








Statistics of Financial Markets

Chapter 7: Efficient markets, random walk and variance-ratio tests



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Acronyms used in this material (1/2)

- **ACF**: Autocorrelation Function
- **ADF**: Augmented Dickey–Fuller (test)
- **AI**: Artificial Intelligence
- **AIC**: Akaike Information Criterion
- **AMH**: Adaptive Markets Hypothesis
- **AR**: AutoRegressive (model)
- **BET**: Bucharest Exchange Trading (index)
- **BET-TR**: BET Total Return (index)
- **BUX**: Budapesti Értéktőzsde index (the Budapest Stock Exchange index)
- **BVB**: Bursa de Valori București (the Bucharest Stock Exchange)
- **CAPM**: Capital Asset Pricing Model
- **CD**: Chow–Denning (multiple variance-ratio test)
- **CME**: Chicago Mercantile Exchange
- **COVID-19**: COronaVIrus Disease 2019
- **EMH**: Efficient Market Hypothesis
- **EODHD**: EOD Historical Data (market-data provider)
- **EU**: European Union
- **FTSE**: Financial Times Stock Exchange (FTSE Russell, index provider)

Acronyms used in this material (2/2)

- **GARCH**: Generalised AutoRegressive Conditional Heteroskedasticity
- **GMM**: Generalized Method of Moments
- **KPSS**: Kwiatkowski–Phillips–Schmidt–Shin (stationarity test)
- **LB**: Ljung–Box (test)
- **MA**: Moving Average (model)
- **MIT**: Massachusetts Institute of Technology
- **OLS**: Ordinary Least Squares
- **PP**: Phillips–Perron (unit-root test)
- **PX**: Prague Stock Exchange index (index Burzy cenných papírů Praha)
- **RW1**: Random Walk 1 (i.i.d. increments)
- **RW2**: Random Walk 2 (independent increments)
- **RW3**: Random Walk 3 (uncorrelated increments)
- **SFE**: Statistics of Financial Markets, Exercises (Quantlet collection of the textbook)
- **VR**: Variance Ratio
- **WIG20**: Warszawski Indeks Giełdowy 20 (the Warsaw Stock Exchange index of the 20 largest companies)

Today's Question and Route

- **Question**: can yesterday's prices tell us anything about tomorrow's return?
  - if yes, a simple rule could beat the market; if no, the market is efficient in the weak sense
  - Chapter 4 defined the random walk and the martingale; here we test them on real prices
- **Route** of the chapter
  - efficient markets: three forms, the joint-hypothesis problem
  - random walk hypotheses RW1, RW2, RW3 and the martingale
  - white noise and the autocorrelation function; Ljung–Box and runs tests
  - unit roots: ADF, Phillips–Perron, KPSS; prices against returns
  - variance-ratio tests; efficiency across markets and over time; anomalies

Learning Outcomes

- State the three forms of market efficiency and explain why every test is a joint test
- Tell RW1, RW2, RW3 and the martingale apart, and say which of them volatility clustering violates
- Read a sample ACF with the right confidence band and compute Box–Pierce, Ljung–Box and runs statistics
- Run and combine ADF, Phillips–Perron and KPSS tests on prices and on returns
- Compute a variance ratio and its robust test, for one horizon and for several horizons together
- Judge, on real data, which markets look efficient, and whether this changes over time

Reading and Tools

- Textbook: Franke, Härdle & Hafner (2019), Ch. 11 (11.3 expectations and efficient markets, 11.5 random walk tests, 11.6 unit root tests)
  - exercises with solutions: Borak, Härdle & López-Cabrera (2013), Ch. 11
- Econometrics of predictability: Campbell, Lo & MacKinlay (1997), Ch. 2 (random walks, variance ratios) and Ch. 4 (event studies)
- Python Quantlets of this chapter: Quantlets/Ch_07
  - ported from the SFE Quantlets SFEtimewn, SFEacfar1, SFEacfar2, SFEacfma1 and SFEacfma2
- Lecture notebook: open in Google Colab
- Video course: Efficiency of cryptocurrency markets – A GMM-based analysis

Returns above the Market

- If past prices predicted future returns, a simple rule would earn money with no risk
  - traders would use the rule; their trades would move prices until the pattern disappears
- The evidence on professional investors
  - mutual funds, on average, did not beat a buy-and-hold portfolio after costs Jensen (1968)
  - few funds show skill large enough to cover their costs Fama & French (2010)
- A survey of the arguments for and against: Malkiel (2003)



The trading floor of the New York Stock Exchange, 1963

Photo: Thomas J. O'Halloran (1963), Library of Congress; public domain; Wikimedia Commons

1900: Prices as a Random Walk

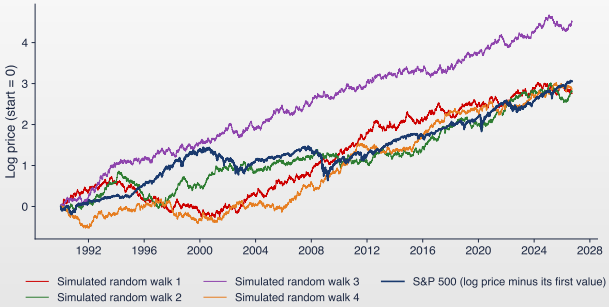
- Louis Bachelier, *Théorie de la spéculation* Bachelier (1900)
  - the first mathematical model of prices: the change of a price is a sum of many small random shocks
  - the expected gain of the speculator is zero
- Kendall (1953): 22 British price series behave like a "wandering" series
  - the changes from one week to the next look like independent draws
- Roberts (1959): simulated random walks show "patterns" that chartists see in real prices



Louis Bachelier (1870–1946)

Photo: unknown author; public domain; Wikimedia Commons

Question for the Room: Which Path Is the S&P 500?



- Four random walks with i.i.d. Normal steps (mean 0.033%, standard deviation 1.13% per day, as the S&P 500) and the S&P 500 log price since 1990
- **Question for the room**: without the legend, could you tell which path is real?
- **Answer**: hardly; booms and long falls appear in pure noise too (end values: 2.76 to 4.50; S&P 500: 3.06)

SFM_ch7_random_walk

1965: Properly Anticipated Prices Fluctuate Randomly

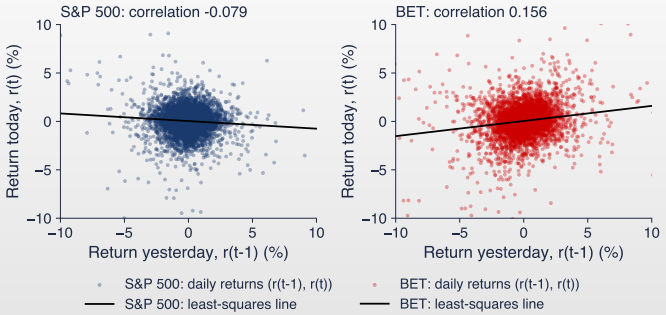
- Paul Samuelson Samuelson (1965): if prices already contain all that traders expect, the next change is unpredictable
  - formally: the price is a **martingale**, $E[P_{t+1} \mid \mathcal{F}_t] = P_t$ (Chapter 4)
  - $\mathcal{F}_t$: the information available at time t
- Randomness of price changes is a sign of a market that works, not of chaos
- Fama (1965): daily returns of the 30 Dow Jones stocks show almost no autocorrelation



Paul A. Samuelson (1915–2009), Nobel Prize 1970

Photo: Bernard Gotfryd (1970–1975); public domain; Wikimedia Commons

Yesterday's Return and Today's Return



- Each point: (return yesterday, return today); the line: least squares; its slope is the first autocorrelation $\hat{\rho}(1)$
- S&P 500: $\hat{\rho}(1) = -0.079$ (a slight reversal); BET: $\hat{\rho}(1) = 0.156$ (a slight continuation); are these numbers different from zero?

SFM_ch7_random_walk

Definition of an Efficient Market

- **EMH** (efficient market hypothesis) Fama (1970): prices "fully reflect" the available information Ω_t
  - no trading rule based on Ω_t earns more than the return that compensates its risk
  - prices move only on **news**: information that was not in Ω_t
- Efficiency is relative to an information set
  - the smaller Ω_t, the weaker the claim and the easier the test
- Efficient does not mean "returns are zero": the expected return pays for risk



Eugene F. Fama, Nobel Prize 2013

Photo: Bengt Nyman (2013); CC BY 2.0; Wikimedia Commons

Three Forms of Efficiency

Form	Information set Ω_t	What cannot earn excess returns	Typical test
Weak	past prices and returns	technical analysis, chart patterns	autocorrelation, runs, variance ratio
Semi-strong	all public information	fundamental analysis of public data	event studies
Strong	all information, also private	insider information	returns of insiders and fund managers

- The forms are nested: strong $\Rightarrow$ semi-strong $\Rightarrow$ weak; a rejection of the weak form rejects all three
- This chapter tests mainly the weak form: past prices are free data for everyone Fama (1970), Fama (1991)

The Fair-Game Model

- Let r_{t+1} be the return of an asset and $E[r_{t+1} \mid \Omega_t]$ its **equilibrium expected return** (the return that pays for its risk)
  - the **excess return** (surprise): $z_{t+1} = r_{t+1} - E[r_{t+1} \mid \Omega_t]$
- **Fair game**: $E[z_{t+1} \mid \Omega_t] = 0$
  - forecast errors are zero on average whatever we know at time t
  - a rule that invests $w(\Omega_t)$ earns $E[w(\Omega_t)\, z_{t+1}] = 0$ in excess of equilibrium
- With a constant expected return μ: $E[r_{t+1} \mid \Omega_t] = \mu$; returns minus μ are a **martingale difference**
- Nothing here says the variance is constant or the distribution is Normal

The Joint-Hypothesis Problem

- To test efficiency we must know the equilibrium expected return $E[r_{t+1} \mid \Omega_t]$
  - this needs a model of equilibrium: constant mean, CAPM (capital asset pricing model), factor models
- Every test of efficiency is a **joint test** of efficiency and of the model Fama (1991)
  - a rejection may mean an inefficient market, or a wrong model of expected returns
  - efficiency alone can never be rejected
- For daily returns the problem is small: daily expected returns are tiny (about 0.033% for the S&P 500), so any reasonable model gives almost the same answer

The Grossman–Stiglitz Paradox

- Information is costly to collect and to analyse Grossman & Stiglitz (1980)
  - if prices reflected all information perfectly, nobody would be paid for collecting it
  - then nobody would collect it, and prices could not reflect it
- Conclusion: a market can only be **nearly** efficient
  - small inefficiencies pay informed traders for their costs
  - the question becomes "how efficient, where and when?", not "yes or no?"
- This view leads to the adaptive markets hypothesis of Section 8

The 2013 Nobel Prize: Fama and Shiller



Eugene F. Fama: prices are hard to predict over days and weeks

Photo: Bengt Nyman (2013); CC BY 2.0; Wikimedia Commons



Robert J. Shiller: prices swing too much compared with fundamentals

Photo: Bengt Nyman (2013); CC BY 2.0; Wikimedia Commons

- Both are right, at different horizons: short-run returns are almost unpredictable; long-run returns are partly predictable from valuation ratios

The Random Walk Model of Log Prices

- Log price $p_t = \ln P_t$; **random walk with drift** μ: $p_t = \mu + p_{t-1} + \varepsilon_t$
  - the return $r_t = p_t - p_{t-1} = \mu + \varepsilon_t$: the increments ε_t have mean 0
  - by recursion: $p_t = p_0 + \mu t + \sum_{s=1}^{t} \varepsilon_s$: every shock is **permanent**
- If the ε_t are uncorrelated with variance σ^2:
  - $E[p_t] = p_0 + \mu t$ and $\mathrm{Var}(p_t - p_0) = t\sigma^2$: the uncertainty grows with the horizon
  - the q-day return $r_t(q) = r_t + \cdots + r_{t-q+1}$ has $\mathrm{Var}(r_t(q)) = q\sigma^2$
- Log prices, not prices: P_t cannot become negative, and log returns add over time (Chapter 1)

Three Random Walk Hypotheses

- **RW1** (i.i.d. increments): ε_t independent and identically distributed, $\varepsilon_t \sim \mathrm{IID}(0, \sigma^2)$ Campbell, Lo & MacKinlay (1997)
  - the strongest: no function of the past predicts any function of the future
- **RW2** (independent increments): ε_t independent, but not identically distributed
  - allows the variance to change over time in a way that is not predictable from the past (regimes)
- **RW3** (uncorrelated increments): $\mathrm{Cov}(\varepsilon_t, \varepsilon_{t-k}) = 0$ for all $k \neq 0$
  - the weakest: allows $\mathrm{Cov}(\varepsilon_t^2, \varepsilon_{t-k}^2) \neq 0$, that is, volatility clustering
- Nested: RW1 $\Rightarrow$ RW2 $\Rightarrow$ RW3; tests of RW3 use only autocorrelations of returns

Comparison of the Random Walk Hypotheses

Property of the returns	RW1	RW2	RW3	**Martingale**
$\mathrm{Corr}(r_t, r_{t-k}) = 0$	yes	yes	yes	yes
$E[r_t \mid \text{past}] = \mu$ (unpredictable mean)	yes	yes	not always	yes
constant variance	yes	no	no	no
volatility clustering allowed	no	no	yes	yes
GARCH returns (Chapter 9) satisfy it	no	no	yes	yes

- **GARCH** (generalised autoregressive conditional heteroskedasticity): the variance of tomorrow depends on today's shocks; the mean stays unpredictable
- Weak-form efficiency needs only a martingale (or RW3), not RW1

Martingale and Random Walk

- $\{p_t - \mu t\}$ is a **martingale** if $E[p_{t+1} - \mu(t+1) \mid \mathcal{F}_t] = p_t - \mu t$ Samuelson (1965)
  - equivalently, $E[r_{t+1} - \mu \mid \mathcal{F}_t] = 0$: the returns minus μ are a martingale difference
  - a martingale difference with finite variance is uncorrelated: martingale $\Rightarrow$ RW3
- RW1 and RW2 imply the martingale; the converse fails: a martingale may have predictable variance
- **Question for the room**: does volatility clustering contradict weak-form efficiency?
  - **Answer**: no; it rejects RW1 and RW2, but a predictable variance gives no predictable return; the martingale and RW3 survive

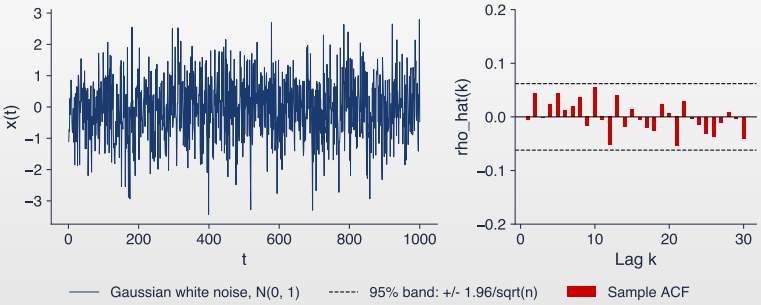
Worked Example: Risk as a Function of the Horizon

- S&P 500 since 1990: daily standard deviation $\hat{\sigma} = 1.1337\%$
  - under a random walk, $\mathrm{sd}(r_t(q)) = \sigma\sqrt{q}$
- Step by step
  - one week, $q = 5$: $1.1337 \times \sqrt{5} = 1.1337 \times 2.236 = 2.53\%$
  - one month, $q = 20$: $1.1337 \times 4.472 = 5.07\%$
  - one year, $q = 250$: $1.1337 \times 15.811 = 17.92\%$
- This is the "square-root-of-time rule" used to scale VaR and volatility
- If returns are autocorrelated, the rule fails: the variance-ratio test of Section 7 measures by how much

White Noise and Autocorrelation

- **Autocovariance** at lag k: $\gamma(k) = \mathrm{Cov}(x_t, x_{t-k})$; **autocorrelation**: $\rho(k) = \gamma(k)/\gamma(0)$
  - the **ACF** (autocorrelation function): $k \mapsto \rho(k)$, $k = 1, 2, \ldots$
- **White noise**: mean 0, constant variance σ^2, $\rho(k) = 0$ for every $k \neq 0$
  - i.i.d. noise is white noise; Gaussian white noise is i.i.d. $N(0, \sigma^2)$
  - white noise need not be independent: GARCH returns are white noise but not i.i.d.
- **Sample ACF**: $\hat{\rho}(k) = \sum_{t=k+1}^{T}(x_t - \bar{x})(x_{t-k} - \bar{x}) \big/ \sum_{t=1}^{T}(x_t - \bar{x})^2$
  - for i.i.d. data, $\hat{\rho}(k) \approx N(0, 1/T)$: 95% band $\pm 1.96/\sqrt{T}$

Gaussian White Noise and Its Sample ACF



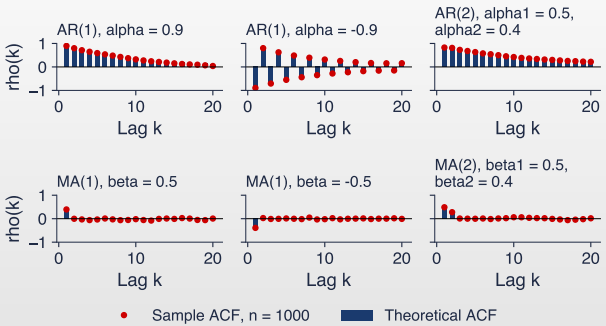
- Left: 1 000 draws from $N(0,1)$ (as in SFEtimewn); right: $\hat{\rho}(1), \ldots, \hat{\rho}(30)$ with the band $\pm 1.96/\sqrt{1000} = \pm 0.062$
- Here 0 of 30 autocorrelations fall outside the band; with 30 lags we expect about 1.5 by chance alone

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The ACF of AR and MA Processes

- **AR(1)** (autoregressive): $x_t = \alpha x_{t-1} + \varepsilon_t$, $|\alpha| < 1$: $\rho(k) = \alpha^k$
  - geometric decay; alternating signs if $\alpha < 0$; example: $\alpha = 0.9 \Rightarrow \rho(3) = 0.9^3 = 0.729$
- **AR(2)**: $x_t = \alpha_1 x_{t-1} + \alpha_2 x_{t-2} + \varepsilon_t$: $\rho(1) = \alpha_1/(1-\alpha_2)$, $\rho(k) = \alpha_1\rho(k-1) + \alpha_2\rho(k-2)$
  - $\alpha_1 = 0.5$, $\alpha_2 = 0.4$: $\rho(1) = 0.5/0.6 = 0.833$, $\rho(2) = 0.5 \times 0.833 + 0.4 = 0.817$
- **MA(1)** (moving average): $x_t = \varepsilon_t + \beta\varepsilon_{t-1}$: $\rho(1) = \beta/(1+\beta^2)$, $\rho(k) = 0$ for $k \geq 2$
  - $\beta = 0.5$: $\rho(1) = 0.5/1.25 = 0.40$; an MA($q$) has an ACF that cuts off after lag q
- The shape of the ACF tells the type of dependence: slow decay (AR), cut-off (MA), nothing (white noise)

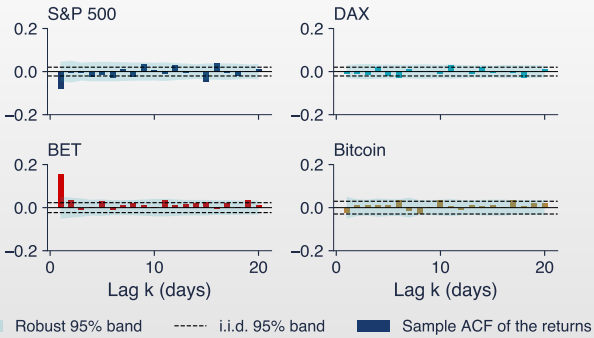
Theoretical and Sample ACF of Six Processes



- Bars: theoretical $\rho(k)$; points: sample ACF of one simulated path of 1 000 values (SFEacfar1, SFEacfar2, SFEacfma1, SFEacfma2)
- A random walk is the limit $\alpha \to 1$ of the AR(1): its ACF would not decay at all

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The ACF of Daily Returns



- Bars: $\hat{\rho}(1), \ldots, \hat{\rho}(20)$; dashed: the i.i.d. band $\pm 1.96/\sqrt{T}$; shaded: the robust band (next slide)

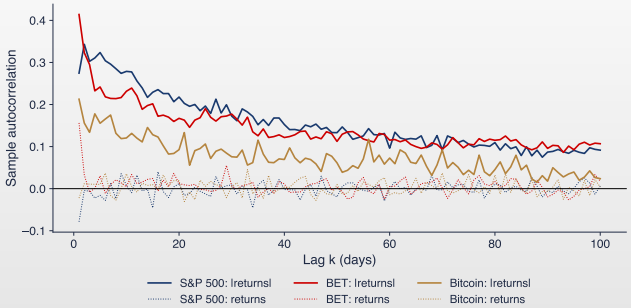
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Reading the ACF of Returns

- First autocorrelations: S&P 500 -0.079, DAX -0.009, BET 0.156, Bitcoin -0.022
  - only the S&P 500 and the BET stand out at lag 1, with opposite signs
- The **robust** band: $\pm 1.96\sqrt{\hat{\delta}(k)/T}$, $\hat{\delta}(k) = T\sum_t e_t^2 e_{t-k}^2 / (\sum_t e_t^2)^2$, $e_t = r_t - \bar{r}$ Lo & MacKinlay (1988)
  - i.i.d. returns: $\hat{\delta}(k) \approx 1$; volatility clustering: large returns follow large returns, so $\hat{\delta}(k) > 1$
  - S&P 500 at lag 1: ± 0.046 instead of ± 0.020
- S&P 500, lags outside the band out of 20: 9 with the i.i.d. band, 3 with the robust band
- The i.i.d. band tests RW1; the robust band tests RW3, the hypothesis that matters for efficiency

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Autocorrelation of Returns and of Absolute Returns



- ACF of $|r_t|$: S&P 500 0.27 at lag 1, 0.22 at lag 20, 0.09 at lag 100; the returns themselves: dotted lines near 0
- Large moves follow large moves: RW1 and RW2 are rejected by eye; RW3 and the martingale are not touched by this chart

SFM_ch7_autocorrelation_tests

Portmanteau Tests: Box–Pierce and Ljung–Box

- H_0: $\rho(1) = \cdots = \rho(m) = 0$; H_1: at least one $\rho(k) \neq 0$; one test for m lags together
  - "portmanteau": a suitcase that carries many autocorrelations at once
- Box–Pierce Box & Pierce (1970): $Q_{BP}(m) = T \sum_{k=1}^{m} \hat{\rho}(k)^2$
  - T: the number of returns; m: the number of lags tested; squares, so positive and negative autocorrelations both count
- **LB** (Ljung–Box) Ljung & Box (1978): $Q_{LB}(m) = T(T+2) \sum_{k=1}^{m} \frac{\hat{\rho}(k)^2}{T-k}$, closer to χ^2 in small samples
  - under i.i.d. returns both $\sim \chi^2(m)$; reject H_0 if $Q > \chi^2_{0.95}(m)$; $\chi^2_{0.95}(10) = 18.31$
- Choice of m: here $m = 10$ (two trading weeks); a larger m dilutes a short-lived effect

Worked Example: Ljung–Box Step by Step

- $T = 1000$ daily returns, $\hat{\rho}(1) = 0.06$, $\hat{\rho}(2) = -0.04$, $\hat{\rho}(3) = 0.03$; test with $m = 3$
  - $T\hat{\rho}(k)^2$: 3.6, 1.6, 0.9; their sum: $Q_{BP}(3) = 6.10$
  - $Q_{LB}(3) = 1000 \times 1002 \times (0.06^2/999 + 0.04^2/998 + 0.03^2/997) = 6.12$
- Decision: $\chi^2_{0.95}(3) = 7.81 > 6.12$: do not reject H_0 at 5%
  - note: $\hat{\rho}(1) = 0.06$ lies just inside the band $\pm 1.96/\sqrt{1000} = \pm 0.062$; the joint test does not reject either
- The same numbers with $T = 5000$: Q five times larger, a clear rejection: small autocorrelations become significant in long samples

A Portmanteau Test Robust to Volatility Clustering

- The χ^2 law of Q_{LB} needs $\mathrm{Var}(\hat{\rho}(k)) = 1/T$: true for i.i.d. returns, false with volatility clustering
  - then Q_{LB} rejects a true martingale far too often
- **Robust** statistic: divide each $\hat{\rho}(k)^2$ by its own variance $\hat{\delta}(k)/T$ Escanciano & Lobato (2009)
  - $\tilde{Q}(m) = \sum_{k=1}^{m} \frac{T\hat{\rho}(k)^2}{\hat{\delta}(k)} \sim \chi^2(m)$ for a martingale difference with finite fourth moment
  - with i.i.d. returns $\hat{\delta}(k) \approx 1$ and $\tilde{Q} \approx Q_{BP}$
- Rule: test RW1 with Q_{LB}; test the martingale (weak-form efficiency) with $\tilde{Q}$

Autocorrelation Tests on Real Data

	T	$\hat{\rho}(1)$	$Q_{LB}(10)$	p	$\tilde{Q}(10)$	p	$Q_{LB}(10)$, r^2	runs z	p
S&P 500	9 240	−0.079	90.8	<0.001	18.8	0.043	8015	2.97	0.003
DAX	9 288	−0.009	21.8	0.016	8.6	0.571	3747	1.73	0.084
BET	7 243	0.156	198.3	<0.001	42.9	<0.001	2500	−7.75	<0.001
Bitcoin	4 384	−0.022	20.2	0.028	11.9	0.294	213	2.68	0.007
TLV	3 788	0.010	29.1	0.001	13.2	0.210	481	−1.68	0.093
SNP	3 815	−0.015	20.7	0.023	9.9	0.446	505	1.72	0.085
BRD	3 771	0.032	19.9	0.030	10.8	0.375	416	−3.04	0.002
TGN	3 725	0.055	28.7	0.001	12.3	0.263	639	1.49	0.136

- Classic LB: all eight series reject at 5%; robust $\tilde{Q}$: only the S&P 500 (p-value: 0.043) and the BET (p-value: <0.001)
- Squared returns: huge Q_{LB} everywhere: RW1 and RW2 are rejected for every series (volatility clustering)
- Start of the series: S&P 500 and DAX 1990; BET 1997; Bitcoin 2014; BVB stocks 2010 (TLV without 30–31 May 2016, a data error, Chapter 2)

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The Runs Test

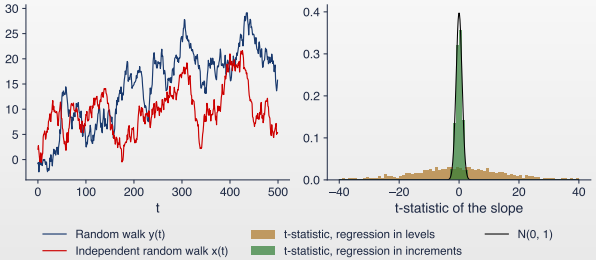
- A **run**: a sequence of returns with the same sign; $++-+++--$ has 4 runs Wald & Wolfowitz (1940)
  - R: the observed number of runs
  - the test uses only the signs: it is **nonparametric**, insensitive to heavy tails and to volatility
- With n_1 positive and n_2 negative returns, $n = n_1 + n_2$, under independence:
  - $E[R] = \dfrac{2n_1n_2}{n} + 1, \quad \mathrm{Var}(R) = \dfrac{2n_1n_2(2n_1n_2 - n)}{n^2(n-1)}, \quad z = \dfrac{R - E[R]}{\sqrt{\mathrm{Var}(R)}} \approx N(0,1)$
- Too few runs ($z < 0$): signs persist, positive dependence; too many runs ($z > 0$): signs alternate
  - BET: 3 278 runs against 3 606 expected, $z = -7.75$; S&P 500: 4 738 against 4 596, $z = 2.97$
  - the same directions as $\hat{\rho}(1)$: continuation in the BET, reversal in the S&P 500

SFM_ch7_autocorrelation_tests

Stationary and Integrated Series

- **(Weakly) stationary** series: constant mean, constant variance, $\mathrm{Cov}(x_t, x_{t-k})$ depends only on k
  - shocks fade away; the series returns to its mean; notation $I(0)$
- **Integrated of order 1**, $I(1)$: not stationary, but its first difference $\Delta x_t = x_t - x_{t-1}$ is
  - the random walk is $I(1)$: p_t wanders, $\Delta p_t = r_t$ is stationary
- **Unit root**: in $p_t = \phi\, p_{t-1} + \varepsilon_t$ the case $\phi = 1$; $|\phi| < 1$ gives a stationary AR(1)
  - the name: the root of $1 - \phi z = 0$ is $z = 1/\phi = 1$
- Question of this section: are log prices $I(1)$ and returns $I(0)$?

Practical Consequence: Spurious Regression



- Regress one random walk on another, independent one ($T = 500$, 2 000 repetitions) Granger & Newbold (1974)
  - $|t| > 1.96$ in 90% of cases, median $R^2 = 0.17$: a "relation" that does not exist
- On the increments: $|t| > 1.96$ in 5.5% of cases, as it should be
  - lesson: test for unit roots before regressing levels ($|t| > 40$ in 2.6% of cases, not drawn)

SFM_ch7_unit_roots

The Dickey–Fuller Test

- Subtract p_{t-1}: $\Delta p_t = c + \gamma\, p_{t-1} + \varepsilon_t$ with $\gamma = \phi - 1$ Dickey & Fuller (1979)
  - $\Delta p_t = p_t - p_{t-1}$; c: a constant; $\gamma < 0$ pulls p_t back towards a level
  - H_0: $\gamma = 0$ (unit root); H_1: $\gamma < 0$ (stationary); a one-sided test
  - statistic: $\tau = \hat{\gamma}/\mathrm{SE}(\hat{\gamma})$, the usual t-ratio of OLS (ordinary least squares)
- Under H_0, τ does **not** follow a t or Normal law: its critical values are more negative
  - 5% critical values MacKinnon (1996): -2.86 with a constant, -3.41 with a constant and a trend (Normal: -1.645)
- Three specifications: no constant; constant c; constant and trend $c + bt$
  - log prices drift upwards: use constant and trend; returns: constant only

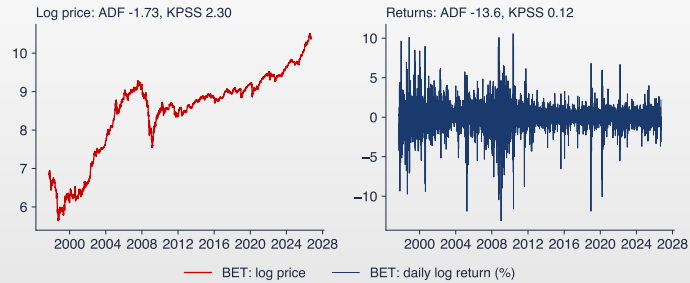
The Augmented Test and a Worked Example

- **ADF** (augmented Dickey–Fuller) Said & Dickey (1984): add lagged differences to remove autocorrelation from ε_t
  - $\Delta p_t = c + bt + \gamma\, p_{t-1} + \sum_{j=1}^{L} \delta_j \Delta p_{t-j} + \varepsilon_t$; same τ, same critical values
  - bt: a linear trend; δ_j: the coefficients of the L lagged differences
  - L chosen by AIC (Akaike information criterion, Chapter 6), at most $12(T/100)^{1/4}$
- **Worked example**: BET log price, constant, no lags, $T = 7\,243$
  - OLS: $\hat{\gamma} = -0.0000138$, $\mathrm{SE}(\hat{\gamma}) = 0.000158$
  - $\tau = -0.0000138/0.000158 = -0.09 > -2.86$: do not reject the unit root
  - here even a usual t-test (critical value -1.645) would not reject; but a τ between -2.86 and -1.645 would be rejected by the wrong (Normal) critical value and kept by the right one

Phillips–Perron and KPSS

- **PP** (Phillips–Perron) Phillips & Perron (1988): the Dickey–Fuller regression without lags; τ is corrected for autocorrelation and heteroskedasticity
  - the correction uses a long-run variance with Bartlett weights Newey & West (1987); same H_0 and critical values as ADF
  - long-run variance $\hat{\lambda}^2 = \hat{\gamma}(0) + 2\sum_{j=1}^{L}\left(1 - \frac{j}{L+1}\right)\hat{\gamma}(j)$, with $\hat{\gamma}(j)$ the autocovariances of the residuals
- **KPSS** (Kwiatkowski–Phillips–Schmidt–Shin) Kwiatkowski et al. (1992): the hypotheses are **reversed**
  - H_0: stationary (around a constant or a trend); H_1: unit root
  - $\mathrm{KPSS} = \sum_{t=1}^{T} S_t^2/(T^2\hat{\lambda}^2)$, S_t the partial sums of the residuals, $\hat{\lambda}^2$ their long-run variance
  - reject stationarity if $\mathrm{KPSS} > 0.463$ (constant) or > 0.146 (trend), at 5%
- Use both: ADF/PP ask "is there evidence against a unit root?"; KPSS asks "is there evidence against stationarity?"
  - ADF does not reject and KPSS rejects: $I(1)$; ADF rejects and KPSS does not: $I(0)$; both or neither reject: inconclusive

The BET: Log Price and Returns



- Log price: ADF does not reject a unit root ($p = 0.736$), KPSS rejects stationarity: $I(1)$
- Returns: ADF rejects, KPSS does not: stationary, $I(0)$; all statistics on the next slide
- The 2008 fall looks like a temporary deviation, but the tests say: permanent shocks, no return to a trend line

SFM_ch7_unit_roots

Unit-Root Tests on Prices and Returns

	log price (constant, trend)			returns (constant)		
	ADF	PP	KPSS	ADF	PP	KPSS
S&P 500	−1.47	−1.57	2.23	−17.61	−105.05	0.12
DAX	−2.69	−2.65	0.94	−23.30	−97.40	0.04
BET	−1.73	−1.54	2.30	−13.65	−74.47	0.12
Bitcoin	−1.48	−1.54	1.81	−20.16	−67.84	0.13
TLV	−3.46	−3.35	0.57	−12.96	−60.79	0.23
SNP	−0.80	−0.74	2.50	−19.50	−62.71	0.29
BRD	−2.18	−2.19	1.64	−59.46	−59.53	0.29
TGN	−0.96	−0.92	0.71	−39.90	−57.83	0.22

- 5% critical values: ADF and PP −3.41 (trend), −2.86 (constant); KPSS 0.146 (trend), 0.463 (constant)
- Log prices: no ADF/PP rejection except TLV (ADF p = 0.044); KPSS rejects stationarity for all: prices are $I(1)$
- Returns: ADF and PP reject strongly; KPSS never rejects (p $>$ 0.10): returns are $I(0)$

SFM_ch7_unit_roots

Unit Roots and Market Efficiency

- A unit root in prices is **necessary** for a random walk, but **not sufficient**
  - example: $r_t = 0.3\, r_{t-1} + \varepsilon_t$ makes p_t an $I(1)$ series whose increments are predictable
  - ADF cannot distinguish it from a random walk; the BET has a unit root and autocorrelated returns
- A stationary price (no unit root) would contradict efficiency: prices would return to a predictable level
  - TLV: ADF rejects at 5% with a trend, KPSS also rejects: an inconclusive case, not a profitable rule
- Unit-root tests have low power against ϕ close to 1: the efficiency question is about the increments, and needs tests on returns

The Variance Ratio: Definition

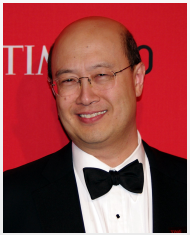
- Under a random walk, $\mathrm{Var}(r_t(q)) = q\,\mathrm{Var}(r_t)$; the **VR** (variance ratio) compares the two sides
  - $\mathrm{VR}(q) = \dfrac{\mathrm{Var}(r_t(q))}{q\,\mathrm{Var}(r_t)} = 1 + 2\sum_{k=1}^{q-1}\left(1 - \dfrac{k}{q}\right)\rho(k)$ Lo & MacKinlay (1988), Campbell, Lo & MacKinlay (1997)
- Reading the number
  - $\mathrm{VR}(q) = 1$: random walk (RW1, RW2, RW3 all give 1)
  - $\mathrm{VR}(q) > 1$: positive autocorrelation, **momentum** (moves continue)
  - $\mathrm{VR}(q) < 1$: negative autocorrelation, **mean reversion** (moves are partly undone)
- One number summarises $q - 1$ autocorrelations with declining weights: more power against persistent, small autocorrelations

Worked Example: VR from the ACF of the S&P 500

- $\hat{\rho}(1), \ldots, \hat{\rho}(4)$ of the S&P 500: -0.079, -0.004, -0.006, -0.022
  - $q = 2$: $\mathrm{VR}(2) = 1 + \hat{\rho}(1) = 0.921$
  - $q = 5$: weights $2(1 - k/5) = 1.6, 1.2, 0.8, 0.4$: $\mathrm{VR}(5) = 1 + 1.6\hat{\rho}(1) + 1.2\hat{\rho}(2) + 0.8\hat{\rho}(3) + 0.4\hat{\rho}(4) = 0.855$
- The Lo–MacKinlay estimator on the same data: $\mathrm{VR}(2) = 0.922$, $\mathrm{VR}(5) = 0.856$
  - almost the same numbers: the two forms differ only by small-sample corrections
- Meaning: the weekly variance is about 0.856 of five daily variances: daily moves of the S&P 500 are partly reversed within a week

Lo and MacKinlay (1988): Estimator and Tests

- With T returns, mean $\hat{\mu}$ and overlapping q-day sums Lo & MacKinlay (1988):
  - $\hat{\sigma}_a^2 = \frac{1}{T-1}\sum_t (r_t - \hat{\mu})^2, \quad \hat{\sigma}_c^2(q) = \frac{1}{m}\sum_t (r_t(q) - q\hat{\mu})^2$
  - $\hat{\sigma}_a^2$: the variance of one-day returns; $\hat{\sigma}_c^2(q)$: the variance of q-day sums, divided by q
  - $m = q(T-q+1)(1-q/T)$ corrects the bias; $\widehat{\mathrm{VR}}(q) = \hat{\sigma}_c^2(q)/\hat{\sigma}_a^2$
- **Homoskedastic** test (RW1): $Z(q) = \dfrac{\widehat{\mathrm{VR}}(q) - 1}{\sqrt{2(2q-1)(q-1)/(3qT)}} \approx N(0,1)$
- **Robust** test (RW3, martingale): $Z^*(q) = \dfrac{\widehat{\mathrm{VR}}(q) - 1}{\sqrt{\hat{\theta}(q)/T}} \approx N(0,1)$
  - $\hat{\theta}(q) = \sum_{j=1}^{q-1} [2(q-j)/q]^2 \hat{\delta}(j)$, with the $\hat{\delta}(j)$ of the robust ACF band
  - $\hat{\theta}(q)/T$: the variance of $\widehat{\mathrm{VR}}(q)$ that allows for volatility clustering

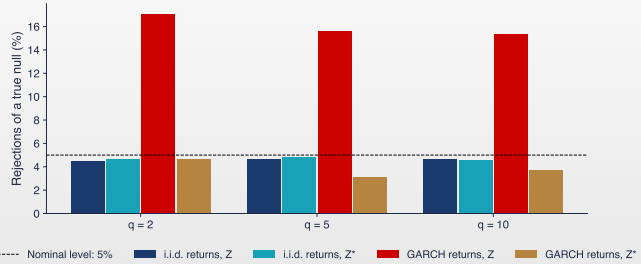


Andrew W. Lo, MIT

Photo: David Shankbone (2012); CC BY 3.0; Wikimedia Commons

SFM_ch7_variance_ratio

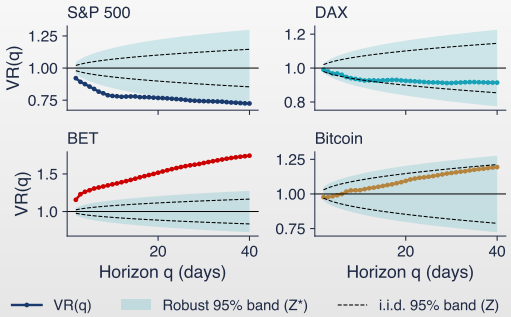
The Need for a Robust Test: a Monte Carlo Study



- 1 000 samples of $T = 2000$ returns with H_0 true: i.i.d. Normal, and GARCH(1,1) ($\alpha = 0.12$, $\beta = 0.86$)
  - GARCH: uncorrelated returns, with volatility clustering (α: reaction to yesterday's shock, β: persistence)
- i.i.d.: both reject about 5%; GARCH: $Z(2)$ rejects 17.1% of the time, $Z^*(2)$ only 4.7%
- The **size** of a test: how often it rejects a true H_0; Z finds "inefficiency" that is only volatility clustering

SFM_ch7_variance_ratio

Variance Ratios for Horizons of 2 to 40 Days



- S&P 500: VR falls below 1 at every horizon (mean reversion); BET: VR keeps rising with the horizon, 1.517 at 20 days and 1.743 at 40 days (momentum)
- DAX and Bitcoin: inside the robust band (shaded) at every horizon; the i.i.d. band (dashed) is much narrower

SFM_ch7_variance_ratio

Variance Ratios and Robust Tests

	VR(2)	VR(5)	VR(10)	VR(20)	$Z^*(2)$	$Z^*(5)$	$Z^*(10)$	$Z^*(20)$
S&P 500	0.922	0.856	0.783	0.767	−3.37	−2.76	−2.73	−2.06
DAX	0.990	0.968	0.932	0.925	−0.61	−0.88	−1.21	−0.91
BET	1.156	1.283	1.362	1.517	5.99	5.25	4.74	4.94
Bitcoin	0.977	0.991	1.027	1.094	−0.88	−0.19	0.37	0.91
TLV	1.009	0.997	0.954	0.995	0.34	−0.04	−0.48	−0.04
SNP	0.985	0.960	0.975	1.038	−0.53	−0.68	−0.29	0.32
BRD	1.032	1.083	1.078	1.107	1.28	1.55	0.97	0.96
TGN	1.055	1.150	1.142	1.166	2.05	2.60	1.69	1.44

- $|Z^*| > 1.96$: reject the random walk at 5% for that horizon
- S&P 500: significant mean reversion at all four horizons; BET: significant momentum at all four
- Transgaz: $Z^*(2) = 2.05$ and $Z^*(5) = 2.60$; DAX, Bitcoin, TLV, SNP, BRD: no rejection
- But four tests per series, 32 in the table: some "significant" horizons appear by chance

SFM_ch7_variance_ratio

Joint Test over Several Horizons: the Chow–Denning Test

- Testing $q = 2, 5, 10, 20$ separately at 5% gives a much higher chance of at least one false rejection
  - for 4 independent tests: $1 - 0.95^4 = 18.5\%$
- **CD** (Chow–Denning) statistic Chow & Denning (1993): $\mathrm{CD} = \max_i |Z^*(q_i)|$, $i = 1, \ldots, m$
  - critical value from the studentised maximum modulus law: $P(\mathrm{CD} \leq c) = (2\Phi(c) - 1)^m$
  - $m = 4$, 5%: $c = 2.49$ instead of 1.96; Φ: the standard Normal distribution function
- Reject the random walk if the largest $|Z^*|$ exceeds 2.49: one decision for all horizons

An Automatic Choice of the Horizon

- The horizon q can be chosen from the data: the automatic VR test Choi (1999)
  - $\mathrm{VR}(k) = 1 + 2\sum_{i=1}^{T-1} w(i/k)\,\hat{\rho}(i)$, with smoothly declining weights w (quadratic spectral kernel) Andrews (1991)
  - the bandwidth k grows with the first-order autocorrelation and with T; statistic $\sqrt{T/k}\,(\mathrm{VR}(k) - 1)/\sqrt{2} \approx N(0,1)$
- **Wild bootstrap** p-value Kim (2009): resample $r_t^* = \eta_t(r_t - \bar{r})$ with $\eta_t \sim N(0,1)$, 500 times
  - each r_t^* keeps the size of r_t (its volatility) but loses any autocorrelation: a robust null distribution
- Three tools, one question: $Z^*(q)$ for a chosen horizon, CD for a set of horizons, the automatic test for a horizon chosen by the data

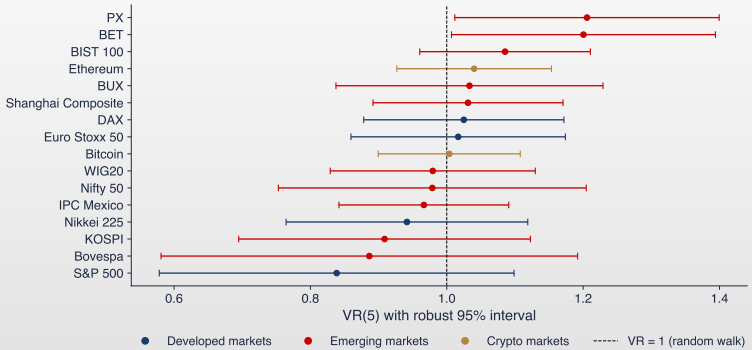
Joint and Automatic Tests on Real Data

	T	CD	p	$\hat{k}$	automatic VR	p (bootstrap)
S&P 500	9 240	3.37	0.003	3.7	−5.21	0.006
DAX	9 288	1.21	0.644	1.7	−0.39	0.608
BET	7 243	5.99	< 0.001	5.6	8.86	< 0.001
Bitcoin	4 384	0.91	0.835	2.0	−1.01	0.352
TLV	3 788	0.48	0.981	1.5	0.31	0.718
SNP	3 815	0.68	0.937	1.7	−0.53	0.562
BRD	3 771	1.55	0.403	2.3	1.58	0.152
TGN	3 725	2.60	0.037	3.0	3.78	0.004

- Both tests reject for the S&P 500, the BET and Transgaz; neither rejects for DAX, Bitcoin, TLV, SNP and BRD
- $\hat{k}$ is large where the first autocorrelation is large (BET 5.6), small where it is near 0 (DAX 1.7)
- Interpretation: rejection does not mean profit
  - an autocorrelation of -0.079 explains $\hat{\rho}(1)^2$, less than 1%, of the variance of tomorrow's return
  - trading costs absorb most of it

SFM_ch7_variance_ratio

Sixteen Markets over the Last Ten Years



⊡ VR(5) with its robust 95% interval, from 19 September 2016 to 18 September 2026: developed, emerging and crypto markets

SFM_ch7_markets_efficiency

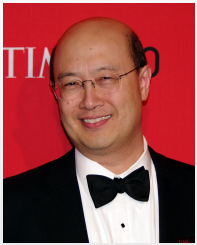
Reading the Cross-Market Comparison

- Only two intervals exclude 1: the PX ($Z^*(5) = 2.08$) and the BET ($Z^*(5) = 2.03$), both above 1
  - two smaller Central European markets, with less liquid index stocks
- **Many markets, many tests**: 16 tests at 5% give 0.8 false rejections on average
  - if the tests were independent, the chance of at least one false rejection would be $1 - 0.95^{16} = 56\%$
  - Chow–Denning over four horizons: no market rejects at 5% (BET p = 0.159, PX p = 0.142)
- S&P 500: VR(5) = 0.84, the lowest, but with a wide interval ($Z^* = -1.22$): the large swings of 2020 inflate $\hat{\theta}$
- Bitcoin and Ethereum: VR(5) close to 1, as for developed markets

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The Adaptive Markets Hypothesis

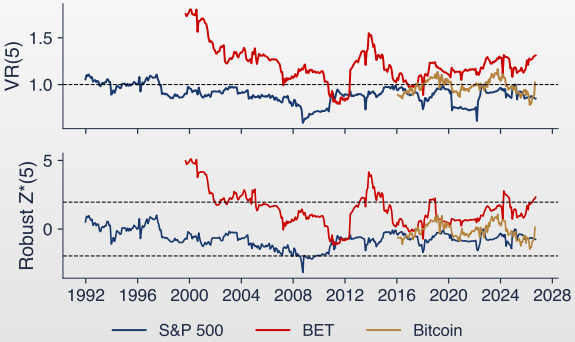
- **AMH** (adaptive markets hypothesis) Lo (2004): markets are ecosystems of traders who learn and adapt
  - profit opportunities appear, are exploited and disappear; efficiency changes with the environment
  - crises, new technology, new participants and regulation move the market away from efficiency or back towards it
- Testable consequence: predictability varies over time
  - evidence: Kim, Shamsuddin & Lim (2011) (U.S., a century of data); Urquhart & McGroarty (2016); survey: Lim & Brooks (2011)
- Tool: the same test on rolling windows



Andrew W. Lo, author of the AMH

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Rolling Variance Ratios



⊡ VR(5) and $Z^*(5)$ on windows of 500 trading days (about two years), a new window every 21 days, dated at the window end

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Reading the Rolling Variance Ratios

- BET: VR(5) up to 1.80 in the window ending in 2000; closer to 1 after 2008, apart from short episodes
  - 29% of the windows reject at 5%, all of them with VR > 1 (momentum)
- S&P 500: VR(5) mostly below 1; the lowest, 0.59, in the window ending in 2008
  - 4% of the windows reject, all with VR < 1: reversals are strongest in crises
- Bitcoin: no window rejects since 2016 (185 windows)
- Caution: consecutive windows share 479 of 500 days; the share of rejecting windows is a description, not a test

SFM_ch7_adaptive_markets

Three Sub-Periods per Market

	period	T	$\hat{\rho}(1)$	VR(5)	$Z^*(5)$	CD p
BET	1997–2006	2 306	0.270	1.50	6.15	< 0.001
BET	2007–2016	2 516	0.066	1.08	0.82	0.262
BET	2017–2026	2 421	0.064	1.20	2.04	0.154
S&P 500	1990–2001	3 025	0.017	0.96	−0.62	0.334
S&P 500	2002–2013	3 019	−0.100	0.80	−2.53	0.010
S&P 500	2014–2026	3 196	−0.124	0.85	−1.30	0.080
Bitcoin	2014–2017	1 201	0.028	0.98	−0.23	0.983
Bitcoin	2018–2021	1 461	−0.058	0.98	−0.21	0.374
Bitcoin	2022–2026	1 722	−0.026	1.01	0.18	0.924

- BET: strong momentum in 1997–2006 ($\hat{\rho}(1) = 0.270$), much weaker afterwards
  - since 2017 VR(5) is still above 1, but the joint test does not reject
- S&P 500: the negative autocorrelation appears after 2002
  - Bitcoin: no rejection in any period since 2014; Urquhart (2016) found inefficiency in its earliest years

SFM_ch7_adaptive_markets

Predictability of the BET in Its Early Years

- **Thin trading**: few trades per day in many index stocks
  - a closing price may be hours old; news enters the index over several days
  - the index then shows positive autocorrelation even if each stock is efficient
- Changes after 2007
  - more liquidity, foreign investors, EU membership, the FTSE Russell upgrade to Secondary Emerging (2020) FTSE Russell (2019)
- Crashes on the Bucharest Stock Exchange: Pele & Mazurencu-Marinescu (2012)



The Stock Exchange Palace, Bucharest

Photo: Neoclassicism Enthusiast (2019); CC BY-SA 4.0; Wikimedia Commons

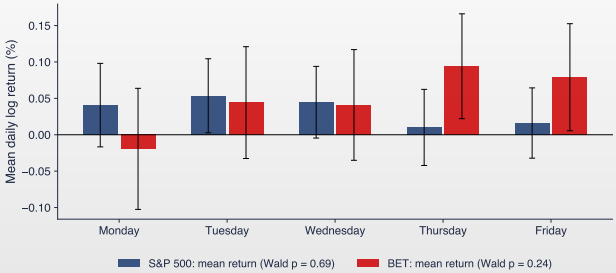
Event Studies and the Strong Form

- **Event study** (semi-strong form) MacKinlay (1997), Campbell, Lo & MacKinlay (1997), Ch. 4: how fast does a price absorb public news?
  - abnormal return: the return minus the return of a model (for example the market); cumulated over a window around the announcement
  - efficient: a jump on the day of the news, no drift afterwards
  - recent research: how fast futures prices absorb news, measured in nanoseconds at Eurex and CME Tak, Schlamp, Osterrieder & Pele (2026); market responses to Ethereum upgrades Lin, Găman, Wang & Pele (2025)
- **Strong form**: can anyone with private information earn excess returns?
  - insiders (managers trading their own shares) do: insider trading is illegal for this reason
  - professional fund managers mostly do not Jensen (1968), Fama & French (2010)

Calendar Anomalies

- An **anomaly**: a pattern in returns that the equilibrium model does not explain
  - **weekend effect**: low or negative Monday returns French (1980)
  - **January effect**: higher returns in January, mostly for small stocks Rozeff & Kinney (1976)
  - **turn-of-the-month effect**: returns concentrated around the first trading days of the month Ariel (1987)
- Do anomalies last?
  - returns of published anomalies fall after publication: traders learn McLean & Pontiff (2016)
  - hundreds of patterns have been tested: a new one needs $|t| > 3$, not 2 Harvey, Liu & Zhu (2016)
- Data mining: with enough calendar splits, some will look significant by chance

Day-of-Week Returns: S&P 500 and BET



- Mean daily return by weekday with 95% intervals (Newey–West standard errors Newey & West (1987))
  - Wald test of equal means across weekdays: S&P 500 p = 0.69, BET p = 0.24
- BET Monday: −0.019%, the only negative mean, yet not significant
  - January: BET 0.185% per day against 0.036% ($t = 1.72$, p = 0.09); S&P 500: p = 0.59
- No calendar effect survives at 5% in our data

SFM_ch7_calendar_anomalies

An Open Research Question

- **Did the BET become more efficient after Romania's upgrade to Secondary Emerging market (September 2020)?** FTSE Russell (2019)
  - five years before: $\hat{\rho}(1) = -0.002$, $\mathrm{VR}(5) = 1.12$ ($Z^* = 0.80$)
  - five years after: $\hat{\rho}(1) = 0.087$, $\mathrm{VR}(5) = 1.18$ ($Z^* = 1.78$)
  - difference of the two VR(5): $z = 0.34$: no evidence of a change
- Why it is open: COVID-19, new large listings and higher volumes happened at the same time
  - is the index or each stock the right unit?
- AI tools can speed up such a study; they do not replace checking it Wang et al. (2023)

SFM_ch7_adaptive_markets

Possible Uses of AI Tools

- **Literature**: list studies of efficiency in Central and Eastern European markets and the tests they use
- **Code**: a first draft of rolling Lo–MacKinlay and automatic VR tests for BET stocks, with wild-bootstrap p-values
- **Robustness**: other windows, other horizons, BET-TR instead of BET, trading volume as a control
- Example prompt
  - `Write a Python function that computes the Lo-MacKinlay variance ratio VR(5) and the heteroskedasticity-robust z-statistic for daily BET log returns on rolling 500-day windows, and test whether the mean VR differs before and after 21 September 2020.`

What to Check

- The statistic: robust Z^*, not the homoskedastic Z; a draft that uses Z will "find" inefficiency
- Log returns of the index, not price levels; no unit-root test on returns used as an efficiency test
- Rolling windows overlap: a t-test on the mean of rolling statistics treats dependent numbers as independent
- The event date and the window are fixed before looking at the result
- References: every cited paper must exist; check the DOI

Project Idea

- **Question**: is the Bucharest market more efficient today than in 2015?
  - a second question: does the change appear in the index or in the individual stocks?
  - data: BET since 1997, BET-TR since 2014, BVB stocks (TLV, SNP, BRD, TGN) since 2010 (EODHD)
- Steps
  - rolling VR(2), VR(5) with Z^* and the Chow–Denning test for the index and for each stock
  - compare 2015–2020 with 2020–2025 by a block bootstrap of the difference
  - repeat for the WIG20 and the BUX as controls: did all of them change at the same time?
- Deliverable: one table, one chart, and a paragraph on what the data can and cannot show
- Declare any AI use, and list the errors of the AI that you corrected

Key Takeaways

- Efficient market: prices reflect information; weak form = past prices cannot be used to earn excess returns
- The right null is the martingale (RW3), not RW1: volatility clustering is not inefficiency
- Log prices are $I(1)$ and returns $I(0)$ in all our series; a unit root alone does not prove efficiency
- Use robust tests: $\tilde{Q}$ and $Z^*(q)$; join horizons with Chow–Denning or let the data choose (automatic VR)
- Evidence: S&P 500 slightly mean-reverting, early BET trending, most markets close to efficient today; efficiency varies over time
- Statistical predictability is not profit: costs and risk decide

Key Formulas

Quantity	Formula
Random walk	$p_t = \mu + p_{t-1} + \varepsilon_t, \quad \mathrm{Var}(r_t(q)) = q\sigma^2$
Sample ACF	$\hat{\rho}(k) = \sum_t e_t e_{t-k} / \sum_t e_t^2, \quad$ band $\pm 1.96\sqrt{\hat{\delta}(k)/T}$
Ljung–Box	$Q_{LB}(m) = T(T+2)\sum_{k=1}^{m} \hat{\rho}(k)^2/(T-k) \sim \chi^2(m)$
Runs test	$E[R] = 2n_1 n_2/n + 1, \quad z = (R - E[R])/\mathrm{sd}(R)$
ADF	$\Delta p_t = c + bt + \gamma p_{t-1} + \sum_j \delta_j \Delta p_{t-j} + \varepsilon_t, \quad \tau = \hat{\gamma}/\mathrm{SE}(\hat{\gamma})$
KPSS	$\sum_t S_t^2/(T^2\hat{\lambda}^2), \quad H_0$: stationarity
VR	$\mathrm{VR}(q) = 1 + 2\sum_{k=1}^{q-1}(1 - k/q)\rho(k)$
Robust VR test	$Z^*(q) = (\widehat{\mathrm{VR}}(q) - 1)/\sqrt{\hat{\theta}(q)/T}, \quad \hat{\theta}(q) = \sum_{j<q}[2(q-j)/q]^2\hat{\delta}(j)$
Chow–Denning	$\max_i \lvert Z^*(q_i)\rvert, \quad P(\mathrm{CD} \le c) = (2\Phi(c) - 1)^m$

Self-Assessment

- **Question**: $\hat{\rho}(1) = 0.10$ and $\hat{\rho}(k) = 0$ for $k \geq 2$; what does VR(2) tell us?
  - **Answer**: $1 + 0.10 = 1.10$: the two-day variance is 10% larger than under a random walk: momentum
- **Question**: ADF does not reject on log prices and KPSS rejects; is the market efficient?
  - **Answer**: we only know that prices are $I(1)$; efficiency needs tests on the increments (ACF, VR)
- **Question**: for the S&P 500 the homoskedastic $Z(q)$ is about twice the robust $Z^*(q)$; which one should we report?
  - **Answer**: $Z^*(q)$: volatility clustering raises the variance of $\widehat{\mathrm{VR}}(q)$, which $Z(q)$ ignores, so $Z(q)$ rejects a true martingale too often
- Next: Chapter 8, volatility estimators and volatility clustering

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