



Statistics of Financial Markets

Chapter 0: Introduction



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It's the financial market!

[Image placeholder: Clinton meme – “It’s the economy, Stupid!”]

Why Study Statistics of Financial Markets (SFM)?

- Financial markets are highly complex, dynamic, and volatile.
- Risk and return trade-offs must be carefully analyzed to make informed investment decisions.
- Pricing, hedging, and portfolio management depend on quantitative and statistical models.
- Data-driven decision-making is fundamental in finance, especially with the rise of big data and machine learning.
- Modern finance relies heavily on statistical techniques for
 - ▶ Asset pricing models (CAPM, APT)
 - ▶ Risk management (VaR, Expected Shortfall)
 - ▶ Portfolio optimization (Mean-Variance Analysis, Black-Litterman)
 - ▶ Algorithmic trading and market microstructure analysis

Real-World Applications of SFM

▣ Asset Pricing & Return Prediction

- ▶ Determine the fair price of financial assets using statistical models.
- ▶ Capital Asset Pricing Model (CAPM), Fama-French multifactor models.
- ▶ Machine learning-based return prediction models.
- ▶ Statistical arbitrage and factor-based investing.

▣ Risk Management & Financial Stability

- ▶ Quantify financial risk, measure volatility, and manage portfolio risk.
- ▶ Value at Risk (VaR) & Conditional Value at Risk (CVaR).
- ▶ Risk-neutral pricing and Monte Carlo simulations.
- ▶ Extreme Value Theory (EVT) for tail risk estimation.
- ▶ Stress testing and scenario analysis for regulatory compliance (Basel III, Solvency II).

Real-World Applications of SFM

- Algorithmic & High-Frequency Trading
 - ▶ Goal: Use quantitative models to develop profitable trading strategies.
 - ▶ Statistical arbitrage using co-integration models.
 - ▶ Market impact modeling and execution strategies.
 - ▶ Market-making and liquidity provision models.
 - ▶ High-frequency trading using machine learning (LSTMs, deep reinforcement learning).
- Credit Risk & Financial Stability
 - ▶ Goal: Assess the probability of default and financial system stability.
 - ▶ Credit scoring models (e.g., Logistic Regression, Random Forests).
 - ▶ Structural models of credit risk (Merton Model).
 - ▶ Reduced-form models (Jarrow-Turnbull model).
 - ▶ Systemic risk indicators and contagion modeling.

How Statistics Helps in Financial Markets

- Detecting Market Trends and Anomalies
 - ▶ Identifying arbitrage opportunities, mean reversion patterns, and seasonal trends.
- Building Predictive Models for Asset Prices
 - ▶ Applying machine learning, time series analysis, and econometric methods.
- Measuring and Managing Financial Risk
 - ▶ Using statistical risk metrics, stress testing, and scenario analysis.
- Validating Financial Theories with Empirical Data
 - ▶ Efficient Market Hypothesis (EMH)
 - ▶ Behavioral Finance theories
 - ▶ Asset Pricing Models

Rise of the machines

- ▣ Cray 2: world's fastest supercomputer:
1985–1990
- ▣ 1.9 GFLOPs
- ▣ 5,500 pounds
- ▣ \$49,863,243.49 million (current \$)
- ▣ $10^{(G=9, T=12, P=15, E=18)}$

[Image: Cray 2 supercomputer]

G = Giga, T = Tera, P = Peta, E = Exa

Rise of the machines

- ▣ 2016 iPhone 7, 178 GFLOPs
- ▣ 2021 iPhone 13, 1200 GFLOPs
- ▣ 2025 iPhone 16, 2.15 TFLOPS
- ▣ 170 g = 1K EUR
- ▣ $10^{(G9, T12, P15, E18)}$

[Image: A18 chip, Cray board, iPhone]

Cryptocurrency Milestones

[Chart: S&P Cryptocurrency MegaCap Index, 2017–2025]

- ▣ 2017 Boom & 2018 Bubble Crash
- ▣ 2020 May – Bitcoin 3rd Halving
- ▣ 2020 Boom & 2021 Bubble Crash
- ▣ 2021 NFT Boom
- ▣ 2022 Apr – SEC Regulation
- ▣ 2022 Nov – FTX Belly Up
- ▣ 2024 Jan – BTC Spot ETF
- ▣ 2024 Apr – BTC 4th Halving
- ▣ 2024 May – ETH Spot ETF
- ▣ 2024 Nov – Trump Elected

Data Source: S&P Cryptocurrency MegaCap Index – Jan 14, 2025.

Speculative Bubbles

Back in the spring of 1720, Sir Isaac Newton owned shares in the South Sea Company, the hottest stock in England. As a rumor among the people appeared that ore of silver/gold was found, this was “fake news” as we call them today. The great physicist muttered that he ‘**could calculate the motions of the heavenly bodies, but not the madness of the people.**’ Newton dumped his South Sea shares, pocketing a 100% profit totaling £7,000. But just months later, swept up in the wild enthusiasm of the market, Newton jumped back in at a much higher price — and lost £20,000 (or more than £5 million in 2025’s money.) For the rest of his life, he forbade anyone to speak the words ‘South Sea’ in his presence.

[Chart: South Sea Stock, Dec 1718 – Dec 1721]

[Image: Newton portrait]

[Image: Newton’s apple tree]

Andrew Odlyzko (2019)

Intrinsic values of CC

- Eventually zero or further adoption?
- Metcalfe's law: p market cap,
 - ▶ $p = e^{\alpha_0} u^{\beta_0}$ and $\beta_0 = 2$.
 - ▶ u : # active users, i.e. addresses
- (Generalized) Metcalfe's law:

$$\log(p) = \alpha + \beta \log(u) + \varepsilon$$

[Chart: Market Cap vs Active Users, log-log scale]



Wheatley S et al (2018)

Bubbles

- Market-to-Metcalfe value ratio

$$\text{MMV}_i = \frac{p_i}{e^{\alpha} u_i^{\beta}}$$

after log-log regression fit,

$$(\alpha, \beta) = (-1.75, 2.12)$$

- Six bubbles identified

Wheatley S et al (2018)

[Chart: MMV ratio, 2010–2024, six bubbles highlighted]



The New Gold

▣ Market Cap Gold (Apr 27, 2023)	13,192 Billion USD
▣ Market Cap BTC (Apr 27, 2023)	559 Billion USD
	} 4.2 %

[Image: Gold coin / BTC]

▣ Market Cap Gold (Jan 16, 2025)	18.318 Trillion USD
▣ Market Cap BTC (Jan 16, 2025)	1.972 Trillion USD
	} 9.29 %

If BTC, CCs are seen as store of value then price can still grow a lot.

[Bitcoin's Market Capitalization History](#)

The Stochastic Volatility with Correlated Jumps Model

$$\frac{dS_t}{S_t} = r dt + \sqrt{V_t} dW_t^s + Z_t^y dN_t$$

$$dV_t = \kappa(\theta - V_t) dt + \sigma_v \sqrt{V_t} dW_t^v + Z_t^v dN_t$$

$$\text{Cov}(dW_t^s, dW_t^v) = \rho dt, \quad P(dN_t = 1) = \lambda dt$$

$$Z_t^v \sim \text{Exp}(\mu_v), \quad Z_t^y | Z_t^v \sim N(\mu_y + \rho_j Z_t^v, \sigma_y^2)$$

[Charts: RMSE comparison – BS, SV, SVJ, SVCJ]

	BS	SV	SVJ	SVCJ
σ	0.0278			
ρ		0.1462	-0.3310	-0.6245
κ		0.2203	1.5459	0.7425
θ		0.0006	0.0008	0.0008
V_0		0.0013	0.0009	0.0009
σ_v		0.0216	0.0368	0.0489
λ			0.0040	0.0033
μ_v			0.2359	0.3166
ρ_j				-1.0821
σ_y			0.1890	0.1267
μ_y				0.0206

Teng and Haerdle (2022) Financial analytics of inverse BTC options in a stochastic volatility world

Outline

1. Motivation ✓
2. Course overview
3. Resources
4. Grading
5. References

Course Objectives

- Understand fundamental statistical methods applied in finance.
- Develop expertise in probability theory and its applications in financial markets.
- Learn time series models (ARIMA, GARCH) for financial data analysis.
- Study the Efficient Market Hypothesis (EMH) and its empirical validation.
- Explore the Fractal Market Hypothesis (FMH) and long-memory processes.
- Understand volatility clustering and its implications for risk modeling.
- Analyze risk indicators such as VaR, CVaR, and systemic risk measures.
- Learn about credit risk models and their role in financial stability.
- Work with real-world financial datasets and Python/R for implementation.

Key Topics Covered

- ▣ Probability & Statistical Methods in Finance
 - ▶ Probability distributions of asset returns (Normal, Log-normal, t-distribution, Stable distributions).
 - ▶ Monte Carlo simulations for pricing and risk assessment.
 - ▶ Stochastic processes in finance (Brownian motion, Lévy processes, Fractional Brownian Motion).
- ▣ Time Series Analysis & Forecasting
 - ▶ Autoregressive models (AR, ARMA, ARIMA) for return prediction.
 - ▶ Volatility modeling: GARCH, EGARCH, TGARCH.
 - ▶ Volatility clustering: Why volatility is persistent and clustered in financial markets.
- ▣ Efficient Market Hypothesis (EMH) & Fractal Market Hypothesis (FMH)
 - ▶ EMH: Weak-form, semi-strong, and strong-form efficiency.
 - ▶ Empirical tests of EMH using statistical techniques (random walk, autocorrelation, variance ratio tests).
 - ▶ FMH: Non-stationary behavior of financial markets, self-similarity, and long-memory processes.
- ▣ Volatility Clustering & Financial Stability
 - ▶ Volatility Clustering: Why financial markets exhibit persistent periods of high/low volatility.
 - ▶ Leverage Effect: How negative shocks increase volatility more than positive shocks.
 - ▶ Risk Indicators: Value at Risk (VaR), Conditional VaR (CVaR).
- ▣ Credit Risk & Default Prediction
 - ▶ Credit scoring models
 - ▶ Stress testing and Basel III regulatory framework.

Resources

-  Franke, J., Härdle, W.K., Hafner, C.M. (2019). *Statistics of Financial Markets: An Introduction*. 4th ed., Springer.
-  Roman, M. (2003). *Statistica financiar-bancara*. ASE Bucharest.
-  Campbell, J.Y., Lo, A.W., MacKinlay, A.C. (1997). *The Econometrics of Financial Markets*. Princeton University Press.
-  Hand, D.J. (2010). *Statistics in Finance*. In: International Encyclopedia of Statistical Science, Springer.
-  Pele, D.T. (2013). *Criza piețelor de capital. O abordare interdisciplinară*. ASE Bucharest.
-  Tsay, R.S. (2010). *Analysis of Financial Time Series*. 3rd ed., Wiley.
-  Quantinar – <https://quantinar.com>

Grading

Final Grade

$$\text{Final Grade} = 0.7 \times \text{Exam} + \text{Seminar}$$

Seminar Grade

$$\text{Seminar} = 0.2 \times \text{Project} + 0.1 \times \text{Attendance}$$

References



Wheatley, S., Sornette, D., Huber, T., Reppen, M., Gantner, R.N. (2018). Are Bitcoin bubbles predictable? Combining a generalized Metcalfe's Law and the Log-Periodic Power Law Singularity model. *Royal Society Open Science*, 6(6), 180538.



Odlyzko, A. (2019). Newton's financial misadventures in the South Sea Bubble. *Notes and Records: The Royal Society Journal of the History of Science*, 73(1), 29–59.



Teng, H., Haerdle, W.K. (2022). Financial analytics of inverse BTC options in a stochastic volatility world. *Review of Derivatives Research*, 25, 223–253.