



Time Series Analysis and Forecasting

Seminar 5: GARCH and Volatility Models



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Seminar Outline

- ▣ **Multiple Choice Quiz** – Knowledge check
- ▣ **True/False** – Conceptual checks
- ▣ **Practice Problems** – Applied practice
- ▣ **Python Workflow** – Hands-on coding
- ▣ **AI-Assisted Exercise** – Critical thinking
- ▣ **Summary** – Key takeaways

Quiz 1: Volatility Clustering

Question

What is “volatility clustering”?

Answer choices

- (A) Volatility is constant over time
- (B) Periods of high volatility tend to be followed by periods of high volatility
- (C) Returns are correlated over time
- (D) The return distribution is normal

Answer on next slide...

Quiz 1: Answer

Answer: B – Periods of high volatility tend to be followed by periods of high volatility

Answer choices

- (A) Volatility is constant over time ✗
- (B) **Periods of high volatility tend to be followed by periods of high volatility** ✓
- (C) Returns are correlated over time ✗
- (D) The return distribution is normal ✗

- ☐ **Volatility clustering** is a stylized fact observed in financial time series
- ☐ “Turbulent” periods (with large movements) tend to persist
- ☐ This implies that conditional variance σ_t^2 is **predictable**
- ☐ GARCH models capture exactly this phenomenon!

Quiz 2: GARCH Parameters

Question

In the GARCH(1,1) model: $\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$, what does the parameter α represent?

Answer choices

- (A) Volatility persistence
- (B) Baseline volatility level
- (C) Reaction to recent shocks (news coefficient)
- (D) Unconditional variance

Answer on next slide...

Quiz 2: Answer

Answer: C – Reaction to recent shocks (news coefficient)

Answer choices

- (A) Volatility persistence ✗
- (B) Baseline volatility level ✗
- (C) **Reaction to recent shocks (news coefficient)** ✓
- (D) Unconditional variance ✗

- ☐ ω = baseline (floor) volatility level
- ☐ α = **reaction** to squared innovations (“news”)
- ☐ β = volatility **persistence** (memory)
- ☐ $\alpha + \beta$ = total persistence
- ☐ A large α means volatility reacts strongly to recent shocks

Quiz 3: Stationarity Condition

Question

What is the stationarity condition for GARCH(1,1)?

Answer choices

- (A) $\omega > 0$
- (B) $\alpha + \beta = 1$
- (C) $\alpha + \beta < 1$
- (D) $\alpha > \beta$

Answer on next slide...

Quiz 3: Answer

Answer: $C - \alpha + \beta < 1$

Answer choices

- (A) $\omega > 0$ ✗
- (B) $\alpha + \beta = 1$ ✗
- (C) $\alpha + \beta < 1$ ✓
- (D) $\alpha > \beta$ ✗

- ☐ $\omega > 0$ (ensures positive variance)
- ☐ $\alpha \geq 0, \beta \geq 0$ (non-negativity)
- ☐ $\alpha + \beta < 1$ (**covariance stationarity**)
- ☐ If $\alpha + \beta = 1 \Rightarrow$ IGARCH (shocks have permanent effect)

Quiz 4: Unconditional Variance

Question

What is the formula for unconditional variance in GARCH(1,1)?

Answer choices

- (A) $\bar{\sigma}^2 = \omega$
- (B) $\bar{\sigma}^2 = \frac{\omega}{1 - \alpha}$
- (C) $\bar{\sigma}^2 = \frac{\omega}{1 - \alpha - \beta}$
- (D) $\bar{\sigma}^2 = \frac{\omega}{\alpha + \beta}$

Answer on next slide...

Quiz 4: Answer

$$\text{Answer: } C - \bar{\sigma}^2 = \frac{\omega}{1 - \alpha - \beta}$$

Answer choices

- (A) $\bar{\sigma}^2 = \omega$ ✗
- (B) $\bar{\sigma}^2 = \frac{\omega}{1 - \alpha}$ ✗
- (C) $\bar{\sigma}^2 = \frac{\omega}{1 - \alpha - \beta}$ ✓
- (D) $\bar{\sigma}^2 = \frac{\omega}{\alpha + \beta}$ ✗

- ☐ Taking unconditional expectation of GARCH(1,1):
- ☐ $\mathbb{E}[\sigma_t^2] = \omega + \alpha\mathbb{E}[\varepsilon_{t-1}^2] + \beta\mathbb{E}[\sigma_{t-1}^2]$
- ☐ $\bar{\sigma}^2(1 - \alpha - \beta) = \omega$
- ☐ $\bar{\sigma}^2 = \frac{\omega}{1 - \alpha - \beta}$

Quiz 5: Leverage Effect

Question

What is the “leverage effect”?

Answer choices

- (A) Positive shocks increase volatility more than negative shocks
- (B) Negative shocks increase volatility more than positive shocks
- (C) Volatility is independent of shock sign
- (D) Returns are asymmetric

Answer on next slide...

Quiz 5: Answer

Answer: B – Negative shocks increase volatility more than positive shocks

Answer choices

- (A) Positive shocks increase volatility more than negative shocks ✗
 - (B) **Negative shocks increase volatility more than positive shocks** ✓
 - (C) Volatility is independent of shock sign ✗
 - (D) Returns are asymmetric ✗
-
- ☐ Empirically observed in stock markets
 - ☐ When prices fall, firm leverage increases (debt becomes larger relative to equity)
 - ☐ This makes the firm riskier $\Rightarrow$ higher volatility
 - ☐ Standard GARCH **cannot** capture this effect (depends on ε^2)
 - ☐ Solutions: **EGARCH, GJR-GARCH, TGARCH**



Quiz 6: EGARCH Leverage

Question

In the EGARCH model, a negative γ parameter indicates:

Answer choices

- (A) Absence of leverage effect
- (B) Presence of leverage effect
- (C) Constant volatility
- (D) Non-stationary model

Answer on next slide...

Quiz 6: Answer

Answer: B – Presence of leverage effect

Answer choices

- (A) Absence of leverage effect ✗
- (B) **Presence of leverage effect** ✓
- (C) Constant volatility ✗
- (D) Non-stationary model ✗

- ☐ EGARCH(1,1): $\ln(\sigma_t^2) = \omega + \alpha(|z_{t-1}| - \mathbb{E}[|z|]) + \gamma z_{t-1} + \beta \ln(\sigma_{t-1}^2)$
- ☐ $\gamma < 0$: negative shock ($z < 0$) $\Rightarrow$ increases $\ln(\sigma_t^2)$
- ☐ $\gamma > 0$: inverse effect (less common)
- ☐ $\gamma = 0$: symmetric effect (like GARCH)

Quiz 7: EGARCH Advantages

Question

What is the main advantage of EGARCH over GARCH?

Answer choices

- (A) Faster to estimate
- (B) No non-negativity constraints needed
- (C) Fewer parameters
- (D) Easier to interpret

Answer on next slide...

Quiz 7: Answer

Answer: B – No non-negativity constraints needed

Answer choices

- (A) Faster to estimate ✗
- (B) **No non-negativity constraints needed** ✓
- (C) Fewer parameters ✗
- (D) Easier to interpret ✗

- ☐ Models $\ln(\sigma_t^2)$, not σ_t^2
- ☐ $\sigma_t^2 = e^{\ln(\sigma_t^2)} > 0$ **automatically**, regardless of parameter values
- ☐ GARCH requires $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$
- ☐ During estimation, these constraints can cause convergence problems

Quiz 8: ARCH Effects Test

Question

Which test do we use to detect ARCH effects in residuals?

Answer choices

- (A) Dickey-Fuller test
- (B) Ljung-Box test on residuals
- (C) Engle's ARCH-LM test
- (D) Breusch-Pagan test

Answer on next slide...

Quiz 8: Answer

Answer: C – Engle's ARCH-LM test

Answer choices

- (A) Dickey-Fuller test ✗
- (B) Ljung-Box test on residuals ✗
- (C) Engle's ARCH-LM test ✓
- (D) Breusch-Pagan test ✗

- ☐ Estimate mean model, obtain residuals $\hat{\varepsilon}_t$
- ☐ Regress: $\hat{\varepsilon}_t^2 = \beta_0 + \beta_1 \hat{\varepsilon}_{t-1}^2 + \cdots + \beta_q \hat{\varepsilon}_{t-q}^2 + u_t$
- ☐ Test statistic: $LM = T \cdot R^2 \sim \chi^2(q)$ under H_0
- ☐ H_0 : No ARCH effects H_1 : ARCH effects present

Quiz 9: Volatility Persistence

Question

For S&P 500, typical values of $\alpha + \beta$ in GARCH(1,1) are:

Answer choices

- (A) 0.50 – 0.70
- (B) 0.70 – 0.85
- (C) 0.95 – 0.99
- (D) Greater than 1

Answer on next slide...



Quiz 9: Answer

Answer: C – 0.95 – 0.99

Answer choices

- (A) 0.50 – 0.70 ✗
- (B) 0.70 – 0.85 ✗
- (C) **0.95 – 0.99** ✓
- (D) Greater than 1 ✗

- ☐ Financial time series exhibit very persistent volatility
- ☐ $\alpha + \beta \approx 0.98$ for S&P 500; Half-life ≈ 35 –60 days

Series	$\alpha + \beta$
S&P 500	0.97–0.99
Bitcoin	0.90–0.98
EUR/USD	0.96–0.99

Quiz 10: Innovation Distributions

Question

Which distribution is most commonly used for GARCH innovations to capture fat tails?

Answer choices

- (A) Normal
- (B) Uniform
- (C) Student-t
- (D) Exponential

Answer on next slide...

Quiz 10: Answer

Answer: C – Student-t

Answer choices

- (A) Normal ✗
- (B) Uniform ✗
- (C) **Student-t** ✓
- (D) Exponential ✗

- ▣ **Normal**: standard, but underestimates extreme risk
- ▣ **Student-t**: fat tails, parameter ν (degrees of freedom)
- ▣ **GED**: Generalized Error Distribution, flexible
- ▣ **Skewed Student-t**: asymmetry + fat tails
- ▣ For S&P 500: $\nu \approx 5-8$ (significantly fatter tails than normal)

Quiz 11: Component GARCH (CGARCH)

Question

In the Component GARCH (CGARCH) model, what does the **permanent component** q_t represent?

Answer choices

- (A) Short-run reaction to news shocks
- (B) The long-run (time-varying) level of variance to which volatility mean-reverts
- (C) The leverage effect coefficient
- (D) The unconditional kurtosis of returns

Answer on next slide...

Quiz 11: Answer

Answer: B – The long-run (time-varying) level of variance to which volatility mean-reverts

Answer choices

- (A) Short-run reaction to news shocks ✗
- (B) **The long-run (time-varying) level of variance to which volatility mean-reverts** ✓
- (C) The leverage effect coefficient ✗
- (D) The unconditional kurtosis of returns ✗

- ☐ CGARCH decomposes variance: $\sigma_t^2 = q_t + \alpha(\varepsilon_{t-1}^2 - q_{t-1}) + \beta(\sigma_{t-1}^2 - q_{t-1})$
- ☐ $q_t =$ **permanent component** (slow-moving trend in volatility)
- ☐ $(\sigma_t^2 - q_t) =$ **transitory component** (short-run deviations)
- ☐ In standard GARCH, $q_t = \bar{\sigma}^2$ is constant; CGARCH allows it to evolve

Quiz 12: APARCH Nesting Property

Question

The Asymmetric Power ARCH (APARCH) model nests many GARCH variants. Which of the following is **NOT** a special case of APARCH?

Answer choices

- (A) GARCH(1,1) when $\delta = 2$ and $\gamma_i = 0$
- (B) TGARCH(1,1) when $\delta = 1$
- (C) EGARCH(1,1)
- (D) GJR-GARCH(1,1) when $\delta = 2$

Answer on next slide...

Quiz 12: Answer

Answer: C – EGARCH(1,1)

Answer choices

- (A) GARCH(1,1) when $\delta = 2$ and $\gamma_i = 0$ ✗
- (B) TGARCH(1,1) when $\delta = 1$ ✗
- (C) **EGARCH(1,1)** ✓
- (D) GJR-GARCH(1,1) when $\delta = 2$ ✗

- APARCH: $\sigma_t^\delta = \omega + \sum_{i=1}^p \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta$
- $\delta = 2, \gamma_i = 0 \Rightarrow$ **GARCH**; $\delta = 1 \Rightarrow$ **TGARCH**
- $\delta = 2, \gamma_i \neq 0 \Rightarrow$ **GJR-GARCH**
- **EGARCH** models $\ln(\sigma_t^2)$ — a fundamentally different structure, not nested

Quiz 13: FIGARCH and Long Memory

Question

What does the fractional differencing parameter d in FIGARCH control?

Answer choices

- (A) The asymmetry between positive and negative shocks
- (B) The rate of decay of the autocorrelation of squared returns
- (C) The number of ARCH lags
- (D) The degrees of freedom of the innovation distribution

Answer on next slide...

Quiz 13: Answer

Answer: B – The rate of decay of the autocorrelation of squared returns

Answer choices

- (A) The asymmetry between positive and negative shocks ✗
- (B) **The rate of decay of the autocorrelation of squared returns** ✓
- (C) The number of ARCH lags ✗
- (D) The degrees of freedom of the innovation distribution ✗

- FIGARCH: $(1 - \beta L)(1 - L)^d \varepsilon_t^2 = \omega + (1 - \beta L - \phi L)(1 - L)^d \varepsilon_t^2$
- $d = 0 \Rightarrow$ **GARCH** (short memory); $d = 1 \Rightarrow$ **IGARCH** (unit root)
- $0 < d < 1 \Rightarrow$ **long memory**: autocorrelations of ε_t^2 decay hyperbolically
- Empirically, $d \approx 0.3\text{--}0.5$ for most financial series

True or False? — Questions

Statement	T/F?
1. ARIMA models can capture volatility clustering.	?
2. In GARCH(1,1), if $\alpha + \beta = 1$, the model is called IGARCH.	?
3. GJR-GARCH uses an indicator variable for negative shocks.	?
4. GARCH volatility forecasts converge to zero in the long run.	?
5. EGARCH can have negative parameters without generating negative variance.	?
6. Value at Risk (VaR) can be calculated using GARCH volatility forecasts.	?
7. EWMA volatility forecasts mean-revert to a long-run level.	?
8. Markov-switching GARCH can explain spurious IGARCH effects.	?
9. GARCH-M allows volatility to directly affect expected returns.	?

True or False? — Answers

Statement	T/F	Explanation
1. ARIMA models can capture volatility clustering.	F	Assumes constant variance
2. In GARCH(1,1), if $\alpha + \beta = 1$, the model is called IGARCH.	T	Volatility has unit root
3. GJR-GARCH uses an indicator variable for negative shocks.	T	$I_{t-1} = 1$ if $\varepsilon_{t-1} < 0$
4. GARCH volatility forecasts converge to zero in the long run.	F	Converges to $\bar{\sigma}^2$
5. EGARCH can have negative parameters without generating negative variance.	T	Models $\ln(\sigma_t^2)$
6. Value at Risk (VaR) can be calculated using GARCH volatility forecasts.	T	$\text{VaR}_\alpha = z_\alpha \cdot \sigma_{t+1}$
7. EWMA volatility forecasts mean-revert to a long-run level.	F	EWMA = IGARCH ($\omega = 0$), no mean-reversion
8. Markov-switching GARCH can explain spurious IGARCH effects.	T	Breaks $\Rightarrow$ upward-biased $\alpha + \beta$
9. GARCH-M allows volatility to directly affect expected returns.	T	$r_t = \mu + \delta \cdot g(\sigma_t^2) + \varepsilon_t$

Exercise 1: Calculating Unconditional Variance

Problem

A GARCH(1,1) model has estimated parameters:

$$\omega = 0.000002, \alpha = 0.08, \beta = 0.90$$

Calculate: (a) Daily unconditional variance; (b) Daily unconditional volatility (%); (c) Annualized volatility (252 trading days); (d) Volatility half-life

Solution

$$\text{(a)} \quad \bar{\sigma}^2 = \frac{\omega}{1 - \alpha - \beta} = \frac{0.000002}{0.02} = 0.0001$$

$$\text{(b)} \quad \bar{\sigma} = \sqrt{0.0001} = 0.01 = 1\% \text{ per day}$$

$$\text{(c)} \quad \sigma_{\text{annual}} = 0.01 \times \sqrt{252} = 15.87\% \text{ per year}$$

$$\text{(d)} \quad HL = \frac{\ln(0.5)}{\ln(0.98)} = \frac{-0.693}{-0.0202} \approx 34 \text{ days}$$

Exercise 2: Volatility Forecast

Problem

- ▣ Using GARCH(1,1): $\omega = 0.000002$, $\alpha = 0.08$, $\beta = 0.90$
- ▣ At time T : $\varepsilon_T = -0.03$ (3% drop), $\sigma_T^2 = 0.0004$
- ▣ Calculate: (a) σ_{T+1}^2 ; (b) σ_{T+5}^2 ; (c) σ_{T+100}^2

Solution

- ▣ **(a)** $\sigma_{T+1}^2 = 0.000002 + 0.08 \times (0.03)^2 + 0.90 \times 0.0004 = 0.000434$; Vol: 2.08%
- ▣ **(b)** $\mathbb{E}_T[\sigma_{T+5}^2] = \bar{\sigma}^2 + (0.98)^4(\sigma_{T+1}^2 - \bar{\sigma}^2) = 0.000408$; Vol: 2.02%
- ▣ **(c)** $\mathbb{E}_T[\sigma_{T+100}^2] = 0.0001 + (0.98)^{99} \times 0.000334 \approx 0.000145$; Vol: 1.20%

Exercise 3: Value at Risk

Problem

- Portfolio: 1,000,000 EUR; $\sigma_{T+1} = 2\%$ daily; Normal distribution, zero mean
- Calculate: (a) VaR 95% (1 day); (b) VaR 99% (1 day); (c) VaR 99% (10 days)
- Quantiles: $z_{0.05} = 1.645$, $z_{0.01} = 2.326$

Solution

- (a) $\text{VaR}_{95\%} = 1.645 \times 0.02 \times 1,000,000 = 32,900$ EUR
- (b) $\text{VaR}_{99\%} = 2.326 \times 0.02 \times 1,000,000 = 46,520$ EUR
- (c) $\text{VaR}_{99\%,10d} = 46,520 \times \sqrt{10} = 147,100$ EUR

Note: the $\sqrt{T}$ scaling rule assumes i.i.d. returns, which contradicts the GARCH dependence structure.

Caution

In practice, for Student-t distribution, quantiles are larger (fatter tails)!

Exercise 4: Model Identification

Problem

Analyze the following estimation results and identify the model:

Parameter	Estimate	Std. Error
ω	0.0000015	0.0000005
α	0.0550	0.0120
γ	0.0850	0.0180
β	0.9100	0.0150

(a) What model is this? (b) Is leverage effect present? (c) Impact of negative vs positive shocks? (d) Is it stationary?

Solution

- (a) **GJR-GARCH(1,1,1)** — presence of γ parameter (threshold/asymmetry)
- (b) Yes: $\gamma = 0.085 > 0$ and significant
- (c) Positive shock: $\alpha = 0.055$; Negative shock: $\alpha + \gamma = 0.140$ (2.5x greater!)
- (d) $\alpha + \gamma/2 + \beta = 0.055 + 0.0425 + 0.91 = 1.0075$ — **slightly above 1** — technically non-stationary (covariance), very close to IGARCH

Exercise 5: IGARCH and EWMA Persistence

Problem

An analyst uses EWMA with $\lambda = 0.94$ to forecast volatility. At time T :

- ▣ Current variance estimate: $\sigma_T^2 = 0.0004$ (i.e., $\sigma_T = 2\%$)
- ▣ Today's return: $r_T = -0.05$ (5% drop)

Calculate: (a) σ_{T+1}^2 under EWMA; (b) The equivalent IGARCH parameters; (c) Compare with GARCH(1,1) where $\omega = 0.000002$, $\alpha = 0.06$, $\beta = 0.93$; (d) Which model produces a higher long-horizon forecast and why?

Solution

- ▣ (a) $\sigma_{T+1}^2 = 0.94 \times 0.0004 + 0.06 \times (-0.05)^2 = 0.000376 + 0.000150 = 0.000526$; Vol: 2.29%
- ▣ (b) EWMA = IGARCH with $\omega = 0$, $\alpha = 1 - \lambda = 0.06$, $\beta = \lambda = 0.94$, $\alpha + \beta = 1$
- ▣ (c) GARCH: $\sigma_{T+1}^2 = 0.000002 + 0.06 \times 0.0025 + 0.93 \times 0.0004 = 0.000524$; Vol: 2.29%
- ▣ (d) EWMA: forecast stays at σ_{T+1}^2 forever (no mean-reversion). GARCH: converges to $\bar{\sigma}^2 = \frac{0.000002}{0.01} = 0.0002$ (1.41%). EWMA gives **higher** long-horizon forecasts after a large shock.

Step 1: Load and Prepare Data

```
import pandas as pd
import numpy as np
import yfinance as yf
from arch import arch_model
from arch.unitroot import ADF

# Download S&P 500 data
data = yf.download('^GSPC', start='2010-01-01', end='2024-01-01')
returns = 100 * data['Adj Close'].pct_change().dropna()

# Check stationarity
adf = ADF(returns)
print(f'ADF statistic: {adf.stat:.4f}')
print(f'p-value: {adf.pvalue:.4f}')
```

Step 2: Test for ARCH Effects

```
from statsmodels.stats.diagnostic import het_arch

# ARCH-LM test on residuals
residuals = returns - returns.mean()
lm_stat, lm_pvalue, f_stat, f_pvalue = het_arch(residuals, nlags=10)

print(f'ARCH-LM statistic: {lm_stat:.4f}')
print(f'p-value: {lm_pvalue:.4f}')

if lm_pvalue < 0.05:
    print('=> ARCH effects present! GARCH modeling justified.')
```

 TSA_ch5_btc_arch

Step 3: Estimate Models

```
# GARCH(1,1) with Student-t distribution
model_garch = arch_model(returns, vol='Garch', p=1, q=1, dist='t')
res_garch = model_garch.fit(dispatch='off')
print(res_garch.summary())

# GJR-GARCH(1,1,1)
model_gjr = arch_model(returns, vol='Garch', p=1, o=1, q=1, dist='t')
res_gjr = model_gjr.fit(dispatch='off')

# EGARCH(1,1)
model_egarch = arch_model(returns, vol='EGARCH', p=1, q=1, dist='t')
res_egarch = model_egarch.fit(dispatch='off')

# Compare AIC
print(f'GARCH AIC: {res_garch.aic:.2f}')
print(f'GJR AIC: {res_gjr.aic:.2f}')
print(f'EGARCH AIC: {res_egarch.aic:.2f}')
```

Step 4: Diagnostics

```
# Standardized residuals
std_resid = res_gjr.std_resid

# Ljung-Box test on squared residuals
from statsmodels.stats.diagnostic import acorr_ljungbox
lb_test = acorr_ljungbox(std_resid**2, lags=10, return_df=True)
print(lb_test)

# Check for remaining ARCH effects
lm_stat2, lm_pval2, _, _ = het_arch(std_resid, nlags=5)
print(f'ARCH-LM residuals: stat={lm_stat2:.2f}, p={lm_pval2:.4f}')

if lm_pval2 > 0.05:
    print('=> No remaining ARCH effects. Model OK!')
```

Step 5: Forecast and VaR

```
# Forecast 10 days ahead
forecasts = res_gjr.forecast(horizon=10)
vol_forecast = np.sqrt(forecasts.variance.values[-1, :])

print('Volatility forecast (%):', vol_forecast)

# Value at Risk 99%
portfolio_value = 1_000_000
VaR_99 = 2.326 * vol_forecast[0] / 100 * portfolio_value
print(f'VaR 99% (1 day): {VaR_99:,.0f} EUR')

# 10-day VaR
VaR_99_10d = VaR_99 * np.sqrt(10)
print(f'VaR 99% (10 days): {VaR_99_10d:,.0f} EUR')
```

AI Exercise: Critical Thinking

Prompt to test in ChatGPT / Claude / Copilot

"Download BTC-USD daily data from Yahoo Finance (last 5 years). Fit a GARCH(1,1) model to log returns. Show the news impact curve, forecast volatility for the next 30 days, and calculate 1-day 99% VaR."

Exercise:

1. Did the AI check for ARCH effects before fitting GARCH?
2. Is the model adequate? (Check Ljung-Box on standardized residuals)
3. Does the news impact curve show asymmetry? Should EGARCH/GJR be used instead?
4. Is the VaR calculation based on the conditional or unconditional distribution?
5. Try changing to EGARCH — does the AI explain why results differ?

Warning: AI-generated code may run without errors and look professional. *That does not mean it is correct.*

Summary: Chapter 5

Key Concepts

1. **ARCH/GARCH**: conditional variance depends on past shocks and lagged variance
2. **IGARCH/EWMA**: unit root in variance ($\alpha + \beta = 1$); no mean-reversion
3. **EGARCH/GJR/TGARCH**: capture leverage effect (asymmetry)
4. **CGARCH/FIGARCH**: permanent vs transitory volatility; long memory
5. **APARCH**: nests GARCH, TGARCH, GJR-GARCH as special cases
6. **Key formulas**: $\bar{\sigma}^2 = \frac{\omega}{1-\alpha-\beta}$, $HL = \frac{\ln(0.5)}{\ln(\alpha+\beta)}$, $\text{VaR}_\alpha = z_\alpha \cdot \sigma \cdot V$

Practical Tip

Use Student-t distribution to capture fat tails. Verify absence of ARCH effects in residuals!

Questions?

Homework Exercises

Exercise 1

Download daily returns for BET (BVB index) and estimate a GARCH(1,1) model. Compare persistence ($\alpha + \beta$) with S&P 500.

Exercise 2

For Bitcoin, estimate GARCH, EGARCH, and GJR-GARCH. Is leverage effect present for cryptocurrencies?

Exercise 3

Calculate daily VaR for a portfolio of 100,000 EUR invested in EUR/USD, using GARCH-forecasted volatility.

Exercise 4

Compare GARCH(1,1) volatility forecast with realized volatility (sum of squared returns) for a 20-day period.

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Online Resources and Code

- ▣ **Quantlet**: <https://quantlet.com> — Code repository for statistics
- ▣ **Quantinar**: <https://quantinar.com> — Quantitative methods learning platform
- ▣ **GitHub TSA**: https://github.com/QuantLet/TSA/tree/main/TSA_ch5 — Python code for this seminar

Thank You!

Questions?

Seminar materials are available at: <https://danpele.github.io/Time-Series-Analysis/>



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