2020\}$ by OLS; test H_0: slope $= 0$ at 5% with ordinary and with Newey–West (50 lags) standard errors Newey & West (1987).

B8 [Proposed]: Did Connectedness Change after 2020? — Task (2/2)

- **Tasks (continued)**
  - (b) For a distinct, more parsimonious full-period comparison, estimate a VAR(1) separately on 2010–2019 and on 2020–2026 and compute C ($H = 10$) of each, with 95% intervals from a residual block bootstrap (blocks of 20 days, 999 draws) in each period.
  - (c) Build a 95% interval for the difference $C_{\mathbf{2020-26}} - C_{\mathbf{2010-19}}$ from the differences of independent draws in the two periods, and decide at 5%.
- **Report**
  - The two means, the slope, both t statistics and decisions; C for both periods with intervals; the difference with its interval
- **Interpretation**
  - Why are consecutive values of the rolling index nearly copies of each other?
  - Why is (b)–(c) a cleaner test than (a)?
  - What does it still assume about the break date?
- **Model**: B2, B3 [Solved]; notebook section B8

C2 [Proposed]: Quantile Connectedness of Bank Losses — Task (1/2)

- **Question**: Do large bank losses spread more than small ones?
- **Data and assumptions**
  - Case study of Lecture 18: Ando, Greenwood-Nimmo & Shin (2022), spillover index of a quantile factor VAR (QFVAR) at quantile τ and Relative Tail Dependence (RTD), $\mathrm{RTD} = S^{(5)}_{0.95} - S^{(5)}_{0.05}$ (Sections 2 and 4.3)
  - Daily adjusted closes of the 13 banks of B2 (course dataset), 2010–2026, aligned on common days before computing returns; $y_{it} = -100 \ln(P_{it}/P_{i,t-1})$, the daily loss in %, so that the right tail is adverse, as for the Credit Default Swap (CDS) changes of the paper
  - Generalized FEVD of Eq. (17), rows normalised, $h = 5$ days
- **Tasks**
  (a) Build the loss panel, choose the VAR order by BIC on the full sample and report it together with the number of factors selected by Eq. (8) at the mean and at each τ of (b) (the number is quantile-specific).
  (b) Estimate the factor VAR at the mean and at $\tau \in \{0.05, 0.10, 0.50, 0.90, 0.95\}$ on the full sample, and compute the spillover index $S^{(5)}_{\tau}$ of Eq. (20) at each.

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C2 [Proposed]: Quantile Connectedness of Bank Losses — Task (2/2)

- **Tasks (continued)**
  - (c) Repeat (b) without factors, as a plain quantile VAR, and compare the indices; interpret their difference as sensitivity to common-factor adjustment, not as an identified common-shock contribution.
  - (d) On rolling windows of 250 days moved by one day, with the number of factors fixed at its full-sample value, compute $S^{(5)}$ at the mean, 0.05, 0.5 and 0.95, and RTD; this is descriptive: the full-sample choices use future data.
- **Report**
  - Indices of (b)–(c) beside Figure 4 of the paper; correlations of the rolling indices of (d) beside Table 3, with block-bootstrap intervals; a chart of RTD with March 2020 and March 2023 marked; the data limitation (equity losses, 13 banks, different closing times), and explain the differences from the CDS network of the paper
- **Interpretation**
  - Do bank losses spread more in both tails than in the centre, as in the credit risk network?
  - Does RTD rise before the stress of 2020 and 2023 or only with it?
- **Model**: B2, B3, Lecture 18 (case study); notebook section C2

B9 [Solved]: The Financial Risk Meter (FRM) Penalty — Task

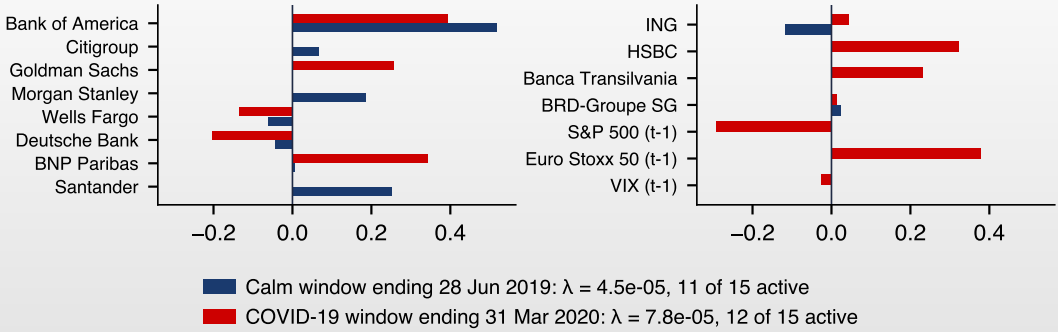
- **Question**: Which banks carry JPMorgan's left tail, in a calm and in a crisis window?
- **Data and assumptions**
  - Daily returns (fractions) of the 13 banks and of the S&P 500, Euro Stoxx 50 and VIX (Cboe Volatility Index), lagged one day
  - FRM (Financial Risk Meter) regression Mihoci et al. (2020): JPMorgan on the other 12 banks and the three lagged indices (15 regressors), quantile level $\tau = 5\%$, L_1 penalty λ, windows of 63 days
  - GACV (generalized approximate cross-validation): 20 values of λ, log-spaced from 10^{-6} to $10^{-1.5}$; $\mathrm{GACV}(\lambda) = \sum_t \rho_\tau(\hat{u}_t)/(n - \mathrm{df})$, df = number of interpolated points ($\hat{u}_t = 0$) $= 1+$ number of non-zero slopes
- **Tasks**
  (a) In the calm window ending 28 June 2019, fit the penalised quantile regression for each λ on the grid, choose the λ that minimises GACV and count the active coefficients.
  (b) Repeat (a) in the crisis window ending 31 March 2020, and list the four largest coefficients in absolute value in each window.
- **Report**
  - $\hat{\lambda}$, the number of active coefficients and the four largest coefficients in each window
- **Interpretation**
  - Which banks carry JPMorgan's tail in each window, and why is the selected λ so small?
- Notebook: section B9

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B9 [Solved]: Solution (1/3)

- Steps: (1) take the 63 days of the window; (2) for each λ solve the penalised quantile regression; (3) compute the residuals, df and GACV; (4) keep the λ with the smallest GACV.
- (a) Calm window (2019-03-21 to 28 June 2019): $\hat{\lambda} = 4.5 \times 10^{-5}$, 11 of 15 coefficients active.
- Largest: Bank of America (0.52), Santander (0.25), Morgan Stanley (0.19), ING (-0.12).
- (b) Crisis window (2019-12-23 to 31 March 2020): $\hat{\lambda} = 7.8 \times 10^{-5}$, 12 of 15 active.
- Largest: Bank of America (0.39), Euro Stoxx 50 (t-1) (0.38), BNP Paribas (0.34), HSBC (0.32).

B9 [Solved]: Solution (2/3)



- The 15 coefficients of the 5% quantile regression at the GACV-selected penalty

B9 [Solved]: Solution (3/3)

- Interpretation: Bank of America leads in both windows; in the crisis the lagged Euro Stoxx 50 and two European banks join the largest coefficients: the tail of JPMorgan becomes international.
- $\hat{\lambda}$ rises from calm to crisis, the direction the FRM reads as higher tail risk.
- λ is small because 15 regressors for 63 days make the unpenalised fit feasible: GACV prefers little shrinkage.
- The FRM is designed for many more firms than days, where the penalty is indispensable Mihoci et al. (2020), Toma et al. (2026).

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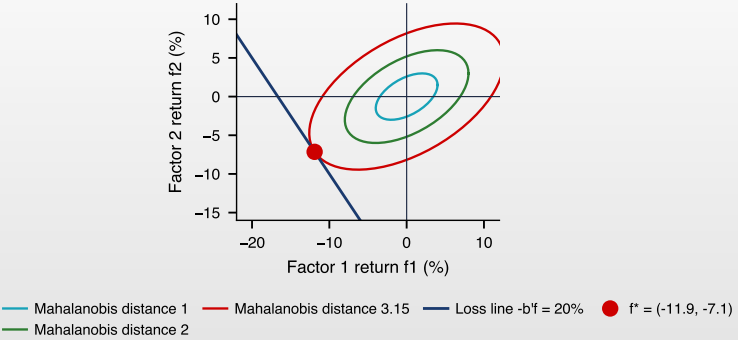
A7 [Solved]: Reverse Stress Test: the Lagrangian — Task

- **Question**: What is the most plausible scenario behind a given portfolio loss?
- **Data and assumptions**
  - Two risk factors $f = (f_1, f_2)^\top \sim N(0, \Sigma)$ (four-week returns, %); portfolio loss $L = -b^\top f$ with exposures b
  - Reverse stress test: the most plausible scenario (highest density) that produces a given loss ℓ^*
  - Numbers for (c): $b = (1.2, 0.8)$, $\Sigma = \begin{pmatrix} 16 & 6 \\ 6 & 9 \end{pmatrix}$ (%²), $\ell^* = 20\%$
- **Tasks**
  (a) Minimise $\frac{1}{2} f^\top \Sigma^{-1} f$ subject to $-b^\top f = \ell^*$ with a Lagrangian and derive f^*.
  (b) Show that the Mahalanobis distance of f^* is $d^* = \ell^* / \sqrt{b^\top \Sigma b}$ and that $P(L \geq \ell^*) = \Phi(-d^*)$.
  (c) Compute Σb, $b^\top \Sigma b$, f^*, d^* and $P(L \geq \ell^*)$, and check that f^* produces the target loss.
- **Report**
  - Every intermediate vector and scalar, f^*, d^* and the probability
- **Interpretation**
  - Which factor carries the larger fall in the scenario?
  - How should a risk committee read a probability of this size?
- Notebook: section A7

A7 [Solved]: Solution (1/3)

- (a) Maximum density is minimum $\frac{1}{2} f^\top \boldsymbol{\Sigma^{-1}} f$; Lagrangian $\mathcal{L} = \frac{1}{2} f^\top \boldsymbol{\Sigma^{-1}} f + \lambda(b^\top f + \ell^*)$.
- First-order condition: $\boldsymbol{\Sigma^{-1}} f + \lambda b = 0$, so $f = -\lambda \Sigma b$.
- Constraint: $-b^\top f = \lambda\, b^\top \Sigma b = \ell^*$, so $\lambda = \ell^*/(b^\top \Sigma b)$ and $f^* = -\ell^* \Sigma b/(b^\top \Sigma b)$.
- (b) $f^{*\top} \boldsymbol{\Sigma^{-1}} f^* = \boldsymbol{\lambda^2}\, b^\top \Sigma b = \ell^{*\boldsymbol{2}}/(b^\top \Sigma b)$, so $d^* = \ell^*/\sqrt{b^\top \Sigma b}$.
- $L = -b^\top f \sim N(0, b^\top \Sigma b)$, so $P(L \geq \ell^*) = 1 - \Phi(\ell^*/\sqrt{b^\top \Sigma b}) = \Phi(-d^*)$.

A7 [Solved]: Solution (2/3)



- Ellipses: points of equal density of f; the most plausible scenario is where the loss line touches the smallest ellipse

A7 [Solved]: Solution (3/3)

- (c) $\Sigma b = (16 \cdot 1.2 + 6 \cdot 0.8,\ 6 \cdot 1.2 + 9 \cdot 0.8) = (24, 14.4)$.
- $b^\top \Sigma b = 1.2 \cdot 24 + 0.8 \cdot 14.4 = 40.32$, $\sqrt{40.32} = 6.35$; $\lambda = 20/40.32 = 0.496$.
- $f^* = -0.496 \cdot (24, 14.4) = (-11.9\%, -7.1\%)$; check: $1.2 \cdot 11.90 + 0.8 \cdot 7.14 = 20\%$.
- $d^* = 20/6.35 = 3.15$ and $P(L \geq 20\%) = \Phi(-3.15) = 0.082\%$.
- Interpretation: factor 1 falls by 11.90% (2.98 of its standard deviations), factor 2 by 7.14% (2.38): the scenario leans on the factor with the larger exposure and variance.
- Under the Normal model such a four-week loss happens about once in 94 years; A8 shows how fragile this number is.

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A8 [Proposed]: Reverse Stress Test under a Student-t Distribution — Task

- **Question**: Does the reverse stress scenario survive heavy tails?
- **Data and assumptions**
  - Same b and Σ as A7; targets $\ell^* = 20\%$ and $\ell^* = 30\%$
  - f is multivariate Student-t with $\nu = 4$ degrees of freedom and covariance Σ, so its scale matrix is $\Sigma(\nu-2)/\nu$; T_ν is the Student-t cumulative distribution function (cdf)
- **Tasks**
  (a) Show that the most plausible scenario f^* is the same as in A7, using that the density depends on f only through $f^\top \Sigma^{-1} f$ and decreases in it.
  (b) Show that $L = -b^\top f$ is a univariate t_ν with scale $\sqrt{b^\top \Sigma b\,(\nu-2)/\nu}$, so that $P(L \geq \ell^*) = 1 - T_\nu\big(d^* \sqrt{\nu/(\nu-2)}\big)$.
  (c) Compute the probability for both targets under the Normal and the t_4 models, and the ratio of the two.
- **Report**
  - d^*, the t argument, the two probabilities and their ratio for each target
- **Interpretation**
  - What is robust in a reverse stress test?
  - What is not robust?
- **Model**: A7 [Solved]; notebook section A8

B10 [Proposed]: Stress Test of a Romanian Bank Portfolio — Task (1/2)

- **Question**: What would a severe but plausible shock cost a Romanian bank portfolio?
- **Data and assumptions**
  - Portfolio of 1 000 000 RON (Romanian leu), half in Banca Transilvania and half in BRD, rebalanced daily; daily log returns, days with no trades removed
  - Historical windows: the bank-tax shock (10 December 2018 – 31 January 2019) and COVID-19 (15 February – 30 April 2020)
  - Hypothetical scenario: weekly log returns of -25% for the Euro Stoxx 50 and -20% for BET; weekly (Friday) log returns 2010–2026 of both banks, the Euro Stoxx 50 and BET; use the last common trading day of each week if Friday is unavailable
- **Tasks**

(a) The daily log return of the rebalanced portfolio is $r_p = 100 \ln\left(\frac{1}{2} e^{r_{TLV}/100} + \frac{1}{2} e^{r_{BRD}/100}\right)$, not the average of the two log returns; in each window find the worst sum S of 10 consecutive r_p and convert it into a loss $1 - e^{S/100}$, in % and in RON.

Back Exercise list

B10 [Proposed]: Stress Test of a Romanian Bank Portfolio — Task (2/2)

- **Tasks (continued)**
  - (b) Regress each bank's weekly return on the Euro Stoxx 50 ($\beta_{\mathbf{1}}$) and BET ($\beta_{\mathbf{2}}$) weekly returns by OLS with a constant; predict each bank's factor-induced log return $\hat{r}_k = \beta_{\mathbf{1}k}(-25) + \beta_{\mathbf{2}k}(-20)$ (intercept and residual set to zero), convert it to a simple return $e^{\hat{r}_k/\mathbf{100}} - 1$ and apply the 50/50 weights held unchanged over the scenario week; also report the average betas and $R^{\mathbf{2}}$, and the Newey–West standard error (6 lags) and p-value of $\beta_{\mathbf{1}}$ for each bank.
- **Report**
  - Both historical losses with their dates and RON amounts; the betas, $R^{\mathbf{2}}$, the Newey–West standard error and p-value of $\beta_{\mathbf{1}}$, and the hypothetical loss in % and RON
- **Interpretation**
  - Which scenario hurts more: a domestic political shock or a global pandemic?
  - Is the Euro Stoxx 50 beta distinguishable from zero once BET is included, and what would each outcome mean?
- **Model**: Lecture 18: Stress Tests and Historical Scenarios; A7 for factor algebra; notebook section B10

C3: Exposure Has a Direction

- Let Y be a Romanian bank return and X the equally weighted return of the five European banks named in C3. Estimate the conditional quantile $q_\alpha(Y \mid X) = a + bX$.
- Compute the empirical quantiles of the conditioning system X: $q_\alpha(X)$ and $q_{0.5}(X)$. Then $\Delta\mathrm{CoVaR}_{Y|X} = b[q_{0.5}(X) - q_\alpha(X)]$, in positive-loss units.
- For example, if $b = 0.4$, the system median is zero and its tail quantile is -5%, the extra bank loss is 2%. Reversing the regression changes the question and generally changes the answer.
- In each bootstrap draw resample blocks of 20 days of paired bank–system observations, refit the quantile regression and recompute the system quantiles. With BET as the conditioning variable, recompute the same quantities for BET.

C3 [Proposed]: How Exposed Are Romanian Banks to European Banking Stress? — Task (1/2)

- **Question**: Are Romanian banks exposed mainly to European banking stress or to domestic shocks?
- **Data and assumptions**
  - Banca Transilvania and BRD; European system: equal-weighted portfolio of Deutsche Bank, BNP Paribas, Santander, ING and HSBC; BET
  - Sample: 5 January 2010 – 18 September 2026, 3 961 log returns (%) between trading dates common to all 13 banks and the four indices (prices aligned on common dates first)
  - Exposure CoVaR: quantile regression of the Romanian bank on the European system, so ΔCoVaR is the extra loss of the Romanian bank when the European system is in distress (the reverse direction of B1)
- **Tasks**

(a) Estimate the exposure ΔCoVaR of each Romanian bank at 5% and at 1% with block-bootstrap intervals (blocks of 20 days, 999 resamples), and the same measure with BET in place of the European system.

Back Exercise list

C3 [Proposed]: How Exposed Are Romanian Banks to European Banking Stress? — Task (2/2)

- **Tasks (continued)**
  - (b) Split the sample into 2010–2017 and 2018–2026, estimate ΔCoVaR 5% in each, and test the change at 5% with $z = (\hat{\Delta}_2 - \hat{\Delta}_1)/\sqrt{\mathrm{se}_1^2 + \mathrm{se}_2^2}$ from independent block-bootstrap draws (or a dominance test Castro & Ferrari (2014)), not by overlapping intervals.
  - (c) With rolling Diebold–Yilmaz tables (log Parkinson volatilities of the 13 banks, windows of 250 days moved by 5 days, VAR(2), $H = 10$), compute the share of the Romanian banks' forecast error variance coming from European and from US banks.
- **Report**
  - Deliverable: a Quantlet folder (code, a Metainfo.txt description file, charts) and three slides: the numbers of (a)–(c) with intervals and tests, the interpretation, the limitations
- **Interpretation**
  - Are Romanian banks exposed mainly to European banking stress or to domestic shocks?
  - Has the exposure grown?
- Notebook: section C3

Acronyms used in this material (1/2)

- **AI**: Artificial Intelligence
- **BET**: Bucharest Exchange Trading (index)
- **BIC**: Bayesian Information Criterion
- **BNP**: BNP Paribas SA (Euronext Paris ticker; Banque Nationale de Paris)
- **CDS**: Credit Default Swap
- **CI**: Confidence Interval
- **CoVaR**: Conditional Value at Risk (the VaR of the system conditional on the distress of an institution)
- **COVID-19**: COronaVIrus Disease 2019
- **DBK**: Deutsche Bank AG (Xetra ticker)
- **DCC**: Dynamic Conditional Correlation
- **ES**: Expected Shortfall
- **FEVD**: Forecast Error Variance Decomposition
- **FRM**: Financial Risk Meter
- **FWER**: Family-Wise Error Rate
- **GACV**: Generalized Approximate Cross-Validation
- **GARCH**: Generalised AutoRegressive Conditional Heteroskedasticity
- **GJR**: Glosten–Jagannathan–Runkle (asymmetric GARCH)
- **HAC**: Heteroskedasticity and Autocorrelation Consistent (standard errors)

Acronyms used in this material (2/2)

- **HSBC**: HSBC Holdings plc (originally the Hongkong and Shanghai Banking Corporation)
- **i.i.d.**: independent and identically distributed
- **ING**: ING Groep (Internationale Nederlanden Groep) — *the Dutch banking group ING*
- **JPM**: JPMorgan Chase & Co. (NYSE ticker)
- **LR**: Likelihood Ratio (test)
- **LRMES**: Long-Run Marginal Expected Shortfall
- **MES**: Marginal Expected Shortfall
- **OLS**: Ordinary Least Squares
- **QFVAR**: Quantile Factor Vector AutoRegression
- **QML**: Quasi-Maximum Likelihood
- **RTD**: Relative Tail Dependence
- **SG**: Société Générale — *the French banking group Société Générale*
- **SPX**: Standard and Poor's 500 index (column name in the notebook)
- **SRISK**: Systemic RISK (capital shortfall measure)
- **US**: United States
- **VAR**: Vector AutoRegression
- **VaR**: Value at Risk
- **VIX**: CBOE Volatility IndeX

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