








Statistics of Financial Markets

Chapter 14: Crypto assets



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Acronyms used in this material (1/2)

- **ACF**: Autocorrelation Function
- **AI**: Artificial Intelligence
- **AIC**: Akaike Information Criterion
- **API**: Application Programming Interface
- **AR**: AutoRegressive (model); Abnormal Return
- **ATM**: Automated Teller Machine
- **CI**: Confidence Interval
- **COVID-19**: COronaVIrus Disease 2019
- **CRIX**: CRypto IndeX
- **EODHD**: EOD Historical Data (market-data provider)
- **ES**: Expected Shortfall
- **ETF**: Exchange-Traded Fund
- **EWMA**: Exponentially Weighted Moving Average
- **FTX**: FTX Trading Ltd. (crypto exchange, bankrupt in November 2022)
- **GARCH**: Generalised AutoRegressive Conditional Heteroskedasticity

Acronyms used in this material (2/2)

- **GENIUS**: Guiding and Establishing National Innovation for U.S. Stablecoins (Act)
- **GJR**: Glosten–Jagannathan–Runkle (asymmetric GARCH)
- **HAC**: Heteroskedasticity and Autocorrelation Consistent (standard errors)
- **HS**: Historical Simulation
- **IGARCH**: Integrated GARCH
- **LPPL**: Log-Periodic Power Law
- **SEC**: Securities and Exchange Commission
- **SHA-256**: Secure Hash Algorithm, 256 bits
- **SVB**: Silicon Valley Bank
- **US**: United States
- **USDC**: USD Coin
- **UST**: TerraUSD
- **UTC**: Coordinated Universal Time (Temps Universel Coordonné)
- **VR**: Variance Ratio

Today's Question and Route

- **Question**: do the statistical tools of Chapters 2–10 work for a market that never closes, has no cash flows and can lose 80% of its value?
  - the answer is a statistics-of-markets view of crypto assets, not a course in cryptography
- **Route** of the chapter
  - Bitcoin and Ethereum: blockchain, proof of work and of stake, the halving
  - crypto as an asset class: annualisation with 365 days, tails, volatility clustering, GARCH, efficiency, the weekend
  - correlation with equities and gold; bubbles and drawdowns; the spot Bitcoin ETFs of 2024
  - stablecoins: types, supply, peg deviations, USDC in March 2023, the collapse of TerraUSD; regulation; VaR 1% and ES 2.5%

Learning Outcomes

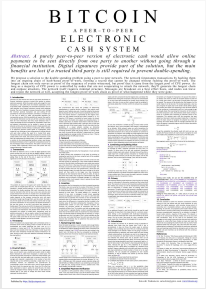
- Explain how a blockchain records ownership, why mining costs energy and how the Bitcoin supply follows from the halving rule
- Annualise crypto returns and volatilities with the actual frequency (365 days) and say what goes wrong with 252
- Estimate the tail index, a GARCH(1,1)-t model and variance-ratio tests for Bitcoin and compare them with the S&P 500
- Measure the correlation of crypto with equities and gold over time, and the drawdowns of a crypto asset
- Describe the three types of stablecoins, measure a peg deviation in basis points and read a depeg episode
- Compute VaR 1% and ES 2.5% of a crypto position and backtest them

Reading and Tools

- Textbook: Franke, Härdle & Hafner (2019), Ch. 23 (cryptocurrencies)
  - surveys: Härdle, Harvey & Reule (2020); crypto index CRIX: Trimborn & Härdle (2018)
  - risks and returns: Liu & Tsyvinski (2021); stablecoins: Gorton et al. (2022), Lyons & Viswanath-Natraj (2023)
- Python Quantlets of this chapter: Quantlets/Ch_14
  - daily data from EODHD (crypto close, 7 days a week); stablecoin supply from DefiLlama
- Lecture notebook: open in Google Colab
- Video courses: Introduction to Blockchain and Cryptocurrencies; Cryptocurrency as an Asset Class

Nakamoto's White Paper (2008)

- 31 October 2008: Nakamoto (2008) proposes "a peer-to-peer electronic cash system"
  - **peer-to-peer**: users pay each other directly, without a bank in the middle
  - the problem solved: **double spending**, i.e. paying twice with the same digital coin
- 3 January 2009: the first block; 2010: the first trades on exchanges
  - **exchange**: a trading platform where crypto assets are bought and sold against dollars or other crypto assets
- **Crypto asset**: a digital asset whose ownership is recorded on a blockchain
  - the author, "Satoshi Nakamoto", is still unknown; the coins mined in 2009–2010 have never moved



BITCOIN

A PEER-TO-PEER

ELECTRONIC

CASH SYSTEM

Abstract. A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution. Digital signatures provide part of the solution, but the main benefits are lost if a trusted third party is still required to prevent double-spending.

The Bitcoin white paper, 2008

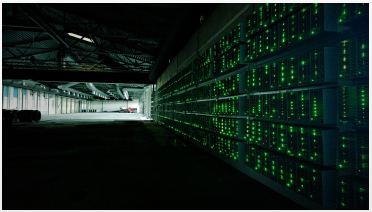
Text: Satoshi Nakamoto (2008); poster: DailyCoinPost (2018); CC0; Wikimedia Commons

The Blockchain: a Public Ledger

- **Ledger**: the list of all transactions; whoever holds the ledger knows who owns what
  - banks keep private ledgers; Bitcoin keeps one public ledger, copied on thousands of computers (**nodes**)
- **Block**: a batch of transactions; **hash**: a short fingerprint of a block (the function SHA-256)
  - changing one character of a block changes its hash completely
  - each block contains the hash of the previous one: the blocks form a **chain**
- Consequence: rewriting an old transaction means rewriting every later block
  - who may add the next block, and at what cost, is decided by the **consensus** rule (next slide)
- Ownership: a coin belongs to whoever holds the private key of its address; a lost key is a lost coin

Proof of Work and Mining

- **Proof of work** (PoW): to add a block, a **miner** must find a number that makes the hash of the block smaller than a target
  - the only method is trial and error: billions of hashes per second, on specialised machines
  - the target is adjusted every 2016 blocks so that a block arrives about every 10 minutes
- The winner receives the **block subsidy** (new coins) plus the fees of the transactions
  - to rewrite history, an attacker would need more computing power than all honest miners together
- The cost of security is electricity: this is why Bitcoin is criticised for its energy use



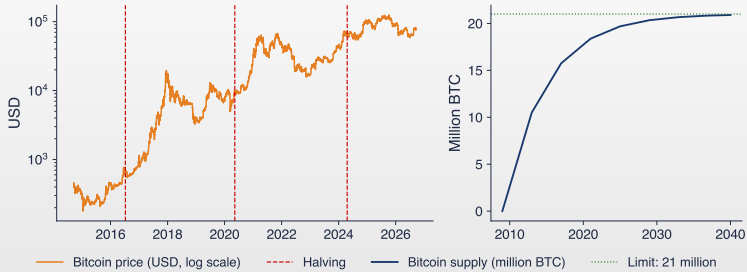
A Bitcoin mining farm, 2014

Photo: Marko Ahtisaari (2014); CC BY 2.0; Wikimedia Commons

The Supply Rule and the Halving

- Block subsidy: 50 BTC from 2009, **halved** every 210 000 blocks (about four years)
  - 2012: 25; 2016: 12.5; 2020: 6.25; April 2024: 3.125 BTC per block
  - the event of halving the subsidy is called the **halving**
- Total supply: a geometric series
  - $210\,000 \times 50 \times (1 + \frac{1}{2} + \frac{1}{4} + \dots) = 210\,000 \times 50 \times 2 = 21$ million BTC
  - about 19.4 million at the start of 2024, 20.4 million in 2030: almost all coins already exist
- The halving dates are known years in advance
  - in an efficient market (Chapter 7), a predictable cut in new supply should already be in the price

Bitcoin Price and Supply



- Left: Bitcoin close in USD (log scale), 457 USD in September 2014 and 80 901 USD on 18 September 2026; dashed: the halvings of 2016, 2020 and 2024. Right: supply from the protocol rule
- Interpretation: one year after the halvings the price was 287%, 559% and 31% higher; three observations are not evidence, and the effect shrinks

SFM_ch14_bitcoin_basics

The Anonymity of Bitcoin's Founder

- Nobody knows who "Satoshi Nakamoto" is; the founder stopped writing in 2011
  - the rules of Bitcoin are changed only if most nodes accept the new software
- Consequences for a statistician
  - no company, no profits, no dividends: no cash flows to discount
  - the price reflects only demand: payments, speculation, belief in scarcity Baur, Hong & Lee (2018)
  - so the usual valuation models do not apply; we study the price as a statistical object
- Trades happen around the clock on hundreds of exchanges, 365 days a year

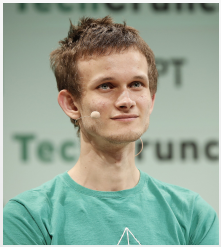


A bust of Satoshi Nakamoto, Budapest

Photo: Fekist (2021); CC BY-SA 4.0; Wikimedia Commons

Ethereum, Smart Contracts and Proof of Stake

- 2014: Buterin (2014) proposes Ethereum, a blockchain that runs programs; launched in July 2015
  - **smart contract**: a program stored on the blockchain that executes a transaction when its conditions are met
  - on top of it: tokens, stablecoins and DeFi (decentralised finance: lending and trading without intermediaries)
- **Proof of stake** (PoS): the right to add a block is drawn in proportion to the coins locked as a deposit (**stake**)
  - cheating is punished by losing the stake; no race of computing power
  - 15 September 2022 ("the Merge"): Ethereum moves from PoW to PoS; energy use falls by more than 99.9% ethereum.org: The Merge



Vitalik Buterin, 2015

Photo: John Phillips (2015); CC BY 2.0; Wikimedia Commons

Prices and Data

- There is no single exchange: each platform has its own price; prices differ across countries Makarov & Schoar (2020)
  - a data provider aggregates the platforms into one daily series; here: EODHD, close at midnight UTC (Coordinated Universal Time)
- Seven trading days a week: no weekend gaps, no holidays
  - Bitcoin: 4 384 daily returns since 2014; the S&P 500: 3 017 returns in the same period
- Market indices of many coins exist, e.g. CRIX (Crypto Index) Trimborn & Härdle (2018)
  - CRIX chooses the number of coins with the AIC criterion (Chapter 6): few coins describe the whole market
- Returns: $r_t = 100(\ln P_t - \ln P_{t-\mathbf{1}})$ in %, each series on its own calendar



A Bitcoin ATM, Prague

Photo: Perituss (2016); CC0; Wikimedia Commons

Annualisation with the Actual Frequency (1/2)

- With A observations a year and i.i.d. daily returns (Chapter 1):
  - annual mean $= A\,\bar{r}$, annual volatility $= \sqrt{A}\,s$
  - $\bar{r}$: the mean daily return; s: the standard deviation of daily returns; A: the number of returns per year
  - i.i.d.: independent and identically distributed; then the variances of the A daily returns add up, hence $\sqrt{A}$
- The right A is the actual number of observations per year
  - equities: $A \approx 252$ trading days; crypto: $A = 365$
  - actual frequency in our data: $A = 365.3$ for Bitcoin, 251.4 for the S&P 500

Annualisation with the Actual Frequency (2/2)

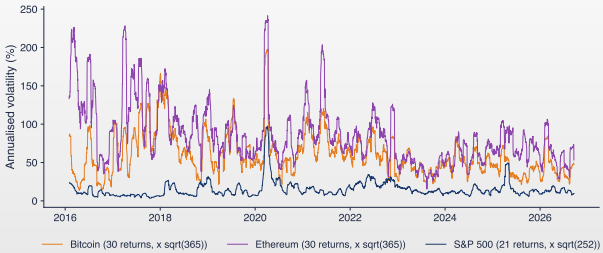
- **Worked example**: Bitcoin, $\bar{r} = 0.1181\%$, $s = 3.485\%$ per day
  - right: $\sqrt{365} \times 3.485 = 19.10 \times 3.485 = 66.6\%$; mean $365 \times 0.1181 = 43.1\%$
  - wrong: $\sqrt{252} \times 3.485 = 55.3\%$ and $252 \times 0.1181 = 29.8\%$
- The 252 rule understates the volatility by 16.9% and the mean by 31.0%
  - mixing the two (mean $\times 252$, volatility $\times \sqrt{365}$) also biases the Sharpe ratio
  - comparisons with equities: annualise each series with its own A, never with a common one

Comparison of Five Assets

	since	n	mean (%)	sd (%)	ann. vol. (%)	skew.	exc. kurt.	min (%)
Bitcoin	2014	4 384	0.118	3.49	66.6	−0.70	12.0	−46.5
Ethereum	2016	3 913	0.202	4.99	95.3	−0.20	9.0	−55.1
Solana	2020	2 351	0.212	6.12	117.1	−0.22	7.9	−55.0
S&P 500	2014	3 017	0.044	1.11	17.6	−0.64	15.8	−12.8
Gold	2014	3 124	0.041	1.01	16.4	−0.41	9.5	−11.1

- Daily log returns until 18 September 2026; annual volatility $= \sqrt{A}\, s$ with the actual frequency A of each series; gold: XAU/USD
  - skewness: the mean of $((r_t - \bar{r})/s)^{\mathbf{3}}$; excess kurtosis: the mean of $((r_t - \bar{r})/s)^{\mathbf{4}}$ minus 3 (both 0 for the Normal distribution)
- Interpretation
  - Bitcoin is 3.8 times as volatile as the S&P 500 (66.6% against 17.6%); Ethereum 5.4 times, Solana 6.7 times
  - on 11.6% of the days Bitcoin moves by more than 5%; the worst day: −46.5% on 12 March 2020
  - all five have negative skewness and excess kurtosis: the stylised facts of Chapter 2 hold, in a stronger form

Volatility Through Time



- Volatility on about one month: 30 daily returns $\times\sqrt{365}$ for crypto, 21 returns $\times\sqrt{252}$ for the S&P 500
- Interpretation: medians 54% (Bitcoin), 77% (Ethereum), 12% (S&P 500); the median ratio Bitcoin/S&P 500 is 3.8; all three peak in March–April 2020
- Bitcoin's volatility trends down, but it was below that of the S&P 500 on only 0.1% of the days

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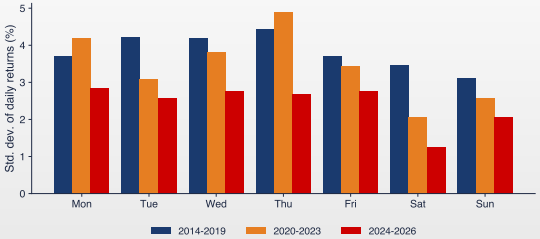
Case Study: Risks and Returns of Cryptocurrency

- Liu & Tsyvinski (2021): Bitcoin, Ripple and Ethereum, daily and weekly returns, 2011–2018
  - design: regress crypto returns on stock-market factors, currencies, commodities and macroeconomic factors
- Findings
  - crypto returns have almost no exposure to the usual factors: a separate asset class
  - strong time-series momentum: high returns this week predict high returns in the next weeks
  - investor attention (Google searches, Twitter posts) predicts returns
- Follow-up Liu, Tsyvinski & Wu (2022): three crypto factors (market, size, momentum) price the cross-section of coins
  - the link with equities has changed since 2020 (Section 5): results depend on the sample period

Day-of-the-Week Effects in Bitcoin

- Stock markets close at weekends; crypto markets do not, but banks, institutional traders and news slow down
  - **weekday effect**: different mean or variance of returns on some days of the week Caporale & Plastun (2019)
- Two tests on Bitcoin, each day labelled by its closing date
  - mean: $r_t = a + b\,W_t + \varepsilon_t$, $W_t = 1$ on Saturday and Sunday, 0 otherwise; HAC (Newey–West) standard errors
  - a: the mean return on weekdays; b: the weekend minus weekday difference; $b = 0$: no weekend effect in the mean
  - variance: the Brown–Forsythe test (Levene's test around the median), robust to heavy tails
- Ratio of the weekend to the weekday variance: equal to 1 if every day had the same risk

Bitcoin Volatility by Day of the Week



- Standard deviation of daily returns by closing day; three periods
- Weekend dummy b (p-value): -0.01 (0.95), -0.11 (0.48), -0.03 (0.83): no weekend effect in the mean
- Interpretation: weekend/weekday variance 0.66, 0.35, 0.39 (Brown–Forsythe p 0.002, < 0.001, < 0.001): crypto is calmer when traditional markets close

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Heavy Tails: the Hill Estimator (1/2)

- Hill (1975) (Chapter 5): the tail index from the k largest losses (or gains)

$$\hat{\alpha} = \Big[\frac{1}{k}\sum_{i=1}^{k} \ln \frac{X_{(i)}}{X_{(k+1)}}\Big]^{-1}, \qquad CI_{95\%} = \hat{\alpha}\Big(1 \pm \frac{1.96}{\sqrt{k}}\Big)$$

- Notation
  - $X_{(1)} \geq X_{(2)} \geq \ldots$: the losses (or gains) sorted from the largest; $X_{(k+1)}$: the threshold of the tail
  - k: the number of tail observations, here $k = 2.5\%$ of n; n: the sample size; CI: confidence interval
- Meaning of α: $P(X > x) \approx c\, x^{-\alpha}$ for large x (c: a constant)
  - a smaller α means a heavier tail
  - moments of order $\geq \alpha$ are infinite: $\alpha < 4$ means an infinite kurtosis

Heavy Tails: the Hill Estimator (2/2)

	left tail $\hat{\alpha}$	95% CI	right tail $\hat{\alpha}$	95% CI
Bitcoin	2.71	[2.20, 3.21]	3.37	[2.74, 4.00]
Ethereum	2.62	[2.10, 3.15]	3.09	[2.48, 3.71]
Solana	2.84	[2.11, 3.57]	3.41	[2.54, 4.29]
S&P 500	2.68	[2.08, 3.29]	2.80	[2.17, 3.43]
Gold	3.11	[2.42, 3.80]	3.04	[2.36, 3.71]

- Interpretation
  - Bitcoin's left tail ($\hat{\alpha} = 2.71$) is heavier than its right tail
  - all left-tail indices are below 4: the sample kurtosis of the table of five assets is not a stable number
  - the intervals of crypto and equities overlap

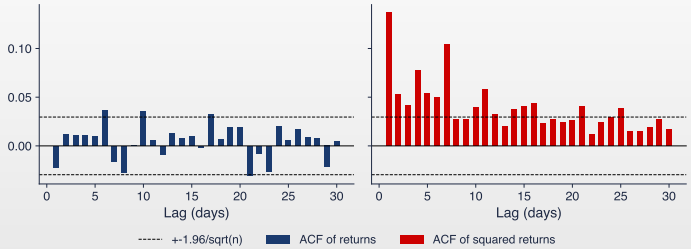
Reading the ACF and the Ljung–Box Test

- **ACF** (autocorrelation function): $\hat{\rho}(k)$, the sample correlation between r_t and r_{t-k}; k: the lag in days
  - under independence, $\hat{\rho}(k)$ lies within $\pm 1.96/\sqrt{n}$ with probability 95% (n: the number of returns)
  - ACF of r_t: is the direction predictable? ACF of r_t^2: is the size of the move predictable (volatility clustering)?
- **Ljung–Box** test of the first 10 autocorrelations together:

$$Q(10) = n(n+2)\sum_{k=1}^{10}\frac{\hat{\rho}(k)^2}{n-k}$$

- Reading
  - without autocorrelation, $Q(10)$ follows a χ^2 distribution with 10 degrees of freedom
  - a large $Q(10)$ and a small p-value: the autocorrelations are jointly significant

Volatility Clustering in Bitcoin



- ACF of r_t and of r_t^2, lags 1–30; dashed: $\pm 1.96/\sqrt{n} = \pm 0.030$
- Returns: $\hat{\rho}(1) = -0.022$, 4 of 30 lags outside the band; squared returns: $\hat{\rho}(1) = 0.137$, 15 of 30 outside
- Interpretation: Ljung–Box $Q(10)$ is 20.2 for r_t (p 0.028) and 212.7 for r_t^2 ($p < 0.001$): the direction is hard to predict, the size of the move is not

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GARCH(1,1)-t for Crypto and Traditional Assets (1/2)

- Bollerslev (1986) (Chapter 9): today's variance is a weighted sum of a constant, yesterday's squared shock and yesterday's variance

$$\sigma_t^2 = \omega + \alpha\,\varepsilon_{t-1}^2 + \beta\,\sigma_{t-1}^2, \qquad \varepsilon_t = \sigma_t z_t$$

- Notation
  - σ_t^2: the conditional variance of day t; ε_t: the shock (the demeaned return); z_t: Student-t with ν degrees of freedom, scaled to variance 1
  - $\omega > 0$: the constant; $\alpha \geq 0$: the reaction to news; $\beta \geq 0$: the memory of past variance; $\alpha + \beta$: the persistence
  - half-life of a shock: $\ln 0.5 / \ln(\alpha + \beta)$ days, the time in which half of its effect on the variance is gone
- GJR: an extra term $\gamma\,\varepsilon_{t-1}^2 \mathbb{1}\{\varepsilon_{t-1} < 0\}$
  - $\mathbb{1}\{\varepsilon_{t-1} < 0\} = 1$ after a fall; $\gamma > 0$: falls raise volatility more than rises, the leverage effect of equities

GARCH(1,1)-t for Crypto and Traditional Assets (2/2)

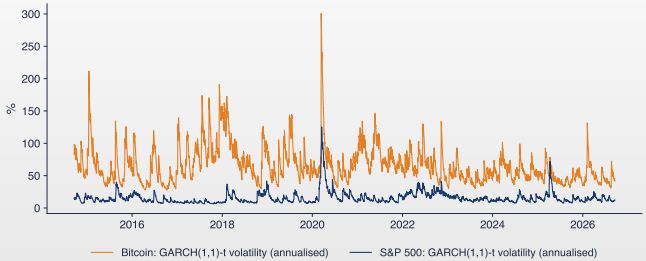
	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}+\hat{\beta}$	half-life (days)	$\hat{\nu}$	GJR $\hat{\gamma}$ (t)
Bitcoin	0.194	0.109	0.891	1.000	–	3.18	−0.013 (−0.72)
Ethereum	1.067	0.175	0.825	1.000	–	3.14	0.032 (0.91)
S&P 500	0.024	0.173	0.820	0.993	100	5.50	0.280 (7.08)
Gold	0.010	0.047	0.947	0.994	113	4.13	−0.051 (−3.33)

- Estimates by maximum likelihood on daily log returns; half-life in days; t: the t-statistic of $\hat{\gamma}$

Interpretation of the GARCH Estimates

- Crypto: $\hat{\alpha} + \hat{\beta} = 1$ at the boundary: an integrated GARCH (IGARCH; the EWMA of Chapter 8 is the case $\omega = 0$)
  - a shock to volatility does not die out within the sample; there is no finite long-run variance to which volatility returns
  - S&P 500 and gold: half-lives of 100 and 113 days
- $\hat{\nu} \approx 3.18$ for Bitcoin: even after GARCH filtering the tails are very heavy
  - the S&P 500 has $\hat{\nu} = 5.50$
- No leverage effect in crypto: $\hat{\gamma} = -0.013$ ($t = -0.72$) for Bitcoin, against 0.280 ($t = 7.08$) for the S&P 500
  - gold has $\hat{\gamma} < 0$ ($t = -3.33$): rises in its price raise volatility more than falls, as for a safe haven
- Katsiampa (2017) compares GARCH-type models for Bitcoin: a model with a short-run and a long-run volatility component fits best

Conditional Volatility: Bitcoin and the S&P 500



- GARCH(1,1)-t volatility $\hat{\sigma}_t$, annualised with $\sqrt{365}$ (Bitcoin) and $\sqrt{252}$ (S&P 500)
- Interpretation: both peak in March 2020 (301% and 125%); on 18 September 2026 the model gives 42% for Bitcoin and 12% for the S&P 500

SFM_ch14_garch_efficiency

Weak-Form Efficiency: the Tested Hypotheses

- Fama (1970): in a weak-form efficient market, past prices do not help to predict future returns (Chapter 7)
  - forms of efficiency: weak (past prices), semi-strong (public information), strong (all information)
- Tests used here
  - first-order autocorrelation $\hat{\rho}(1)$ and Ljung–Box $Q(10)$
  - variance ratio Lo & MacKinlay (1988): $\mathrm{VR}(q) = \mathrm{Var}(r_t + \cdots + r_{t-q+1})/(q\,\mathrm{Var}(r_t))$, equal to 1 under a random walk
  - q: the horizon in days; $r_t + \cdots + r_{t-q+1}$: the q-day log return; without autocorrelation its variance is q times the daily one
  - $Z^*(q) = (\widehat{\mathrm{VR}}(q) - 1)/\sqrt{\hat{\theta}^*(q)}$; $\hat{\theta}^*$: the variance of $\widehat{\mathrm{VR}}$, robust to volatility clustering
  - $Z^* \approx N(0,1)$ under a random walk (Chapter 7); reject at 5% if $|Z^*| > 1.96$
- VR > 1: momentum (trends); VR < 1: mean reversion

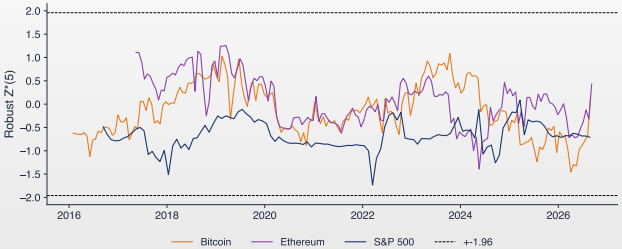
Variance-Ratio Tests on the Whole Sample

	$\hat{\rho}(1)$	$Q(10)$ (p)	VR(2)	VR(5)	VR(10)	$Z^*(5)$ (p)
Bitcoin	−0.022	20.2 (0.028)	0.977	0.991	1.027	−0.19 (0.853)
Ethereum	−0.012	22.7 (0.012)	0.988	1.067	1.137	1.17 (0.241)
S&P 500	−0.126	236.1 (<0.001)	0.875	0.849	0.813	−1.28 (0.201)

	period	n	$\hat{\rho}(1)$	VR(5)	$Z^*(5)$ (p)
Bitcoin	2014–2017	1 201	0.028	0.975	−0.23 (0.815)
Bitcoin	2018–2021	1 460	−0.057	0.981	−0.25 (0.801)
Bitcoin	2022–2026	1 721	−0.026	1.013	0.18 (0.856)
Ethereum	2016–2017	730	0.034	1.127	1.03 (0.303)
Ethereum	2018–2021	1 460	−0.064	0.997	−0.04 (0.971)
Ethereum	2022–2026	1 721	−0.015	1.014	0.20 (0.844)

- Interpretation: no VR test rejects the random walk, in any sub-period; the Ljung–Box test rejects weakly for crypto because it ignores volatility clustering

Efficiency Through Time



- Robust $Z^*(5)$ on windows of 500 observations, one window every 21 observations (as in Chapter 7)
- Interpretation: Bitcoin 0.0% of 185 windows reject; with the i.i.d. statistic $Z(5)$ the share would be 1.6% for Bitcoin and 21.7% for the S&P 500: ignoring volatility clustering creates false rejections

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Evolution of Efficiency

- Urquhart (2016): Bitcoin was inefficient in 2010–2016 as a whole, but moved towards efficiency after 2013
  - Nadarajah & Chu (2017): a power transformation of the same returns passes the tests: the early verdict was fragile
  - Tran & Leirvik (2020): an adjusted measure of inefficiency; crypto markets became more efficient after 2017
- Our data (since 2014): VR(5) between 0.78 and 1.17 on rolling windows; no robust rejection
  - the market grew, arbitrageurs arrived, derivatives and ETFs appeared: the adaptive-markets view of Chapter 11
- Weak-form efficiency is not efficiency in general: prices can be unpredictable and still be far from any fundamental value

Case Study: Trading and Arbitrage Across Exchanges

- Makarov & Schoar (2020): tick data of 34 exchanges in 19 countries, 2017–2018
  - **arbitrage**: buy where the price is low, sell where it is high; in an efficient market the gap closes fast
- Findings
  - large and persistent price gaps between countries (Korea, Japan and the United States), much smaller within a country
  - the gaps open when Bitcoin rises and capital controls make the arbitrage hard
  - a common component explains most of the order flow: prices are driven by one global market
- Lesson: the "law of one price" fails where money cannot move freely; a statistical test on one series cannot see this

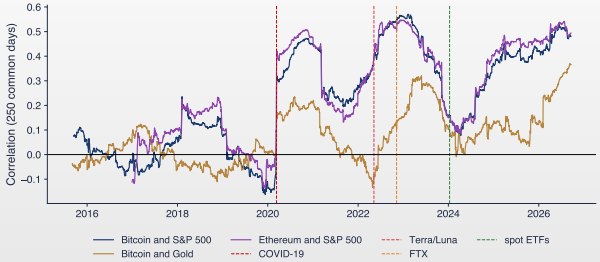
Case Study: Is Bitcoin Really Untethered?

- Griffin & Shams (2020): blockchain records of Bitcoin and of the stablecoin Tether (USDT), 2017–2018
  - question: was new USDT used to buy Bitcoin after price falls?
- Findings
  - purchases with Tether came after market falls and were followed by price rises
  - the flows came mostly from one large account on one exchange (Bitfinex)
  - hours with such flows account for a large share of the 2017 rise
- Related: manipulation of the Mt. Gox exchange in 2013 Gandal et al. (2018). Lesson: blockchain data allow tests that are impossible in equity markets

Measuring Correlation Across Calendars

- Bitcoin trades 7 days a week, the S&P 500 and gold 5: first join the **prices** on common days, then compute returns
  - Monday's Bitcoin return then covers Friday to Monday, like the S&P 500 return
  - joining returns instead would drop the weekend moves of Bitcoin
- **Asynchronous** closes: Bitcoin at midnight UTC, the S&P 500 at 16:00 New York time
  - weekly returns (Friday to Friday) reduce this problem; we report both
- **Rolling correlation**: the sample correlation on the last 250 common days, recomputed every day
  - standard error about $(1 - \rho^2)/\sqrt{250} \approx 0.06$ (ρ: the true correlation, here small): moves below 0.1 are noise

Rolling Correlation with Equities and Gold



- Daily log returns on common days; 250-day windows; dashed: COVID-19, Terra/Luna, FTX and the spot ETFs
- Interpretation: Bitcoin and the S&P 500 were uncorrelated before 2020 (0.02) and positively correlated after (0.39); the maximum, 0.57, came on 9 February 2023; the link with gold, weak until 2025, rose in 2026 (maximum 0.37 on 11 September 2026)

SFM_ch14_correlation

Correlation Year by Year

Bitcoin with	2019	2020	2021	2022	2023	2024	2025
S&P 500	−0.11	0.45	0.26	0.56	0.16	0.36	0.42
gold	0.01	0.21	−0.02	0.15	0.12	0.12	0.12
Ethereum with S&P 500	−0.02	0.47	0.20	0.55	0.15	0.40	0.46

- Whole period: daily 0.24, weekly 0.19 for Bitcoin and the S&P 500
  - the asynchronous closes do not explain the correlation: weekly returns give a similar number
- Interpretation
  - since 2020, crypto behaves as a risk asset: it falls with equities when interest rates rise (2022)
  - the change coincides with the arrival of institutional investors; a diversification benefit measured before 2020 no longer holds

Three Crisis Episodes

Episode	period	Bitcoin	Ethereum	S&P 500	gold	worst Bitcoin day
COVID-19	19 February 2020 – 23 March 2020	−40.6	−65.5	−41.4	0.3	−46.5
Terra/Luna	5 May 2022 – 12 May 2022	−23.0	−33.8	−5.4	−3.0	−11.7
FTX	5 November 2022 – 21 November 2022	−29.9	−38.4	4.6	3.4	−15.5

- Cumulative log returns in % from the first to the last close; worst day: the smallest daily log return of Bitcoin
- Interpretation
  - COVID-19 (a global shock): crypto fell as much as equities, gold did not move
  - Terra/Luna (May 2022) and FTX (an exchange that went bankrupt in November 2022): crypto-specific shocks; crypto fell by a quarter or more, the S&P 500 much less or not at all
  - the dependence is asymmetric: equity crashes reach crypto, crypto crashes rarely reach equities

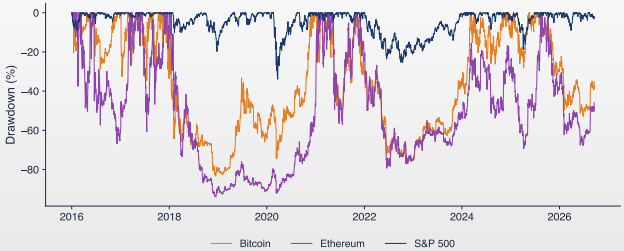
Bitcoin as a Hedge or a Safe Haven

- Baur & Lucey (2010): a **hedge** is uncorrelated or negatively correlated with equities on average; a **safe haven** is so in market crashes
  - test: the mean return on the worst 5% of S&P 500 days (151 common days, S&P 500 below -1.65%, on average -2.70%)
- Bitcoin: -2.77% on those days ($t = -4.89$, p < 0.001); negative on 70% of them
  - gold: 0.10% ($t = 0.86$, p 0.394)
- Interpretation: Bitcoin is not a safe haven; on the worst equity days it falls as much as the S&P 500; gold holds its value
- The "digital gold" narrative is not supported by the data of this sample

Drawdown

- **Drawdown**: $D_t = P_t / \max_{s \le t} P_s - 1$, the loss from the last record high (Chapter 1)
  - P_t: the price on day t; $\max_{s \le t} P_s$: the highest price up to day t; $D_t \le 0$
  - **maximum drawdown**: $\min_t D_t$; **time under water**: the days until a new record high
- A loss of d needs a gain of $d/(1-d)$ to recover, because $(1-d)(1+g) = 1$ gives $g = d/(1-d)$
  - $d = 75\%$: $0.75/0.25 = 300\%$; $d = 80\%$: 400%
  - Bitcoin's deepest drawdown in our data: -83.4% (15 December 2018); recovery needed a gain of 502%
- Drawdowns depend on the path, not only on the distribution of returns: high volatility and clustering produce deep ones

Drawdowns of Bitcoin, Ethereum and the S&P 500



- Daily closes since 2016; maximum drawdowns: Bitcoin −83.4%, Ethereum −94.0%, S&P 500 −33.9%
- Interpretation: Bitcoin spent 76% of the days more than 10% below its record, Ethereum 90%, the S&P 500 16%; on 18 September 2026 Bitcoin was 35.2% below its record

SFM_ch14_bubbles_etf

Bitcoin's Large Drawdowns

Peak	USD	trough	USD	depth (%)	new record	days under water
2014-09-17	457	2015-01-14	178	−61.1	2015-12-15	454
2017-12-16	19 497	2018-12-15	3 237	−83.4	2020-11-30	1080
2021-04-13	63 503	2021-07-20	29 807	−53.1	2021-10-19	189
2021-11-08	67 567	2022-11-21	15 787	−76.6	2024-03-04	847
2025-10-06	124 753	2026-06-30	58 559	−53.1	not yet	–

- Episodes since September 2014 whose drawdown exceeded 50%; dates as year-month-day
- Interpretation
  - 5 falls of more than half in twelve years: a crash of this size is a regular event, not a rare one
  - the 2017 and 2021 peaks were recovered after 1080 and 847 days

Bubbles and Log-Periodic Models

- **Bubble**: a price far above the fundamental value, sustained by the expectation of selling higher
  - Cheah & Fry (2015): the fundamental value of Bitcoin is zero; its price shows speculative bubbles
- Log-periodic power law (LPPL) Johansen, Ledoit & Sornette (2000): before a crash, $\ln P_t \approx A + B(t_c - t)^m + C(t_c - t)^m \cos(\omega \ln(t_c - t) - \phi)$
  - t_c: the critical time (the most likely crash date); A: the log price reached at t_c (here A is not the number of periods per year)
  - $B < 0$, $0 < m < 1$: faster than exponential growth; C, ω, ϕ: amplitude, frequency and phase of oscillations that accelerate towards t_c
- Caution: seven parameters, many local optima, and t_c is the most uncertain of them; fits look convincing only after the crash

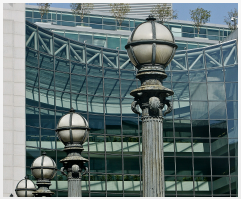


Didier Sornette

Photo: Didier Sornette (2012); CC BY-SA 3.0 DE; Wikimedia Commons

January 2024: Approval of the Spot Bitcoin ETFs

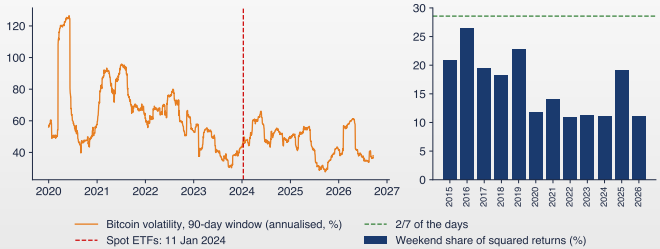
- **ETF** (exchange-traded fund): a fund whose shares trade on a stock exchange; **spot**: it holds the asset itself, not futures
  - 10 January 2024: the SEC (Securities and Exchange Commission) approves 11 spot Bitcoin ETFs SEC (2024); trading starts on 11 January
- Why it matters
  - pension funds and advisers can hold Bitcoin through a regulated security
  - ETF shares trade only on weekdays: part of the trading moves to stock-market hours
- Statistical question: did Bitcoin's volatility, its weekend pattern or its link with equities change?



SEC headquarters, Washington, D.C.

Photo: David (dbking) (2008); CC BY 2.0; Wikimedia Commons

Volatility and the Weekend Share Around the ETFs



- Left: Bitcoin volatility on 90-day windows, annualised with $\sqrt{365}$. Right: weekend share of the sum of squared returns by year (2/7 if every day had the same variance)
- Interpretation: volatility kept falling after January 2024, but it was already falling before; the weekend share has been well below 2/7 since 2020

SFM_ch14_bubbles_etf

Bitcoin before and after the Spot ETFs

Bitcoin	two years before	two years after
annualised volatility (%)	55.2	47.5
weekend/weekday variance	0.30	0.42
correlation with the S&P 500	0.45	0.37
GARCH(1,1)-t $\hat{\alpha} + \hat{\beta}$	1.000	0.97

- Windows: 11 January 2022 – 10 January 2024 (729 days) and 11 January 2024 – 10 January 2026 (730 days)
- Brown–Forsythe test of equal variance: statistic 1.22, p $= 0.27$: the fall in volatility is not significant
- Interpretation
  - a before–after comparison is not a causal effect: interest rates, the halving of April 2024 and the US election changed at the same time
  - a credible study needs a control group (e.g. coins without an ETF) or intraday data around the opening of the stock exchange

Tracking Error of a Spot Bitcoin ETF

- IBIT (iShares Bitcoin Trust), adjusted close, and Bitcoin, prices joined on the 671 trading days of the ETF
  - correlation of daily returns 0.91; slope of IBIT on Bitcoin 0.94
  - **tracking error**: the annualised standard deviation of the return difference, 20.8%
- Why not 1: the two closes are four hours apart (16:00 in New York against midnight UTC)
  - over a week the difference averages out; the fund itself holds Bitcoin one to one, minus a small fee
- Lesson: a high tracking error can be a measurement artefact; align the times before judging a fund

Stablecoins: Definition and Types

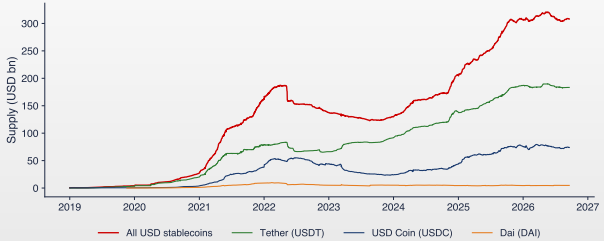
Type	how the peg is held	examples	risk
fiat-backed	reserves of dollars, deposits and Treasury bills; holders redeem 1 coin for 1 USD at the issuer	USDT, USDC	the reserves and the bank that holds them
crypto-backed	over-collateralised loans in crypto, liquidated automatically by smart contracts	DAI	a crash of the collateral
algorithmic	no reserves: an algorithm mints and burns a second coin to keep the price at 1 USD	UST (Terra)	a run that destroys both coins

- **Stablecoin**: a crypto asset designed to keep a fixed price, usually 1 USD (the **peg**)
  - used as cash on crypto exchanges and for payments across borders; DefiLlama classifies today 91.3% of the supply as fiat-backed, 8.7% as crypto-backed

Maintaining the Peg: Arbitrage

- Primary market: authorised traders create a coin for 1 USD at the issuer, or redeem it for 1 USD
  - price 0.99 on exchanges: buy at 0.99, redeem at 1.00, profit 1 cent: demand pushes the price back up
  - price 1.01: create at 1.00, sell at 1.01: supply pushes the price down
- Lyons & Viswanath-Natraj (2023): the peg of Tether holds because of this arbitrage; premiums appear when traders flee into stablecoins in crypto downturns
  - Ma, Zeng & Zhang (2025): with few authorised arbitrageurs, runs are less likely but deviations last longer
- The arbitrage works only while traders trust that redemption at 1 USD will be honoured: a peg is a promise

Supply of Stablecoins



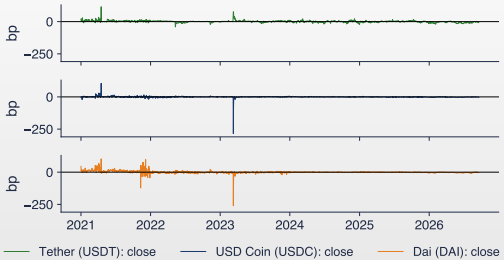
- Circulating supply in billion USD (DefiLlama): all USD stablecoins, USDT, USDC and DAI
- Interpretation: from 4.2 billion USD at the start of 2020 to 308 billion on 18 September 2026, a factor of 74; USDT holds 60%, USDC 24%
- The total fell from 187 billion (2 April 2022) to 123 billion (19 August 2023) after Terra and FTX; USDC fell from 43 to 24 billion in 2023

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Peg Deviation in Basis Points

- **Basis point** (bp): one hundredth of a percent, $0.01\% = 0.0001$
  - **peg deviation**: $d_t = 10\,000\,(P_t - 1)$ bp for a coin pegged to 1 USD; P_t: its price in USD on day t
  - $d_t < 0$: the coin trades below the peg; $d_t > 0$: above it
- **Worked example**: USDC closed at 0.9715 USD on 11 March 2023
  - $d = 10\,000 \times (0.9715 - 1) = -285$ bp, a loss of 2.85%
  - a holder of 1 000 000 USDC who sold at that close lost 28 500 USD; at the daily low (0.8774): 122 600 USD
- Statistics of a peg: the median absolute deviation, the share of days beyond 10 or 50 bp, the worst close and the worst low
  - speed of return: AR(1) $d_t = \phi\, d_{t-1} + e_t$, half-life $\ln 0.5 / \ln \phi$ days
  - $\phi \in (0, 1)$: the share of today's deviation still present tomorrow; e_t: a new shock; a small ϕ means a fast return to the peg

Peg Deviations of USDT, USDC and DAI



- Daily closing deviation from 1 USD in bp since 2021 (EODHD)
- Median absolute deviation: USDT 3.1 bp, USDC 1.1 bp, DAI 2.2 bp; days beyond 10 bp: 11.4%, 1.5%, 10.7%
- Interpretation: half-lives of 2.2, 0.6 and 0.6 days: deviations close within days; the exception is March 2023 (USDC −285 bp, DAI −261 bp)

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March 2023: USDC and Silicon Valley Bank

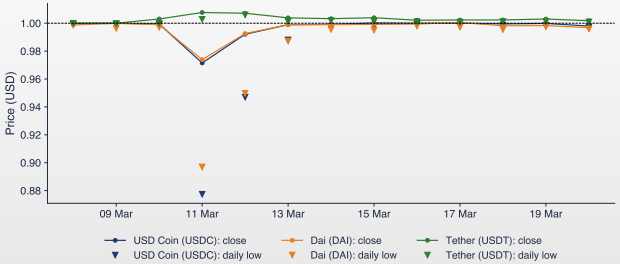
- 10 March 2023: Silicon Valley Bank (SVB) is closed by its regulator FDIC (2023); part of the cash reserves of USDC was deposited there
  - a weekend follows: banks are closed, crypto markets are open
- 11 March: USDC falls to 0.8774 USD (low, −1226 bp) and closes at 0.9715; DAI, backed largely by USDC, follows
  - USDT trades above 1 (close up to 77 bp): money flees from one stablecoin into another
- 12 March, evening: US authorities guarantee all SVB deposits U.S. Treasury, Federal Reserve & FDIC (2023); on 13 March USDC is back at the peg



SVB headquarters, Santa Clara, 13 March 2023

Photo: Minh Nguyen (2023); CC BY-SA 4.0; Wikimedia Commons

The USDC Depeg, Day by Day



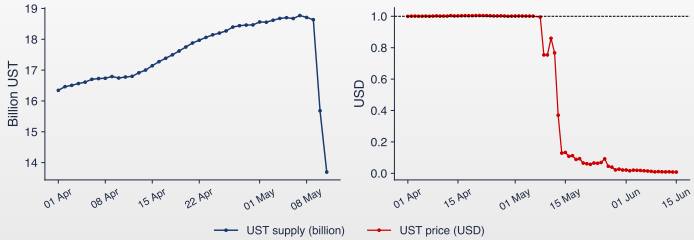
- Daily closes (circles) and lows (triangles), 8–20 March 2023; dashed: the peg
- Interpretation: a fiat-backed coin is as safe as its bank: the peg broke when the reserves were in doubt and returned when the deposits were guaranteed; the run was on the reserves, not on the code

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TerraUSD: an Algorithmic Stablecoin

- UST was kept at 1 USD by a swap with a second coin, LUNA: 1 UST could always be exchanged for 1 USD worth of LUNA
  - UST below 1: buy UST, swap it for 1 USD of new LUNA, sell the LUNA: UST is burned, LUNA is minted
  - the only backing was the market value of LUNA
- Demand came from the Anchor protocol, which paid close to 20% a year on UST deposits Liu, Makarov & Schoar (2023)
  - UST supply peaked at 18.8 billion on 7 May 2022
- The **death spiral**: large withdrawals from Anchor, UST below 1, more LUNA minted, LUNA falls, the backing shrinks, more withdrawals
  - Liu, Makarov & Schoar (2023): large, sophisticated holders withdrew first; small holders were the last to leave

The Collapse of UST, May 2022



- DefiLlama: UST supply until the last published day (11 May 2022) and the daily UST price
- Interpretation: the price was 0.75 USD on 9 May, 0.37 on 13 May, 0.13 on 14 May and 0.02 on 31 May; the supply fell by 27% in four days, from 7 to 11 May
- Unlike USDC, there was no reserve and no lender of last resort: an algorithmic peg cannot be rescued

SFM_ch14_stablecoins

Stablecoins as Private Money

- Gorton & Zhang (2021): stablecoins resemble the banknotes of private US banks in the "wildcat banking" era (1837–1863)
  - notes of different banks traded at different discounts, depending on trust in each bank
  - remedy then: a uniform national currency; remedy proposed now: regulate stablecoins as bank deposits
- Gorton et al. (2022): the peg holds as long as holders value the coin for trading with leverage in crypto markets
  - when that demand disappears, the coin trades below par, like a bank note of an uncertain bank
- Statistical lesson: peg deviations are small and short in normal times and jump in a crisis: a heavy-tailed, regime-dependent series

Regulation in the European Union: MiCA

- MiCA, Markets in Crypto-Assets Regulation (EU) 2023/1114 (MiCA), adopted in 2023
  - rules for stablecoins apply from 30 June 2024; for other crypto-asset services from 30 December 2024 ESMA: MiCA
- Stablecoins pegged to one currency are **e-money tokens**
  - issued only by authorised credit or e-money institutions; redemption at par at any time; no interest to holders
  - reserves kept separate and invested in safe, liquid assets
- Algorithmic stablecoins without reserves have no place in this framework



European Parliament, Strasbourg

Photo: jeffowenphotos (2010); CC BY 2.0; Wikimedia Commons

Regulation in the United States: the GENIUS Act

- GENIUS Act (Guiding and Establishing National Innovation for U.S. Stablecoins Act), signed on 18 July 2025 GENIUS Act, Public Law 119-27
  - payment stablecoins: reserves of at least 1 USD per coin, in cash, deposits and short-term Treasury bills
  - monthly public reports on the reserves; no interest paid to holders
- Common points of MiCA and GENIUS: full reserves, redemption at par, supervision of issuers
  - they address the run on reserves of March 2023 and the algorithmic collapse of May 2022
- A fully reserved stablecoin is close to a narrow bank: safe, but its issuer earns the interest on the reserves



United States Capitol, Washington, D.C.

Photo: Architect of the Capitol; public domain; Wikimedia Commons

VaR 1% and ES 2.5% in Dollars

- Chapter 10: $\mathrm{VaR}_\alpha = -q_\alpha$, the loss exceeded with probability α (VaR 1%); $\mathrm{ES}_\alpha = -E[r \mid r \le q_\alpha]$ (ES 2.5%)
  - q_α: the α-quantile of the daily return r; $E[r \mid r \le q_\alpha]$: the mean return on the days beyond the quantile
  - historical simulation (HS), Normal, Student-t (maximum likelihood), GARCH(1,1)-t for the next day
- From log returns to dollars: a log return of $-v\%$ means a loss of $W(1 - e^{-v/100})$ on a position of value W
  - Bitcoin, HS: VaR 1% $= 10.54\%$; $W = 100\,000$ USD: $100\,000 \times (1 - 0.8999) = 10\,006$ USD
- Horizon: crypto trades every day, so a 10-day VaR covers 10 calendar days, not two trading weeks

VaR 1% and ES 2.5% under Four Methods

	VaR 1% (%)				ES 2.5% (%)			
	HS	Normal	t	GARCH-t	HS	Normal	t	GARCH-t
Bitcoin	10.54	7.99	11.06	7.37	10.84	8.03	13.37	8.11
Ethereum	14.29	11.40	15.39	10.95	14.77	11.45	18.13	12.05
S&P 500	3.26	2.54	3.04	1.78	3.50	2.55	3.45	1.86

Position of 100 000 USD	VaR 1% HS	ES 2.5% HS	VaR 1% Normal	10-day VaR 1% HS
Bitcoin	10 006	10 277	7 679	25 563
Ethereum	13 312	13 732	10 772	31 842
S&P 500	3 204	3 438	2 506	9 049

- Whole samples until 18 September 2026; t: Student-t by maximum likelihood ($\hat{\nu} = 2.23$ for Bitcoin); GARCH-t: the forecast for the next day ($\hat{\sigma} = 2.83\%$)

Interpretation of the Risk Numbers

- A Bitcoin position is about three times as risky as an S&P 500 position: VaR 1% 10.54% against 3.26%
  - the Normal VaR 1% is 24% below the historical one for Bitcoin: heavy tails again
- Conditional against unconditional: GARCH-t gives 7.37% today, below the historical 10.54%, because volatility is now low
  - the historical number is a long-run average; the GARCH number is today's risk
- 10 days: the historical VaR 1% of 10-day returns is 29.52%; the square-root-of-time rule gives 33.34%
  - the square-root-of-time rule: $\mathrm{VaR}(h \text{ days}) = \sqrt{h}\,\mathrm{VaR}(1 \text{ day})$, exact only for i.i.d. Normal returns
  - the rule overstates: with tail index $\alpha > 2$, an extreme quantile of a sum of h returns grows roughly like $h^{1/\alpha}$, more slowly than $\sqrt{h}$
- ES 2.5% (HS) is close to VaR 1% (HS) for Bitcoin, as in Chapter 10 for equities

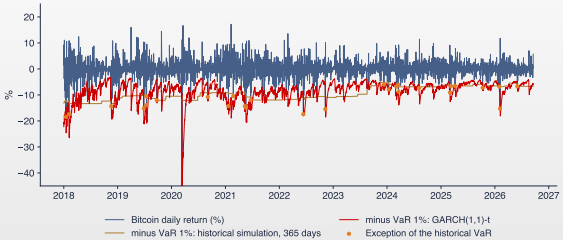
Backtesting: the Kupiec Test

- **Exception**: a day with $r_{t+1} < -\widehat{\mathrm{VaR}}_t$; x exceptions in n days, observed rate $\hat{\pi} = x/n$
  - with a correct VaR 1%, each day is an exception with probability $\alpha = 0.01$: about αn exceptions are expected
- **Kupiec test** Kupiec (1995): is the exception rate equal to α?

$$LR_{uc} = -2 \ln \frac{(1-\alpha)^{n-x}\,\alpha^{x}}{(1-\hat{\pi})^{n-x}\,\hat{\pi}^{x}}$$

- Reading
  - numerator: the likelihood of the x exceptions under the rate α; denominator: under the observed rate $\hat{\pi}$
  - $LR_{uc} \sim \chi^2(1)$ if the rate is α; reject at 5% if $LR_{uc} > 3.84$ (p-value below 0.05)
  - the test counts the exceptions only; their clustering needs the Christoffersen test (Chapter 10)

Backtesting Bitcoin VaR 1%



- One-day forecasts since 1 January 2018: HS on the last 365 days; GARCH(1,1)-t re-estimated every 365 days on all earlier data
- Exceptions (expected 31.8): HS 29 (0.91%), GARCH-t 41 (1.29%); Kupiec: $LR_{uc} = 0.26$ (p $= 0.61$) and 2.45 (p $= 0.12$)
- Interpretation: both pass the Kupiec test; HS needs a higher VaR on average (9.5% against 8.5%), GARCH-t reacts faster but is exceeded more often (2018: 9 exceptions)

SFM_ch14_crypto_var

An Open Research Question

- **Do stablecoin flows move crypto prices?**
  - Griffin & Shams (2020) found that Tether issuance supported Bitcoin in 2017; the stablecoin market is now about 74 times larger than in 2020
  - starter result: weekly log change of the stablecoin supply against Bitcoin's return, 349 weeks since 2020
- Same week: correlation 0.26; next week: 0.07, slope 0.19 ($t = 1.20$, p $= 0.23$, $R^2 = 0.4\%$)
  - flows and prices move together; flows do not predict the next week: cause, effect or common demand?
- AI tools can speed up such a study; they do not replace checking it Wang et al. (2023)

Possible Uses of AI Tools

- **Literature**: a list of studies on stablecoin issuance and crypto prices, with their data and periods
- **Data**: code that downloads the supply of each stablecoin from DefiLlama and aligns it with daily prices
- **Robustness**: daily and weekly data, USDT and USDC separately, sub-periods, Granger causality in both directions
- Example prompt
  - `Write Python code that downloads the daily supply of USDT and USDC from the DefiLlama stablecoins API, computes weekly log changes, joins them with weekly Bitcoin log returns, and tests whether supply changes Granger-cause returns and returns Granger-cause supply changes, with HAC standard errors.`

What to Check

- Annualisation: 365 days for crypto, 252 for equities; never one number for both
- Alignment: prices joined on common days before returns; the same week definition for both series
- Units: basis points against percent (285 bp $= 2.85\%$); billions against millions in supply data
- VaR convention: VaR 1% is a positive loss; never "VaR 99%"
- Look-ahead: the supply series must be known at the date of the forecast; revisions of the data provider count
- References: every cited paper must exist; check the DOI

Project Idea

- **Question**: do weekly changes in USDT and USDC supply help to forecast Bitcoin's return or volatility, and did this change after the spot ETFs?
  - data: DefiLlama supply (public, no key) and the course data for prices
- Steps
  - weekly log changes; correlation and Granger tests in both directions with HAC standard errors
  - out-of-sample forecasts of next week's return and of the absolute return, against a random walk
  - sub-periods: before and after Terra (May 2022) and the spot ETFs (January 2024)
- Deliverable: one table, one chart, and a paragraph on what the data can and cannot show
- Declare any AI use, and list the errors of the AI that you corrected

Key Takeaways

- Bitcoin: a public ledger secured by proof of work, with a fixed supply of 21 million; Ethereum runs smart contracts under proof of stake
- Annualise with 365 days; crypto volatility is 3.8 to 6.7 times that of the S&P 500, with heavy tails and IGARCH-like clustering
- Robust VR tests do not reject weak-form efficiency since 2014; the weekend is calmer in variance, not different in mean
- Correlation with equities rose from about 0 to about 0.39 after 2020; Bitcoin is no safe haven
- Drawdowns of more than 50% are regular; the ETF effect on volatility is not significant
- Stablecoins hold a peg by arbitrage and reserves; USDC 2023 and UST 2022 show the two ways a peg breaks

Key Formulas

Quantity	Formula
Bitcoin supply	$210\,000 \times 50 \times \sum_{j \geq 0} 2^{-j} = 21$ million
Annualisation	mean $A\bar{r}$, volatility $\sqrt{A}\, s$; $A = 365$ (crypto), 252 (equities)
Hill	$\hat{\alpha} = \left[\frac{1}{k} \sum_{i \leq k} \ln(X_{(i)}/X_{(k+1)}) \right]^{-1}$
GARCH(1,1)	$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$, half-life $\frac{\ln 0.5}{\ln(\alpha+\beta)}$
Variance ratio	$\mathrm{VR}(q) = \mathrm{Var}(r_t + \cdots + r_{t-q+1})/(q\,\mathrm{Var}(r_t))$
Drawdown	$D_t = P_t / \max_{s \leq t} P_s - 1$; recovery gain $d/(1-d)$
Peg deviation	$d_t = 10\,000\,(P_t - 1)$ bp; half-life $\ln 0.5 / \ln \phi$
VaR in money	$W(1 - e^{-\mathrm{VaR}/100})$, $\mathrm{VaR}_\alpha = -q_\alpha$

Self-Assessment

- **Question**: a fund reports a Bitcoin volatility of 55% a year from a daily standard deviation of 3.5%. Is it right?
  - **Answer**: no: $3.5 \times \sqrt{252} \approx 55.6\%$; with 365 days it is $3.5 \times \sqrt{365} \approx 66.9\%$
- **Question**: USDT closes at 1.0040 USD. What is the peg deviation?
  - **Answer**: $10\,000 \times 0.0040 = 40$ bp above the peg: demand for USDT exceeds what arbitrage has supplied
- **Question**: why was UST lost while USDC returned to the peg?
  - **Answer**: USDC had dollar reserves whose bank deposits were guaranteed; UST was backed only by LUNA, whose value fell with UST
- Further reading by the author of the course: Pele et al. (2023) (are cryptos alternative assets?) and Pele & Mazurencu-Marinescu-Pele (2019) (Bitcoin VaR)
- Next: Chapter 15, systemic risk

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