



Statistics of Financial Markets

Chapter 8: Stochastic Volatility Models – Part 2



Daniel Traian PELE

Bucharest University of Economic Studies

IDA Institute Digital Assets

Blockchain Research Center

AI4EFin Artificial Intelligence for Energy Finance

Romanian Academy, Institute for Economic Forecasting

MSCA Digital Finance

Outline

Motivation

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PGARCH Model

TGARCH Model

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References

Motivation

- Limitations of symmetric GARCH: inability to capture the leverage effect
- Asymmetric response of volatility to positive vs. negative shocks
- Need for flexible volatility specifications in risk management

EGARCH Model

- Nelson's (1991) Exponential GARCH specification
- Log-volatility equation: $\ln \sigma_t^2 = \omega + \beta \ln \sigma_{t-1}^2 + \gamma z_{t-1} + \alpha(|z_{t-1}| - \mathbb{E}|z_{t-1}|)$
- No positivity constraints required
- Captures asymmetry through the sign parameter γ

PGARCH Model

- Ding, Granger, and Engle's (1993) Power GARCH
- $\sigma_t^\delta = \omega + \sum_{i=1}^q \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^p \beta_j \sigma_{t-j}^\delta$
- Flexible power parameter δ nesting GARCH and AVGARCH
- Asymmetry through leverage parameters γ_i

TGARCH Model

- ▣ Zakoian's (1994) Threshold GARCH specification
- ▣ $\sigma_t = \omega + \sum_{i=1}^q (\alpha_i \varepsilon_{t-i}^+ - \gamma_i \varepsilon_{t-i}^-) + \sum_{j=1}^p \beta_j \sigma_{t-j}$
- ▣ GJR-GARCH as a related threshold model (Glosten, Jagannathan, Runkle, 1993)
- ▣ Piecewise linear response to shocks

GARCH-M Model

- GARCH-in-Mean: risk premium in the conditional mean equation
- $r_t = \mu + \lambda g(\sigma_t^2) + \varepsilon_t$
- Common specifications: $g(\sigma_t^2) = \sigma_t^2$, $g(\sigma_t^2) = \sigma_t$, $g(\sigma_t^2) = \ln \sigma_t^2$
- Testing the risk-return tradeoff

Comparison

- ▣ News impact curves across different GARCH specifications
- ▣ Nesting relationships among models
- ▣ Model selection criteria and diagnostic tests
- ▣ Strengths and weaknesses of each specification

Applications

- ▣ Fitting asymmetric GARCH models to equity returns
- ▣ Comparing forecasts from symmetric vs. asymmetric models
- ▣ Python implementation using `arch` package

 [SFM_ch8_advanced_garch](#)

Conclusions

- ▣ Summary of asymmetric and extended GARCH models
- ▣ Practical guidelines for model selection
- ▣ Connection to VaR estimation (Chapter 9)

References

- Nelson, D.B. (1991). Conditional heteroskedasticity in asset returns: A new approach.
- Ding, Z., Granger, C.W.J., Engle, R.F. (1993). A long memory property of stock market returns and a new model.
- Glosten, L.R., Jagannathan, R., Runkle, D.E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks.